# CSE524 Parallel Algorithms

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30 March 2010

# CSE524 Parallel ~~Algorithms~~ Computation

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# Course Logistics

- Teaching Assistants: Matt Kehrt and Adrienne Wang
- Text: Lin&Snyder, *Principles of Parallel Programming,* Addison Wesley, 2008
  - There will also be occasional readings
- Class web page is headquarters for all data
- Take lecture notes -- the slides will be online sometime **after** the lecture

**Informal class; ask questions immediately**

# Expectations

- Readings: We will cover much of the book; please read the text before class
- Lectures will layout certain details, arguments … discussion is encouraged
- Most weeks there will be graded homework to be submitted electronically PRIOR to class
- Am assuming most students have access to a multi-core or other parallel machine
- Grading: class contributions, homework assignments; no final is contemplated at the moment

# Part I: Introduction

Goal: Set the parameters for studying parallelism

# Why Study Parallelism?

- After all, for most of our daily computer uses, sequential processing is plenty fast
  - It is a fundamental departure from the “normal” computer model, therefore it is inherently cool
  - The extra power from parallel computers is enabling in science, engineering, business, …
  - Multicore chips present a new opportunity
  - Deep intellectual challenges for CS -- models, programming languages, algorithms, HW, …

# Facts

Single Processor

**Opportunity**
Moore's law continues, so use more gates

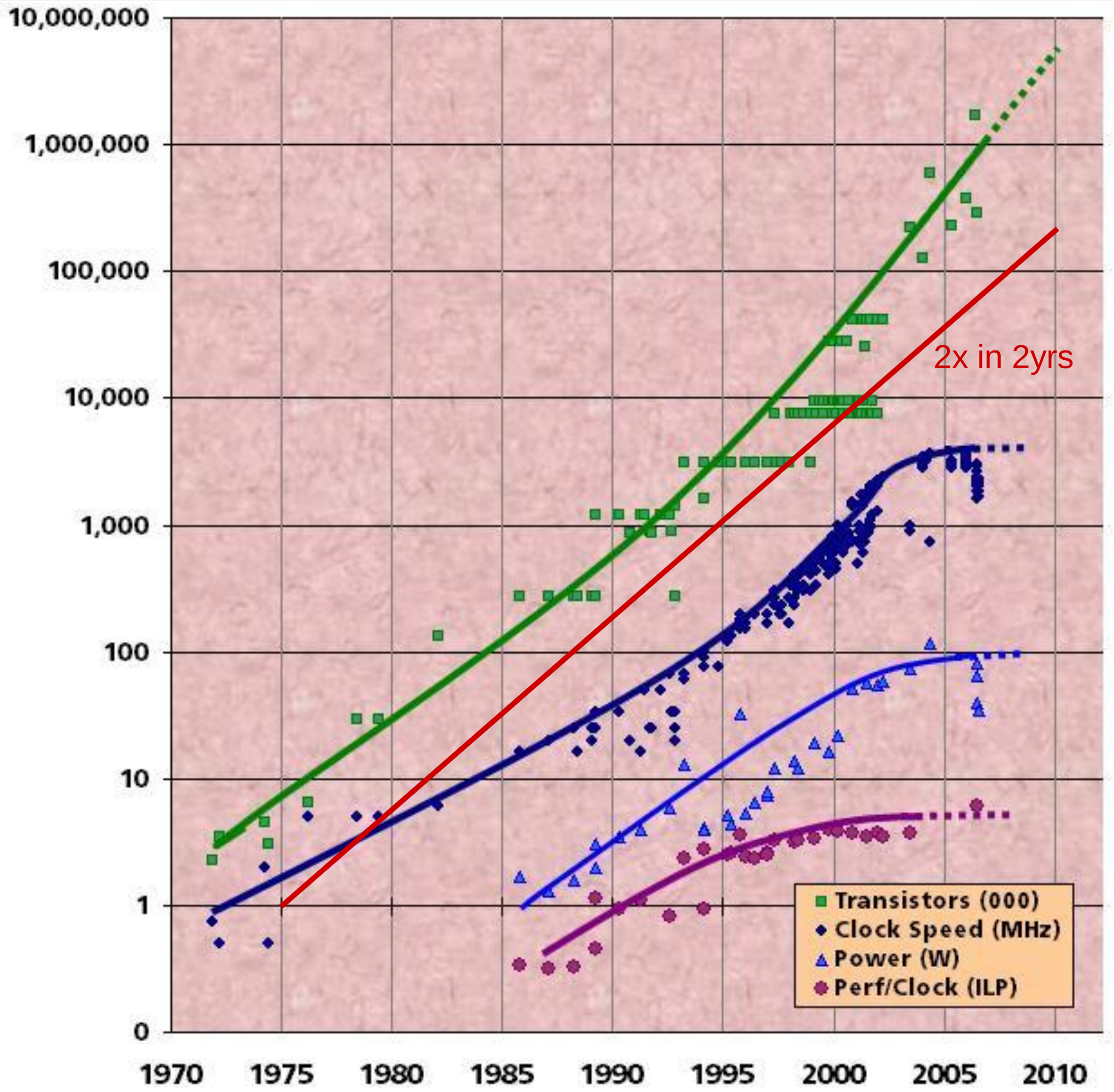
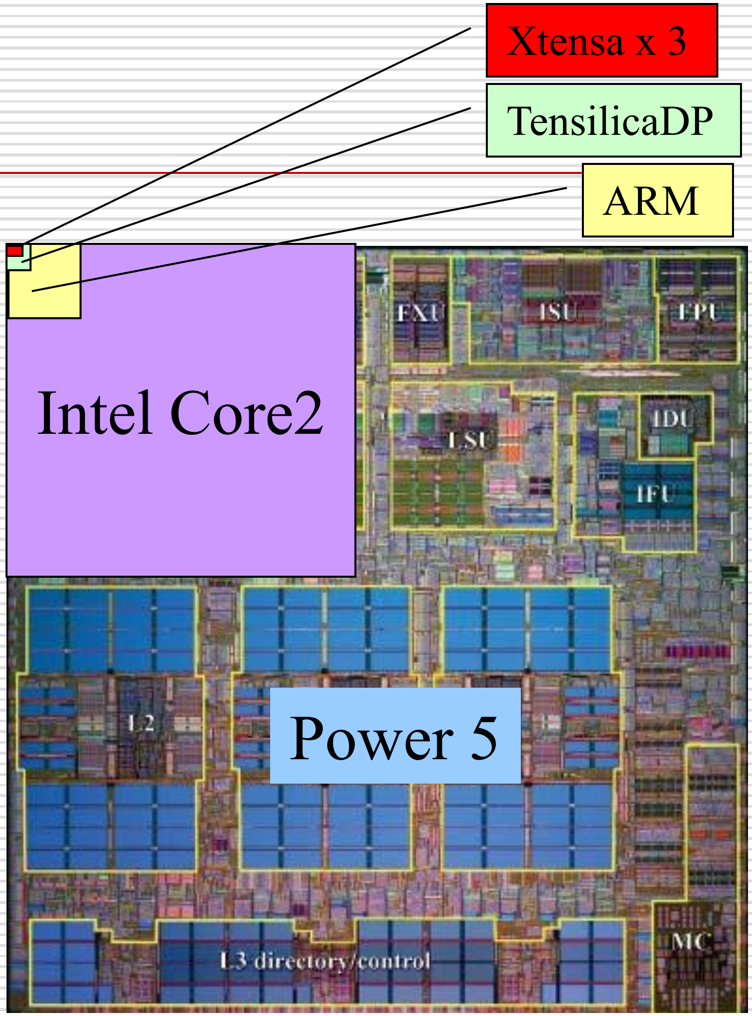



Figure courtesy of Kunle Olukotun, Lance Hammond, Herb Sutter & Burton Smith

# Size vs Power

- Power5 (Server)
  - 389mm^2
  - 120W@1900MHz
- Intel Core2 sc (laptop)
  - 130mm^2
  - 15W@1000MHz
- ARM Cortex A8 (automobiles)
  - 5mm^2
  - 0.8W@800MHz
- Tensilica DP (cell phones / printers)
  - 0.8mm^2
  - 0.09W@600MHz
- Tensilica Xtensa (Cisco router)
  - 0.32mm^2 for 3!
  - 0.05W@600MHz




