Homework 2:
Shading, Raytracing, and Texture Mapping

Assigned: Tuesday, 8 May 2001
Due: Monday, 21 May 2001

DIRECTIONS

Please provide short written answers to the questions in the space provided. If you require extra space, you may staple additional pages to the back of your assignment. Feel free to talk over the problems with classmates, but please answer the questions on your own.

Name: ___________________________________

Problem 1. Phong shading

The Phong shading model for a scene illuminated by global ambient light and a single directional light can be summarized by the following equation:

$$I_{phong} = k_e + k_a I_a + I_l\left[k_d(\mathbf{N}\cdot\mathbf{L})_+ + k_s(\mathbf{V}\cdot\mathbf{R})_+^{ns}\right]$$

Imagine a scene with one gray sphere illuminated by white global ambient light and a single white directional light. Describe the effect of the conditions (a)-(e) on the shading of the object. At each incremental step, assume that all the preceding steps have been applied first. The ambient light is on throughout the problem, and you may assume that all values are set such that no clamping occurs. Also, “white” and “gray” imply R=G=B.

a. The directional light is off. How does the shading vary over the surface of the object?

b. Now turn the directional light on. The specular reflection coefficient k_s of the material is zero, and the diffuse reflection coefficient k_d is non-zero. How does the shading vary over the surface of the object?

c. Now translate the sphere straight towards the viewer. What happens to the shading of the object?

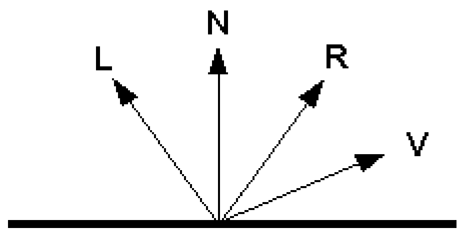
d. Now increase the specular reflection coefficient k_s of the material to be greater than zero. What happens?

e. Now increase the specular exponent n_s. What happens?

Problem 2. Halfway vector specular shading

Blinn and Newell have suggested that if **V** and **L** are assumed to be constants the computation of $\mathbf{V} \cdot \mathbf{R}$ in the Phong shading model can be simplified by associating with each light source a fictitious light source that will generate specular reflections. This second light source is located in a direction **H** halfway between **V** and **L**. The specular component is then computed from $(\mathbf{N} \cdot \mathbf{H})^{ns}$, instead of from $(\mathbf{V} \cdot \mathbf{R})^{ns}$.

a. On the diagram below, assume that V and L are the new constant viewing direction and lighting direction vectors. Draw the new direction H on the diagram.

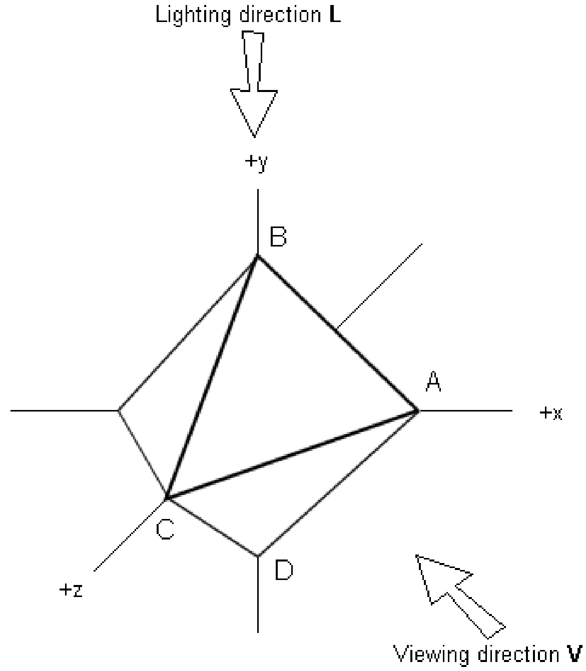


b. Under what circumstances or by making what approximations might **L** and **V** be assumed to be constant for all points on the surface of the object as seen for any pixel on the image plane?

c. What is an advantage of this approximation? A disadvantage?

Problem 3. Interpolated shading

The faceted polyhedron shown in the figure consists of two pyramids connected at the base (an octahedron) drawn with 8 equilateral triangular faces with vertices at (1,0,0), (0,1,0), (0,0,1), (-1,0,0), (0,-1,0), and (0,0,-1). The viewer center of projection is at (10,0,10) looking directly towards the origin, and there is a directional light shining down from above parallel to the y-axis.



a. In order to draw the faces as flat-shaded triangles in OpenGL, you need to specify the normal at each vertex as perpendicular to the face. What is the unit normal perpendicular to the face at each vertex for the triangles ABC and ACD?

Triangle	Vertex	Coords	Unit Normal
ABC	A	(1,0,0)	
ABC	B	(0,1,0)	
ABC	C	(0,0,1)	
ACD	A	(1,0,0)	
ACD	C	(0,0,1)	
ACD	D	(0,-1,0)	

b. If the vertices of triangle ABC are drawn with OpenGL in GL_TRIANGLES mode in the order A, then B, then C, is the viewer looking at the front or the back face of the triangle?

