Computability and Complexity — Winter 2009

Problem Set #1 — Instructor: Anna Karlin

Due on **January 23, 2009** by email to `ncthach@cs.washington.edu`

Reminder: If you haven't done so already, subscribe to class email list by following the link from the course webpage.

Instructions: You are allowed to discuss the problems with fellow students taking the class. However, you must write up your solutions completely on your own. Moreover, if you do discuss the problems with someone else, I am asking, on your honor, that you do not take any written material away from the discussion. In addition, for each problem on the homework, I ask that you acknowledge the people you discussed that problem with, if any.

The problems have been carefully chosen for their pedagogical value and hence might be similar or identical to those given out in past offerings of this course at UW, or similar courses at other schools. Using any pre-existing solutions from these sources, from the Web or other textbooks constitutes a violation of the academic integrity expected of you and is strictly prohibited.

Most of the problems require only one or two key ideas for their solution – spelling out these ideas should give you most of the credit for the problem even if you err in some finer details. So, make sure you clearly write down the main idea(s) behind your solution.

A final piece of advice: Begin work on the problem set early and don't wait till the deadline is a day or two away. This one is not very hard, but it's a good habit to get into.

Readings: Arora/Barak Chapters 1 and 2, Sipser Chapters 3-5.

1. Give a detailed description (including a complete description of the transition function) of a Turing machine such that, on input equal to a number i in binary, moves the head on the work tape to the ith location.

2. Let Q be the set of complex numbers x such that $ax^2 + bx + c = 0$ for some a, b, c integer, with $a \neq 0$. Show that Q is countable.

3. Show that the number of functions $f : \{0,1\}^* \rightarrow \{0,1\}$ is uncountable. You may use any facts we have proved in class without reproving them.

4. Show that the set of decidable languages is closed under concatenation. (Reminder about terminology: A language is a set $L \subseteq \Sigma^*$. L is decidable if there is a Turing machine that decides the language L, i.e., there exists a Turing Machine that accepts $x \in L$ (accepts means outputs 1) and rejects $x \notin L$ (rejects means outputs 0). To say that the set of decidable languages is closed under concatenation means that for any pair of languages L', L'', if L' and L'' are both decidable, then $L = \{w'w'' | w' \in L' \wedge w'' \in L''\}$ is also decidable.

5. *Extra Credit*

   Solve Problem 13 in Chapter 1 of the Arora/Barak book `http://www.cs.princeton.edu/theory/complexity/modelchap.pdf`, page 1.23(31). You do not need to give low-level description of your Turing machine construction. You will need to read at least portions of the chapter to completely understand the question.