Each processor operates with 0.3-0.1 efficiency of the largest chip: more threads, lower power

# Topic Overview

- Goal:  To give a good idea of parallel computation
  - Concepts -- looking at problems with “parallel eyes”
  - Algorithms -- different resources; different goals
  - Languages -- reduce control flow; increase independence; new abstractions
  - Hardware -- the challenge is communication, not instruction execution
  - Programming -- describe the computation without saying it sequentially
  - Practical wisdom about using parallelism

# Everyday Parallelism

- Juggling -- event-based computation
- House construction -- parallel tasks, wiring and plumbing performed at once
- Assembly line manufacture -- pipelining, many instances in process at once
- Call center -- independent tasks executed simultaneously

**How do we describe execution of tasks?**

# Parallel vs Distributed Computing

- Comparisons are often matters of degree

| *Characteristic* | *Parallel* | *Distributed* |
|---|---|---|
| Overall Goal | Speed | Convenience |
| Interactions | Frequent | Infrequent |
| Granularity | Fine | Coarse |
| Reliable | Assumed | Not Assumed |

# Parallel vs Concurrent

- In OS and DB communities execution of multiple threads is logically simultaneous
- In Arch and HPC communities execution of multiple threads is physically simultaneous
- The issues are often the same, say with respect to races
- Parallelism can achieve states that are impossible with concurrent execution because two events happen at once

# Consider A Simple Task …

- Adding a sequence of numbers A[0],…,A[n-1]
- Standard way to express it

```
sum = 0;
for (i=0; i<n; i++) {
   sum += A[i];
}
```

- Semantics require: (…((sum+A[0])+A[1])+…)+A[n-1]
  - That is, <u>sequential</u>
- Can it be executed in parallel?

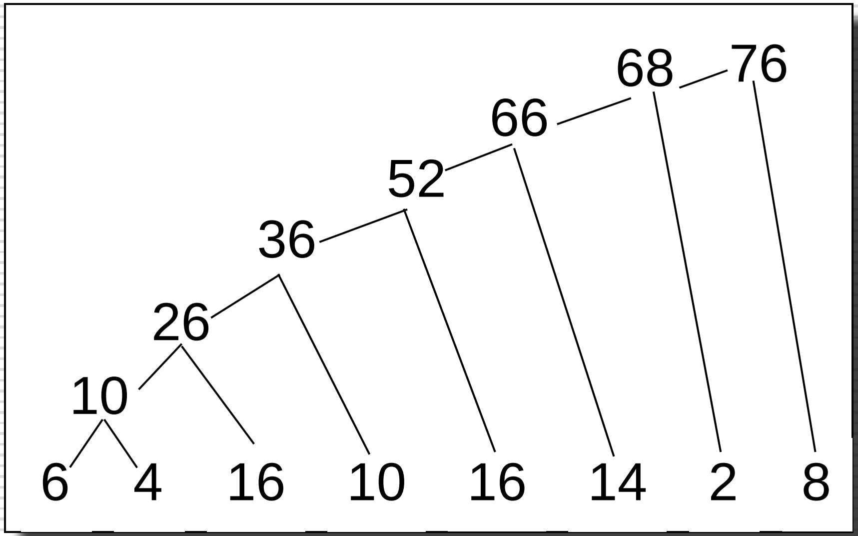
# Parallel Summation

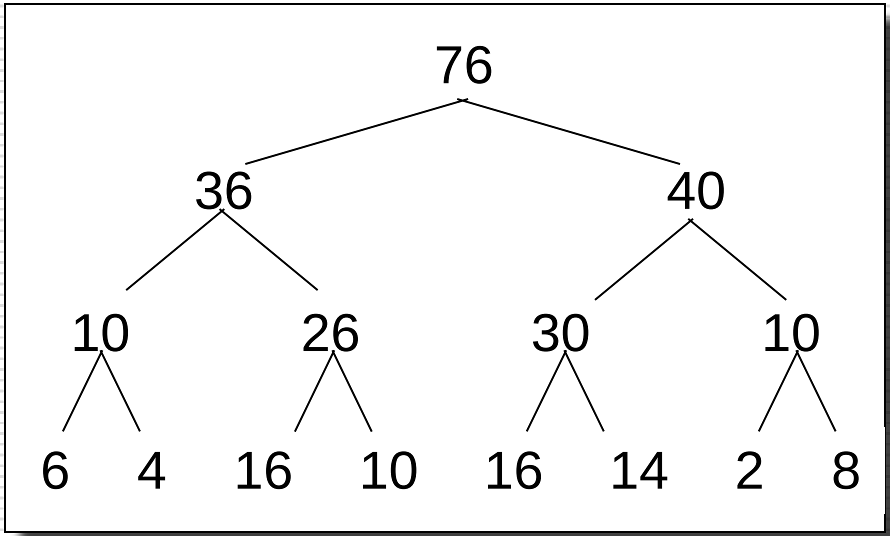
- To sum a sequence in parallel
  - add pairs of values producing 1st level results,
  - add pairs of 1st level results producing 2nd level results,
  - sum pairs of 2nd level results …
- That is,

(…((A[0]+A[1]) + (A[2]+A[3])) + ... + (A[n-2]+A[n-1]))…)

# Express the Two Formulations

- Graphic representation makes difference clear




  - Same number of operations; different order

# The Dream …

- Since 70s (Illiac IV days) the dream has been to compile sequential programs into parallel object code
  - Three decades of continual, well-funded research by smart people implies it's hopeless
    - For a tight loop summing numbers, its doable
    - For other computations it has proved **extremely** challenging to generate parallel code, even with pragmas or other assistance from programmers

# What’s the Problem?

- It’s not likely a compiler will produce parallel code from a C specification any time soon…
- Fact: For most computations, a “best” sequential solution (practically, not theoretically) and a “best” parallel solution are usually fundamentally different …
  - Different solution paradigms imply computations are not “simply” related
  - Compiler transformations generally preserve the solution paradigm

Therefore... the programmer must discover the || solution

# A Related Computation

- Consider computing the prefix sums

```
for (i=1; i<n; i++) {
  A[i] += A[i-1];
}
```

A[i] is the sum of the first i + 1 elements

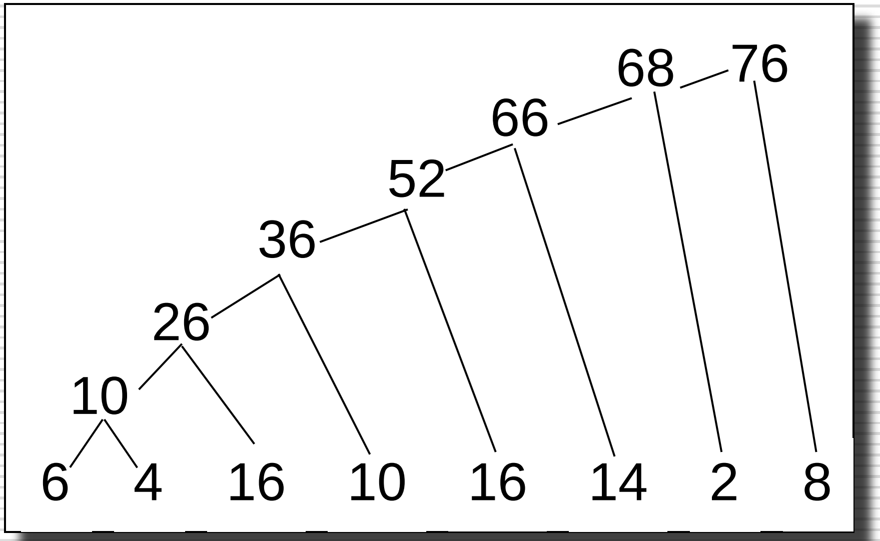
- Semantics ...
  - A[0] is unchanged
  - A[1] = A[1] + A[0]
  - A[2] = A[2] + (A[1] + A[0])

