Assignment 3 -- Solution

Problem 1

H_1: $r_1[y]\ r_1[x]\ r_2[x]\ w_1[y]\ c_1\ w_2[y]\ c_2$

H_1 is normally-strict two-phase locked:

$rl_1[y]\ r_1[y]\ rl_1[x]\ r_1[x]\ rl_2[x]\ r_2[x]\ wl_1[y]\ w_1[y]\ c_1\ ru_1[x]\ wu_1[y]$
$wl_2[y]\ w_2[y]\ c_2\ ru_2[x]\ wu_2[y]$

Note that $ru_1[y]$ isn't needed, since $ru_1[y]$ was converted into $wu_1[y]$, i.e., T_1 holds only one lock on y.

Problem 1 (continued)

H_2: $r_1[y]$ $r_1[x]$ $r_2[x]$ $\mathbf{w_2[x]}$ $w_1[y]$ c_1 $w_2[y]$ c_2

H_2 is two-phase locked, but not strict two-phase locked. To run $w_2[x]$, T_1 must have released its read lock on x before $w_2[x]$, which means it cannot be strict two-phase locked. Moreover, to be two-phase locked, it must have gotten its write lock on y before it released its read lock on x. Thus, we have the following:

$rl_1[y]$ $r_1[y]$ $rl_1[x]$ $r_1[x]$ $rl_2[x]$ $r_2[x]$ $\mathbf{wl_1[y]}$ $\mathbf{ru_1[x]}$ $\mathbf{wl_2[x]}$ $w_2[x]$ $w_1[y]$ c_1 $wu_1[y]$ $wl_2[y]$ $w_2[y]$ c_2 $ru_2[x]$ $wu_2[y]$

Problem 1 (continued)

H_3: $r_1[y]$ $r_1[x]$ $r_2[x]$ $w_1[y]$ $w_2[y]$ c_2 $\mathbf{c_1}$

H_3 is two-phase locked, but not strict two-phase locked because T_1 must have released its write lock before $w_2[y]$ executed.

$rl_1[y]$ $r_1[y]$ $rl_1[x]$ $r_1[x]$ $rl_2[x]$ $r_2[x]$ $wl_1[y]$ $w_1[y]$ $\mathbf{wu_1[y]}$ $wl_2[y]$ $w_2[y]$ c_2 $ru_2[x]$ $wu_2[y]$ c_1 $ru_1[x]$

Problem 1 (continued)

H_4**:** $r_1[y]$ $r_1[x]$ $r_2[x]$ $\mathbf{w_2[x]}$ $\mathbf{r_3[y]}$ $w_1[y]$ c_1 $\mathbf{w_3[z]}$ $\mathbf{c_3}$ $w_2[y]$ c_2

H_4 is not two-phase locked. To see why, consider the following prefix of the history:

$rl_1[y]$ $r_1[y]$ $rl_1[x]$ $r_1[x]$ $rl_2[x]$ $r_2[x]$

The next operation is $w_2[x]$. So as in H_2, T_1 must have released its read lock on x before $w_2[x]$, so again the next few operations must have been $wl_1[y]$ $ru_1[x]$ $wl_2[x]$ $w_2[x]$, as in the following expanded prefix.

$rl_1[y]$ $r_1[y]$ $rl_1[x]$ $r_1[x]$ $rl_2[x]$ $r_2[x]$ $wl_1[y]$ $ru_1[x]$ $wl_2[x]$ $w_2[x]$

H_4 continued

H_4**:** $r_1[y]$ $r_1[x]$ $r_2[x]$ $\mathbf{w_2[x]}$ $\mathbf{r_3[y]}$ $w_1[y]$ c_1 $\mathbf{w_3[z]}$ $\mathbf{c_3}$ $w_2[y]$ c_2

$rl_1[y]$ $r_1[y]$ $rl_1[x]$ $r_1[x]$ $rl_2[x]$ $r_2[x]$ $wl_1[y]$ $ru_1[x]$ $wl_2[x]$ $w_2[x]$

The next operation is $r_3[y]$. To have executed here, T_3 would have to obtain its lock on y, which requires that T_1 had already released its lock on y, which it could not have done at this point because it hasn't yet executed $w_1[y]$.

Nevertheless, this history is SR. We have only the following SG edges:

$T_1 \rightarrow T_2$ because $(r_1[x], w_2[x])$ and $(w_1[y], w_2[y])$

$T_3 \rightarrow T_1$ because $(r_3[y], w_1[y])$

There's no cycle in the SG, so the history is serializable as T_3 T_1 T_2. Note that there are no transaction handshakes in the input, so there are none to preserve.