Problem 3 (cont'd)

c. Assume that this object is really just a crude approximation of a sphere (you are using the octahedron to represent the sphere because your computer is slow). If you want to shade the octahedron so that it approximates the shading of a sphere, what would you specify as the normal at each vertex?

Triangle	Vertex	Coords	Unit Normal
ABC	A	(1,0,0)	
ABC	B	(0,1,0)	
ABC	C	(0,0,1)	
ACD	A	(1,0,0)	
ACD	C	(0,0,1)	
ACD	D	(0,-1,0)	

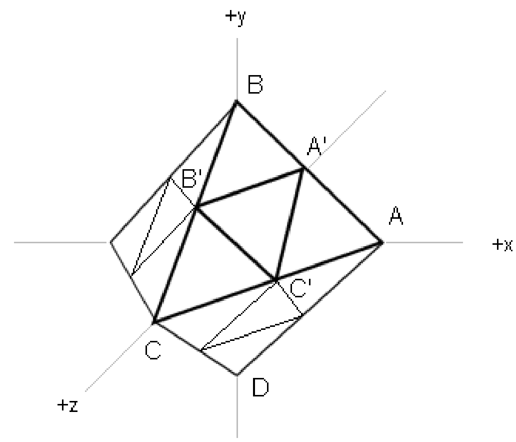
d. Assume that the normals have been specified as in (c). Now switch from OpenGL's Gouraud interpolated shading to Phong interpolated shading. This will affect both the diffuse and specular components of the shading over the surface. How will the diffuse appearance of the image change? How will the specular appearance of the image change?

e. What is one reason that Gouraud shading is standard on most systems and Phong shading is not?

Problem 3 (cont'd)

f. Remember that this object is being used to simulate a sphere. One easy-to-calculate improvement to the geometry of the model is to subdivide each triangular face into four new equilateral triangles and then specify three new additional vertices. If you subdivided triangle ABC this way as shown in the figure below, what would be the best choices for the new coordinates and the normals of the three added vertices A', B', and C' in order to more closely approximate a sphere? Why?

Triangle	Vertex	Coordinates	Normals
A'B'C'	A'		
A'B'C'	B'		
A'B'C'	C'		



g. How would you continue to improve the approximation to the sphere?

Problem 4. Recursive ray termination.

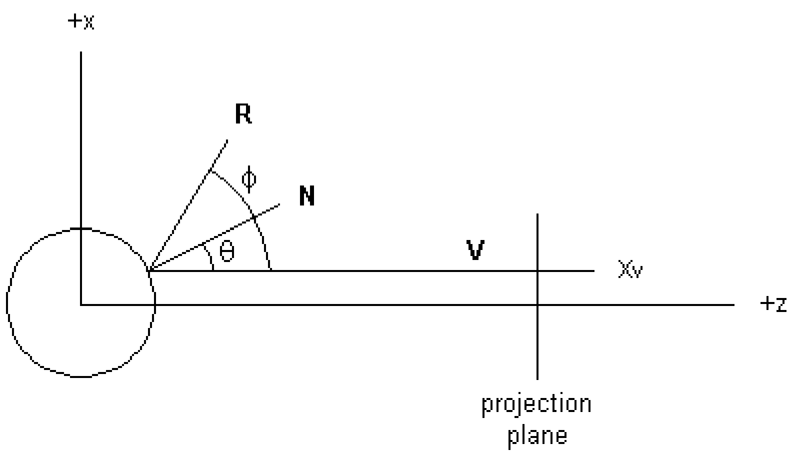
The company you work for has just bought rights to a raytracing engine. Unfortunately, you don't have the source code, just a compiled library. You have been asked to determine how rays are terminated. So, you call the authors you find out even they don't remember for sure. All they can tell you is this: *The termination criteria for tracing rays is either (a) rays are traced to a maximum recursion depth of 5, or (b) rays are adaptively terminated based on their contribution to a pixel color.*

Describe a scene that can be used to determine which method is used. Be specific about all relevant aspects of the scene and what you would look for in the resulting image to determine which termination method is used.

Problem 5. Environment mapping

One method of environment mapping (reflection mapping) involves using a "gazing ball" to capture an image of the surroundings. The idea is to place a chrome sphere in an actual environment, take a photograph of the sphere, and use the resulting image as an environment map. Let's examine this in two dimensions, using a "gazing circle" to capture the environment around a point.

Below is a diagram of the setup. In order to keep the intersection and angle calculations simple, we will assume that each view ray **V** that is cast through the projection plane to the gazing circle is parallel to the z-axis, meaning that the viewer is located at infinity on the z-axis. The circle is of radius 1, centered at the origin.



a. If the x-coordinate of the view ray is x_v, what are the (x,z) coordinates of the point at which the ray intersects the circle? What is the unit normal vector at this point?

b. What is the angle θ between the view ray **V** and the normal **N** as a function of x_v?

Problem 5 (cont'd)

c. Note that the angle ϕ between the view ray **V** and the reflection direction **R** is $2*\theta$. Plot ϕ versus x_v. In what regions do small changes in the intersection point result in large changes in the reflection direction?

d. We can now treat the photograph of the chrome circle as an environment map. If we were to ray-trace a new, shiny object and index into the environment map according to each reflection direction, what types of errors would we expect to see in the resulting image?