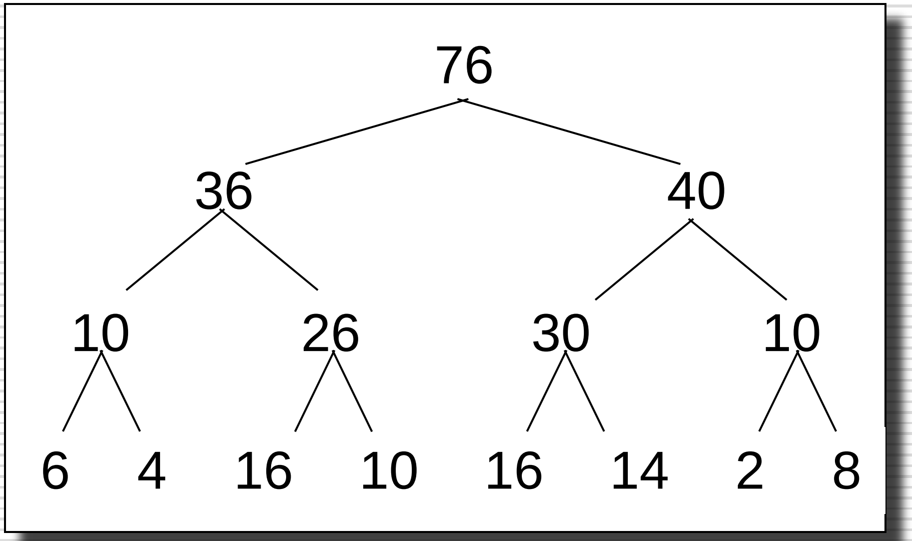
    ...
  - A[n-1] = A[n-1] + (A[n-2] + ( ... (A[1] + A[0]) … )

What advantage can ||ism give?

# Comparison of Paradigms

- The sequential solution computes the prefixes … the parallel solution computes only the last



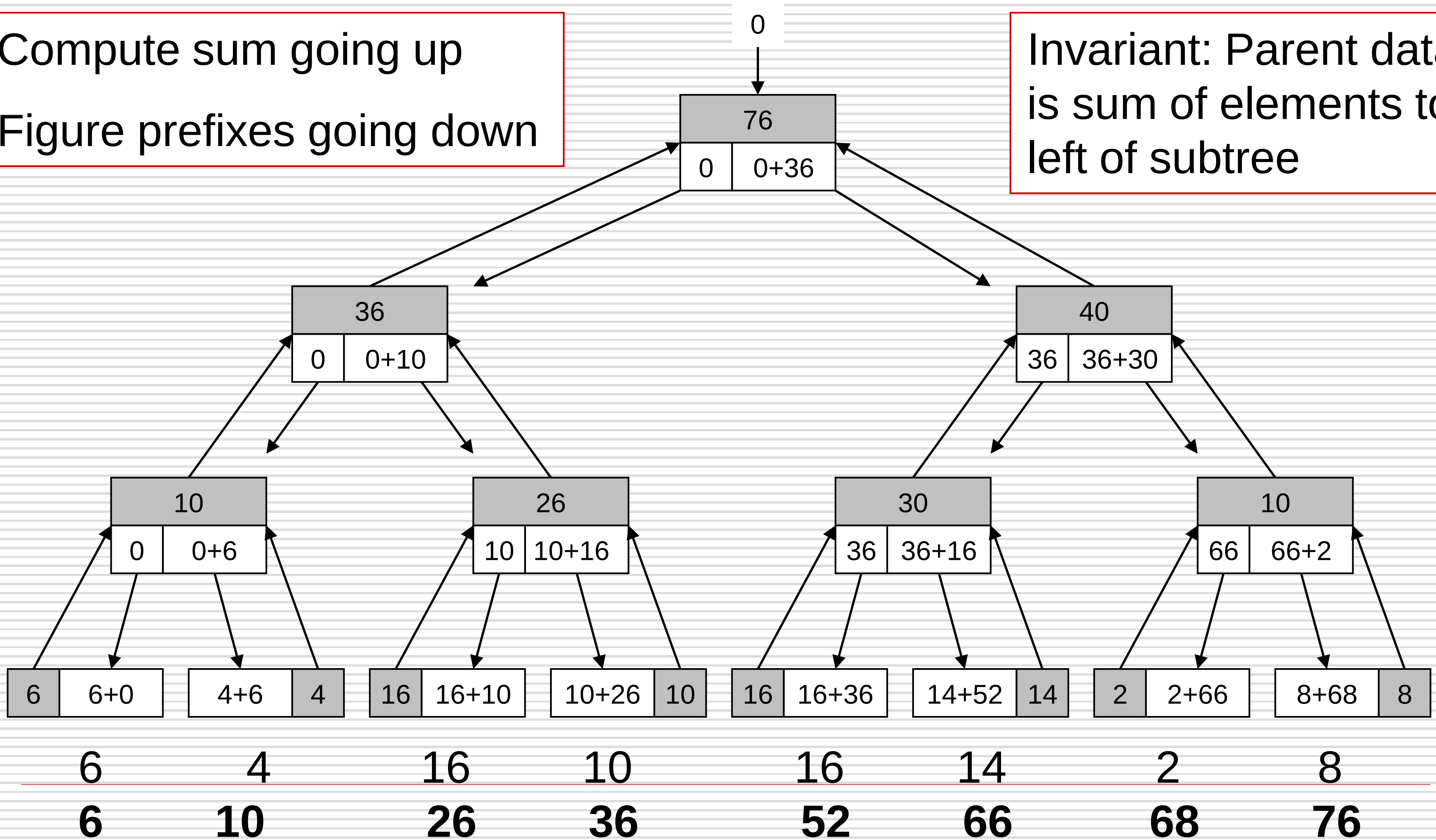

- Or does it?

# Parallel Prefix Algorithm

Compute sum going up

Figure prefixes going down

Invariant: Parent data is sum of elements to left of subtree



# Fundamental Tool of || Pgmming

- Original research on parallel prefix algorithm published by

  R. E. Ladner and M. J. Fischer

  Parallel Prefix Computation

  *Journal of the ACM* 27(4):831-838, 1980

**The Ladner-Fischer algorithm requires *2log n* time, twice as much as simple tournament global sum, not linear time**

**Applies to a wide class of operations**

# Parallel Compared to Sequential Programming

- Has different costs, different advantages
- Requires different, unfamiliar algorithms
- Must use different abstractions
- More complex to understand a program's behavior
- More difficult to control the interactions of the program's components
- Knowledge/tools/understanding more primitive

# Consider a Simple Problem

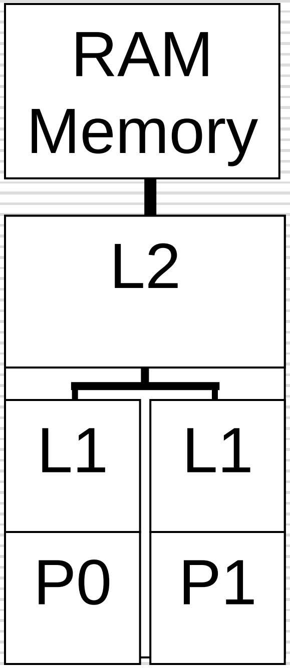
- Count the 3s in `array[]` of `length` values
- Definitional solution …
  - Sequential program

```
count = 0;
for (i=0; i<length; i++)
  {
    if (array[i] == 3)
       count += 1;
  }
```

# Write A Parallel Program

- Need to know something about machine … use multicore architecture

**How would you solve it in parallel?**



# Divide Into Separate Parts

- Threading solution -- prepare for MT procs

**`length=16 t=4`**

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **`array`** | 2 | 3 | 0 | 2 | 3 | 3 | 1 | 0 | 0 | 1 | 3 | 2 | 2 | 3 | 1 | 0 |
| | *Thread 0* | | | | *Thread 1* | | | | *Thread 2* | | | | *Thread 3* | | | |

```
int length_per_thread = length/t;
int start = id * length_per_thread;
for (i=start; i<start+length_per_thread; i++)
   {
      if (array[i] == 3)
               count += 1;
   }
```