Extra credit: Is it possible for a history to be strict two-phase locked but not normally-strict two phase locked?

No. To prove it, let H be a strict 2PL history that has been augmented with lock and unlock operations to demonstrate that it's strict 2PL. We can transform H into a history each of whose lock operations immediately precedes the operation it's synchronizing, as follows.

- Suppose that for some operation $o_i[x]$ in H, the corresponding lock request $ol_i[x]$ does not immediately precede $o_i[x]$.
- The only constraint that prevents moving $ol_i[x]$ to the right in H so that it immediately precedes $o_i[x]$ is an unlock operation by T_i, since that would break 2PL.
- However, since H is strict 2PL, all of T_i's unlock operations follow c_i.
- Therefore, it's possible to move $ol_i[x]$ to the right in H so that it immediately precedes $o_i[x]$.
- This can be done for all offending lock operations in H, thereby transforming it into a demonstration that H is normally-strict 2PL-ed.

Problem 2: Yes, a transaction can be involved in multiple deadlocks.

Consider the following three sequential transactions:

T_1: $r_1[x]$ $r_1[y]$

T_2: r_2 $[x]$ $r_2[y]$

T_3: $w_3[y]$ $w_3[x]$

Suppose they start executing as follows:

H_1: $r_1[x]$ $r_2[x]$ $w_3[y]$

So far, T_1 and T_2 each have a read lock on x, and T_3 has a write lock on y. Next, each transaction tries to set a lock for its second operation: $r_1[y]$, $r_2[y]$, and $w_3[x]$. However, no matter which order the three lock requests are made, none of those lock requests can be granted, because another transaction already owns a conflicting lock. In terms of the waits-for graph, we have:

$T_1 \rightarrow T_3$ because T_1 requests a read lock on y and T_3 owns a write lock on y

$T_2 \rightarrow T_3$ for the same reason as above

$T_3 \rightarrow T_1$ because T_3 requests a write lock on x and T_1 owns a read lock on x

$T_3 \rightarrow T_2$ for the same reason as above.

Thus, there are two deadlock cycles in the graph,

$T_1 \rightarrow T_3 \rightarrow T_1$ and $T_2 \rightarrow T_3 \rightarrow T_2$.

Problem 2 (continued):

Since each transaction is sequential, it can only have one blocked operation. It is therefore tempting to say that *there could only be one outgoing edge* from the transaction in the waits-for graph. But the italicized implication is wrong, because a transaction may issue a write request, thereby waiting for *all* of the transactions holding a read lock. Therefore, it is waiting for each of those read transactions and has more than one outgoing edge. In the above example T_3 is waiting for both T_1 and T_2 to unlock x. Then T_1 and T_2 each request a lock on y, which causes each of them to deadlock (independently) with T_3.

Problem 3

Let's hand execute each sequence by issuing a lock request for each operation as it arrives:

a) H_1: $r_1[x,y]$ $r_2[x]$ $w_1[x]$ $w_2[z]$ $r_3[z]$ $r_3[y]$ $w_3[y]$

$rl_1[x,y]$ $r_1[x,y]$ $rl_2[x]$ $r_2[x]$ {$wl_1[x]$ is blocked} $wl_2[z]$ $w_2[z]$
{T_2 is done so it could have issued commit at this point}
c_2 $wu_2[x]$ $wu_2[z]$ {now we can set $wl_1[x]$} $wl_1[x]$ $w_1[x]$
{T_1 is done so it can commit} c_1 $ru_1[y]$ $wu_1[x]$
{now there are no locks held so T_3 can execute and commit}.

So adding commits to H_1:
H_1: $r_1[x,y]$ $r_2[x]$ $w_1[x]$ $w_2[z]$ $\mathbf{c_2}$ $\mathbf{c_1}$ $r_3[z]$ $r_3[y]$ $w_3[y]$ $\mathbf{c_3}$

Problem 3 (continued)

b) H_2: $r_1[x,y]$ $r_2[x]$ $w_1[x]$ $r_3[z]$ $w_2[z]$ $r_3[y]$ $w_3[y]$

$rl_1[x,y]$ $r_1[x,y]$ $rl_2[x]$ $r_2[x]$ **{$w_1[x]$ is blocked}** $rl_3[z]$ $r_3[z]$ **{$w_2[z]$ is blocked}** $rl_3[y]$ $r_3[y]$ **{$w_3[y]$ is blocked}**

There's a deadlock: $w_1[x]$ is waiting for $rl_2[x]$, $w_2[z]$ is waiting for $rl_3[z]$, and $w_3[y]$ is waiting for $rl_1[y]$.