# Divide Into Separate Parts

- Threading solution -- prepare for MT procs

`length=16 t=4`

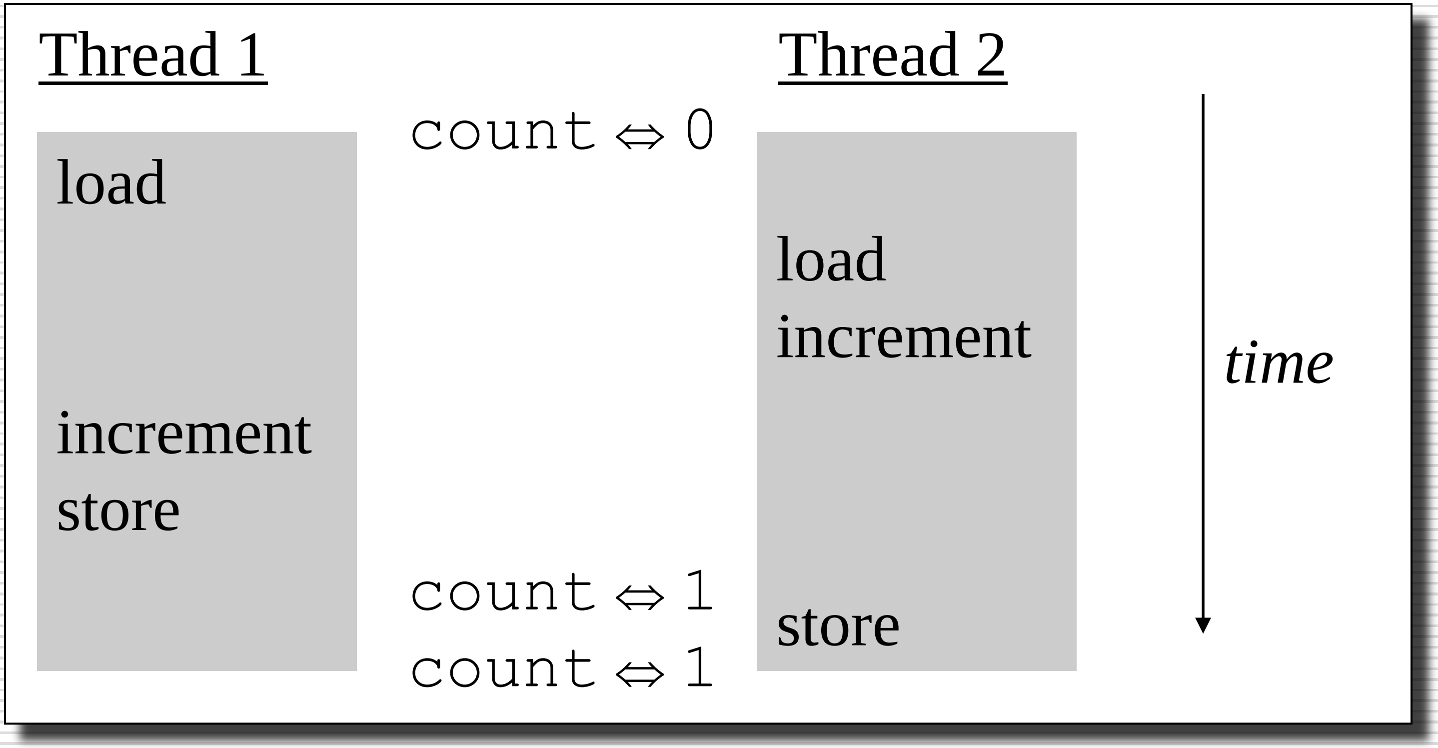
| `array` | 2 | 3 | 0 | 2 | 3 | 3 | 1 | 0 | 0 | 1 | 3 | 2 | 2 | 3 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Thread 0* | | | | *Thread 1* | | | | *Thread 2* | | | | *Thread 3* | | | |

```
int length_per_thread = length/t;
int start = id * length_per_thread;
for (i=start; i<start+length_per_thread; i++)
   {
      if (array[i] == 3)
                   count += 1;
   }
```

**Doesn't actually get the right answer**

# Races

- Two processes interfere on memory writes



# Races

- Two processes interfere on memory writes

| Thread 1 | | Thread 2 | |
|---|---|---|---|
| | `count` $\Leftrightarrow$ `0` | | |
| load | | | |
| | | load | |
| | | increment | *time* |
| increment | | | |
| store | | | |
| | `count` $\Leftrightarrow$ `1` | | |
| | | store | |
| | `count` $\Leftrightarrow$ `1` | | |

**Try 1**

# Protect Memory References

- Protect Memory References

```
mutex m;
for (i=start; i<start+length_per_thread; i++)
    {
    if (array[i] == 3)
      {
        mutex_lock(m);
          count += 1;
        mutex_unlock(m);
      }
    }
```

# Protect Memory References

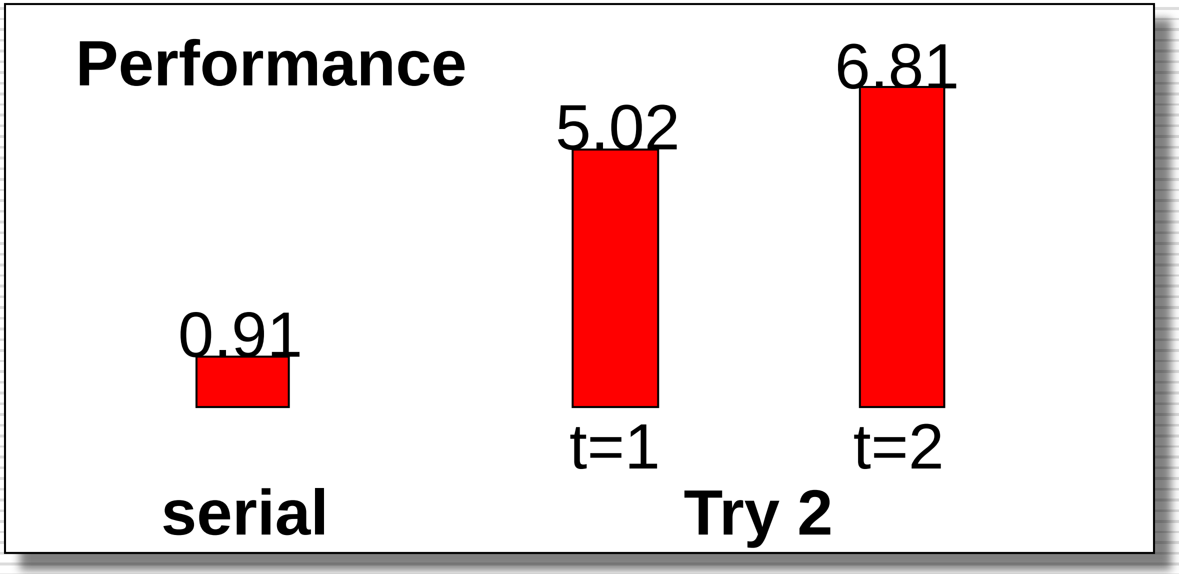
- Protect Memory References

```
mutex m;
for (i=start; i<start+length_per_thread; i++)
     {
     if (array[i] == 3)
        {
           mutex_lock(m);
             count += 1;
           mutex_unlock(m);
        }
     }
```

**Try 2**

# Correct Program Runs Slow

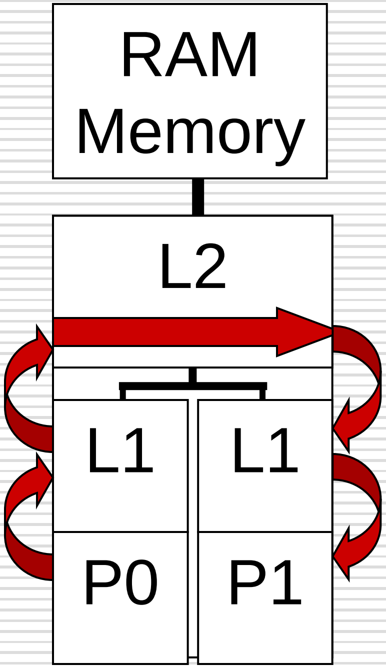
- Serializing at the mutex




  - The processors wait on each other

# Closer Look: Motion of `count, m`

- Lock Reference and Contention




```
mutex m;
for (i=start; i<start+length_per_thread; i++)
    {
    if (array[i] == 3)
      {
        mutex_lock(m);
         count += 1;
        mutex_unlock(m);
      }
    }
```

# Accumulate Into Private Count

- Each processor adds into its own memory; combine at the end

```
for (i=start; i<start+length_per_thread; i++)
    {
       if (array[i] == 3)
          {
             private_count[t] += 1;
          }
    }
mutex_lock(m);
  count += private_count[t];
mutex_unlock(m);
```

# Accumulate Into Private Count

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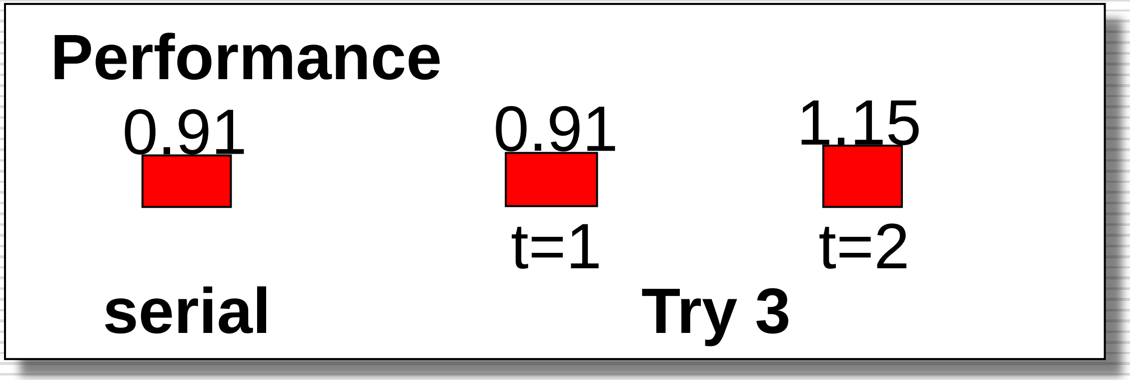
for (i=start; i<start+length_per_thread; i++)
    {
       if (array[i] == 3)
          {
            private_count[t] += 1;
          }
     }
mutex_lock(m);
  count += private_count[t];
mutex_unlock(m);
```

**Try 3**

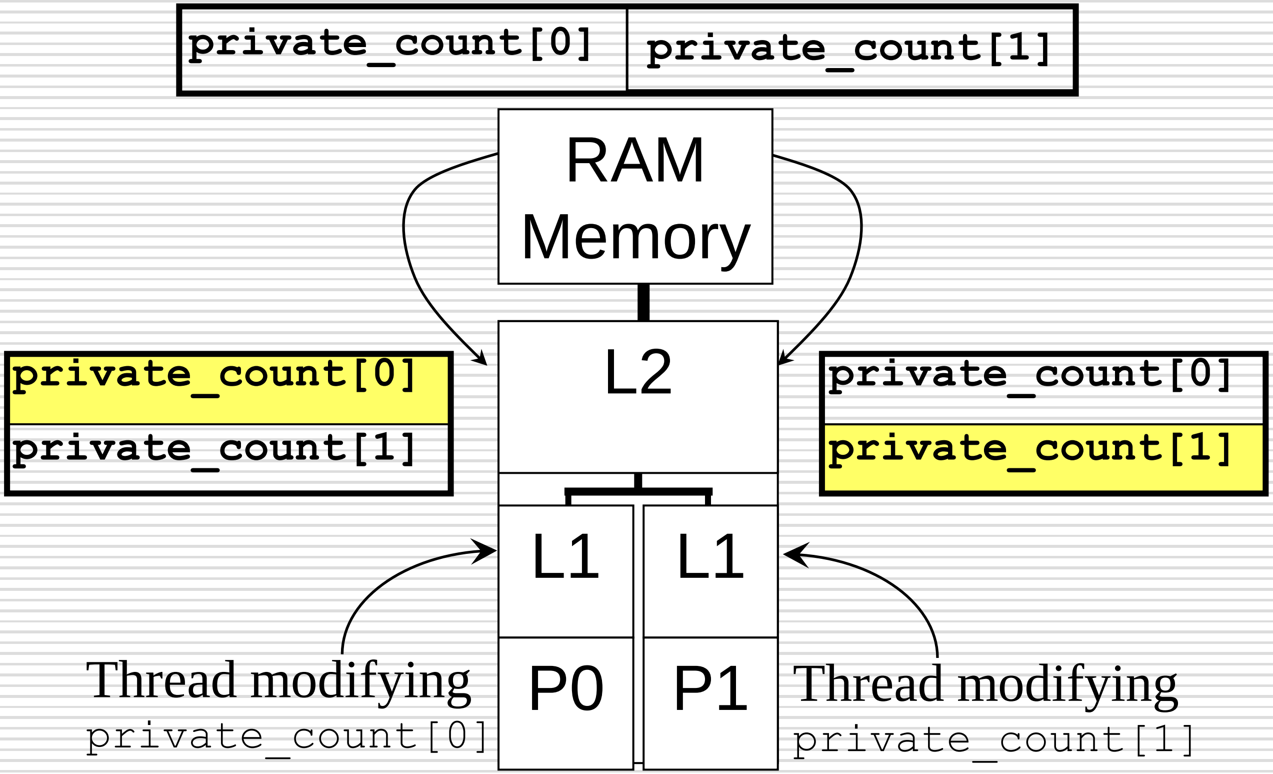
# Keeping Up, But Not Gaining

- Sequential and 1 processor match, but it's a loss with 2 processors



# False Sharing

- Private var ≠ private cache-line



# Force Into Different Lines

- Padding the private variables forces them into separate cache lines and removes false sharing

```
struct padded_int
  {  int value;
     char padding[128];
  }  private_count[MaxThreads];
```

# Force Into Different Lines

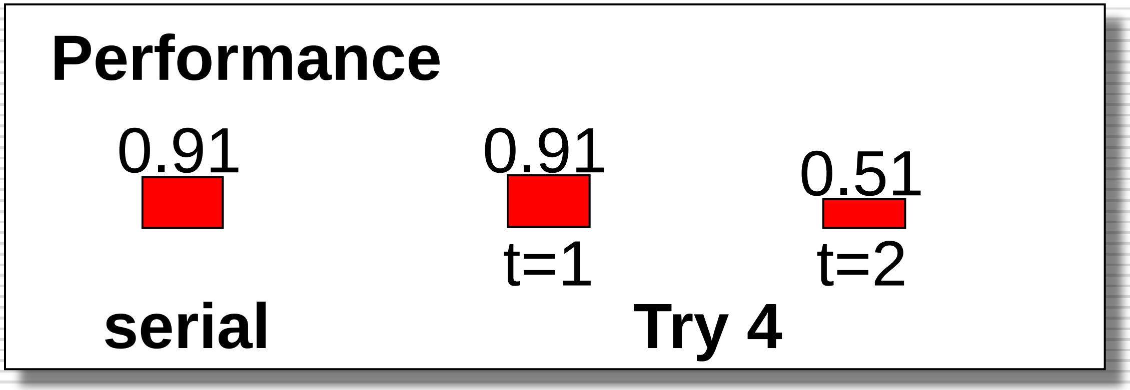
- Padding the private variables forces them into separate cache lines and removes false sharing

```
struct padded_int
  {  int value;
     char padding[128];
  }  private_count[MaxThreads];
```

**Try 4**

# Success!!

- Two processors are almost twice as fast




Is this the best solution???

# Count 3s Summary

- Recapping the experience of writing the program, we
  - Wrote the obvious “break into blocks” program
  - We needed to protect the `count` variable
  - We got the right answer, but the program was slower … lock congestion
  - Privatized memory and 1-process was fast enough, 2- processes slow … false sharing
  - Separated private variables to own cache line

**Finally, success**

# Break

- During break think about how to generalize the “sum n-integers” computation for n>8, and possibly, more processors

# Variations

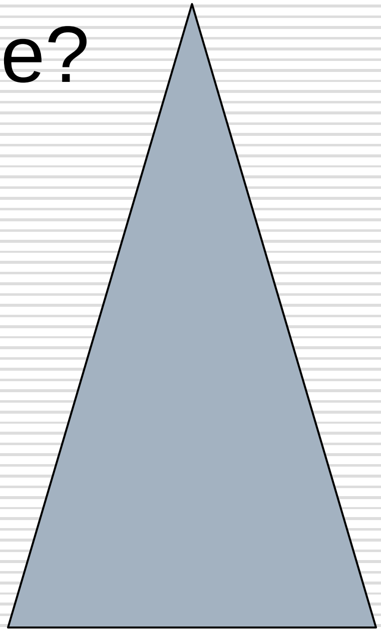
- What happens when more processors are available?
  - 4 processors
  - 8 processors
  - 256 processors
  - 32,768 processors

# Our Goals In Parallel Programming

- Goal: Scalable programs with performance and portability
  - Scalable: More processors can be "usefully" added to solve the problem faster
  - Performance: Programs run as fast as those produced by experienced parallel programmers for the specific machine
  - Portability: The solutions run well on all parallel platforms

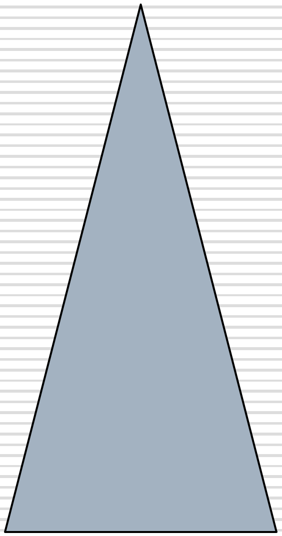
# Program A Parallel Sum

- Return to problem of writing a parallel sum
- Sketch solution in class when $n > P = 8$
- Use a logical binary tree?



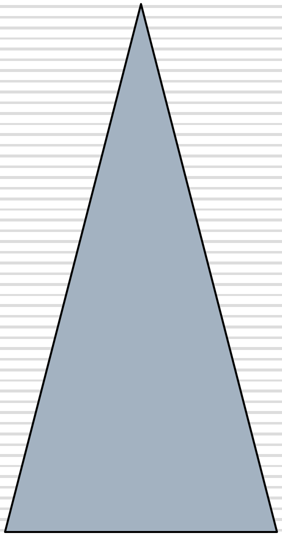
# Program A Parallel Sum

- Return to problem of writing a parallel sum
- Sketch solution in class when $n > P = 8$
- Assume communication time = 30 ticks
- $n = 1024$
- compute performance



# Program A Parallel Sum

- Return to problem of writing a parallel sum
- Sketch solution in class when $n > P = 8$
- and communication time = 30 ticks
- $n = 1024$
- compute performance
- Now scale to 64 processors



# Program A Parallel Sum

- Return to problem of writing a parallel sum
- Sketch solution in class when $n > P = 8$
- and communication time = 30 ticks
- $n = 1024$
- compute performance
- Now scale to 64 processors

**This analysis will become standard, intuitive**

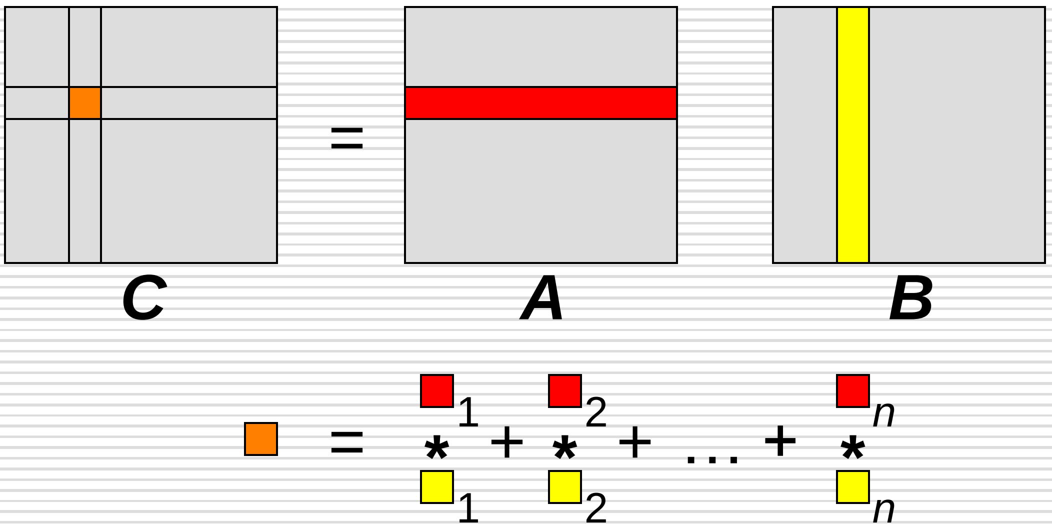
# Matrix Product: || Poster Algorithm

- Matrix multiplication is most studied parallel algorithm (analogous to sequential sorting)
- Many solutions known
  - Illustrate a variety of complications
  - Demonstrate great solutions
- Our goal: explore variety of issues
  - Amount of concurrency
  - Data placement
  - Granularity

**Exceptional by requiring O($n^3$) ops on O($n^2$) data**

# Recall the computation…

- Matrix multiplication of (square n x n) matrices ***A*** and ***B*** producing n x n result ***C*** where $\boldsymbol{C}_{rs} = \sum_{1 \le k \le n} \boldsymbol{A}_{rk} * \boldsymbol{B}_{ks}$

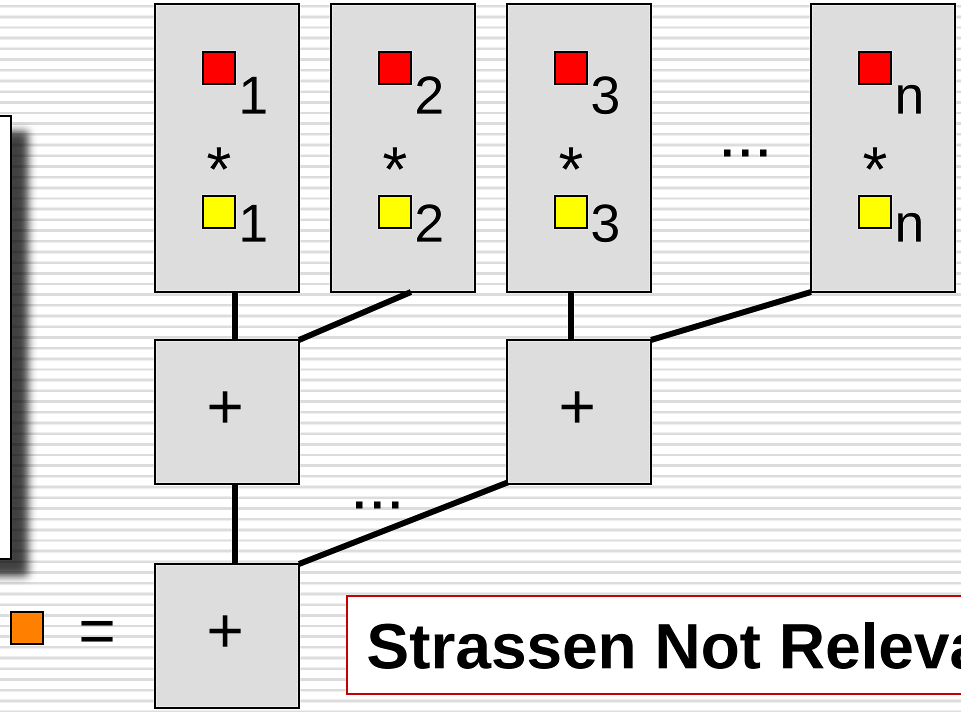


# Extreme Matrix Multiplication

- The multiplications are independent (do in any order) and the adds can be done in a tree

**O(*n*) processors for each result element implies O($n^3$) total**

**Time: O(*log n*)**




**Strassen Not Relevant**

# O(*log n*) MM in the real world …

Good properties

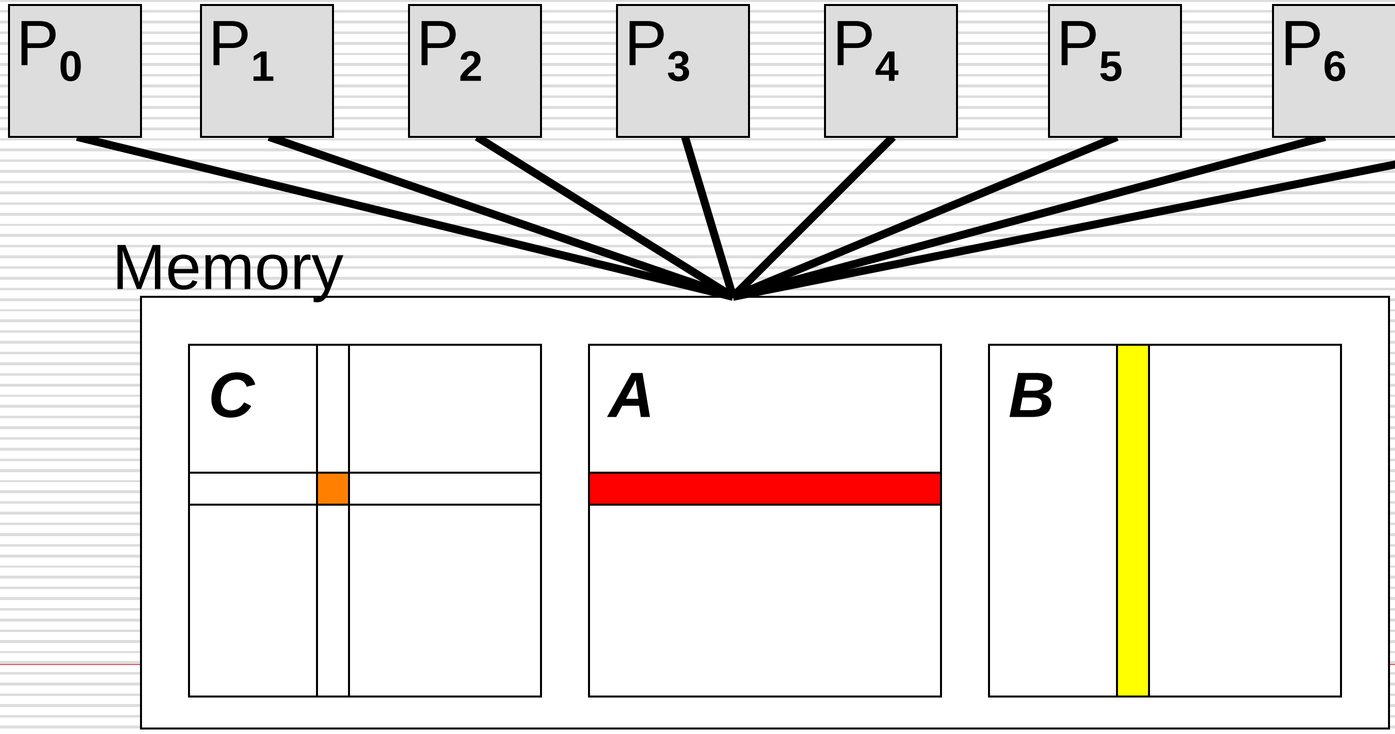
- Extremely parallel … shows limit of concurrency
- Very fast -- $log_2\, n$ is a good bound … faster?

Bad properties

- Ignores memory structure and reference collisions
- Ignores data motion and communication costs
- Under-uses processors -- half of the processors do only 1 operation

# Where is the data?

- Data references collisions and communication costs are important to final result … need a model … can generalize the standard RAM to get PRAM



# Parallel Random Access Machine

- Any number of processors, including $n^c$
- Any processor can reference any memory in "unit time"
- Resolve Memory Collisions
  - Read Collisions -- simultaneous reads to location are OK
  - Write Collisions -- simultaneous writes to loc need a rule:
    - Allowed, but must all write the same value
    - Allowed, but value from highest indexed processor wins
    - Allowed, but a random value wins
    - Prohibited

**Caution: The PRAM is *not* a model we advocate**

# PRAM says O(*log n*) MM is good

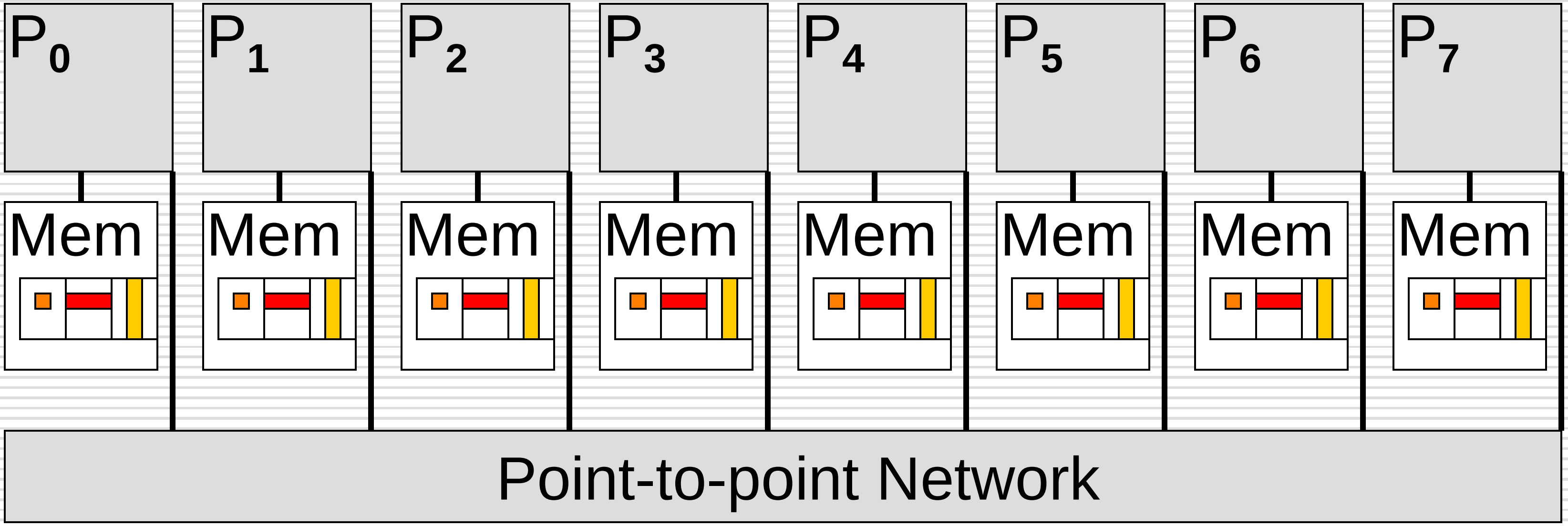
- PRAM allows any # processors => O($n^3$) OK
- ***A*** and ***B*** matrices are read simultaneously, but that's OK
- ***C*** is written simultaneously, but no location is written by more than 1 processor => OK

**PRAM model implies O(*log n*) algorithm is best … but in real world, we suspect not**

**We return to this point later**

# Where else could data be?

- Local memories of separate processors …

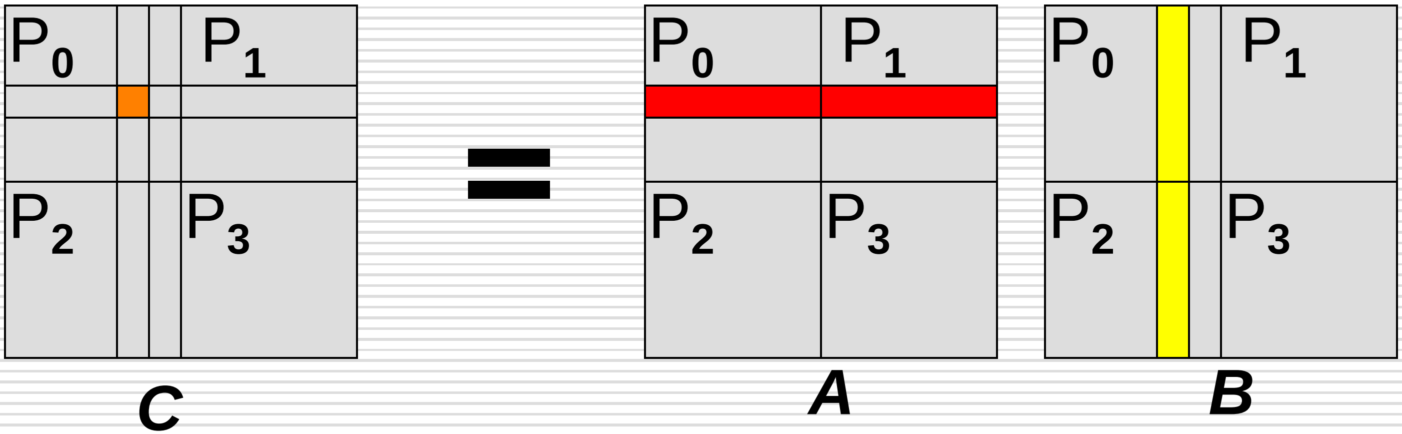



- Each processor could compute block of ***C***
  - Avoid keeping multiple copies of ***A*** and ***B***

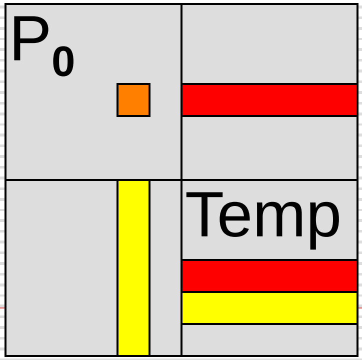
**Architecture common for servers**

# Data Motion

- Getting rows and columns to processors

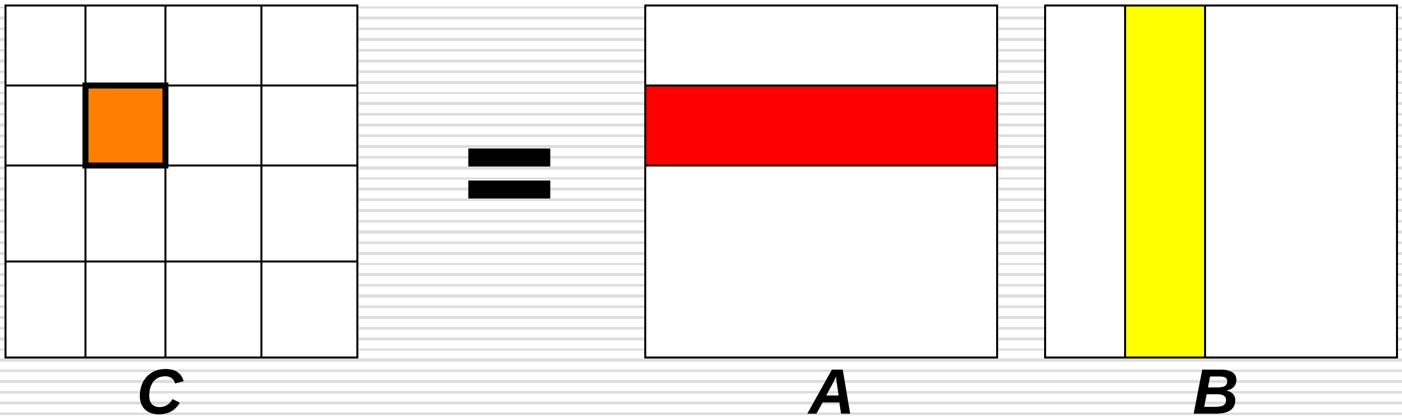



  - Allocate matrices in blocks
  - Ship only portion being used



# Blocking Improves Locality

- Compute a *b* x *b* block of the result



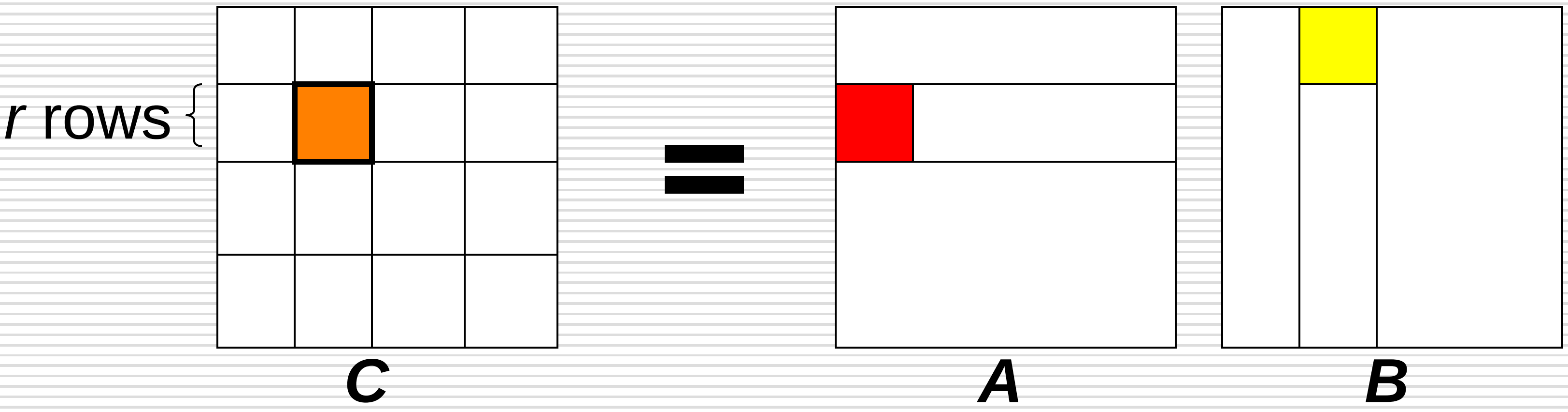

- Advantages
  - Reuse of rows, columns = caching effect
  - Larger blocks of local computation = hi locality

# Caching in Parallel Computers

- Blocking = caching … why not automatic?
  - Blocking improves locality, but it is generally a manual optimization in sequential computation
  - Caching exploits two forms of locality
    - Temporal locality -- refs clustered in time
    - Spatial locality -- refs clustered by address
- *When multiple threads touch the data, global reference sequence may not exhibit clustering features typical of one thread -- thrashing*

# Sweeter Blocking

- It's possible to do even better blocking …




- Completely use the cached values before reloading

# Best MM Algorithm?

- We haven't decided on a good MM solution
- A variety of factors have emerged
  - A processor's connection to memory, unknown
  - Number of processors available, unknown
  - Locality--always important in computing--
    - Using caching is complicated by multiple threads
    - Contrary to high levels of parallelism
- Conclusion: Need a better understanding of the constraints of parallelism

**Next week, architectural details + model of ||ism**

# Assignment for Next Time

- Reproduce the parallel prefix tree labeling to compute the bit-wise & scan
- Try the “count 3s” computation on your multi-core computer
  - Implementation Discussion Board … please contribute – success, failure, kibitzing, …
  - https://catalysttools.washington.edu/gopost/board/snyder/16265/