# 9. Hidden Surface Algorithms



## Reading

**Reading:**

- Watt, 6.6 (esp. intro and subsections 1, 4, and 8–10), 12.1.4.

**Optional reading:**

- Foley, van Dam, Feiner, Hughes, Chapter 15
- I. E. Sutherland, R. F. Sproull, and R. A. Schumacker, A characterization of ten hidden surface algorithms, *ACM Computing Surveys* 6(1): 1-55, March 1974.



## Introduction

In the previous lecture, we figured out how to transform the geometry so that the relative sizes will be correct if we drop the *z* component.

But, how do we decide which geometry actually gets drawn to a pixel?

Known as the **hidden surface elimination problem** or the **visible surface determination problem**.

There are dozens of hidden surface algorithms.

They can be characterized in at lease three ways:

- Object-precision vs. image-precision (a.k.a., object-space vs. image-space)
- Object order vs. image order
- Sort first vs. sort last



## Object-precision algorithms

Basic idea:

- Operate on the geometric primitives themselves. (We'll use "object" and "primitive" interchangeably.)
- Objects typically intersected against each other
- Tests performed to high precision
- Finished list of visible objects can be drawn at any resolution

Complexity:

- For n objects, can take $O(n^2)$ time to compute visibility.
- For an *mxm* display, have to fill in colors for $m^2$ pixels.
- Overall complexity can be $O(k_{obj} n^2 + k_{disp} m^2)$.

Implementation:

- Difficult to implement
- Can get numerical problems

## Image-precision algorithm

Basic idea:

- Find the closest point as seen through each pixel
- Calculations performed at display resolution
- Does not require high precision

Complexity:

- Naïve approach checks all n objects at every pixel. Then, $O(n\, m^2)$.
- Better approaches check only the objects that *could* be visible at each pixel. Let's say, on average, *d* objects are visible at each pixel (a.k.a., depth complexity). Then, $O(d\, m^2)$.

Implementation:

- Very simple to implement.
  - Used a lot in practice.



## Object order vs. image order

Object order:

- Consider each object only once, draw its pixels, and move on to the next object.
- Might draw the same pixel multiple times.

Image order:

- Consider each pixel only once, find nearest object, and move on to the next pixel.
- Might compute relationships between objects multiple times.



## Sort first vs. sort last

Sort first:

- Find some depth-based ordering of the objects relative to the camera, then draw back to front.
- Build an ordered data structure to avoid duplicating work.

Sort last:

- Sort implicitly as more information becomes available.



## Outline of Lecture

- Z-buffer
- Ray casting
- Binary space partitioning (BSP) trees

# Z-buffer

The **Z-buffer**' or **depth buffer** algorithm [Catmull, 1974] is probably the simplest and most widely used.

Here is pseudocode for the Z-buffer hidden surface algorithm:

```
for each pixel (i,j) do
    Z-buffer [i,j] ← FAR
    Framebuffer[i,j] ← <background color>
end for
for each polygon A do
    for each pixel in A do
        Compute depth z and shade s of A at (i,j)
        if z > Z-buffer [i,j] then
            Z-buffer [i,j] ← z
            Framebuffer[i,j] ← s
        end if
    end for
end for
```

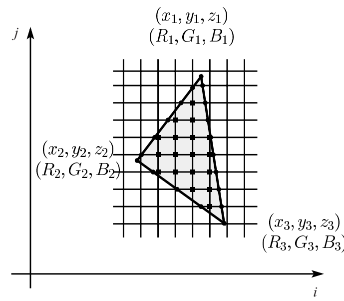
**Q**: What should FAR be set to?



# Z-buffer, cont'd

The process of filling in the pixels inside of a polygon is called **rasterization**.

During rasterization, the *z* value and shade *s* can be computed incrementally (fast!).




<u>Curious fact:</u>

- Described as the "brute-force image space algorithm" by [SSS]
- Mentioned only in Appendix B of [SSS] as a point of comparison for <u>huge</u> memories, but written off as totally impractical.

Today, Z-buffers are commonly implemented in hardware.

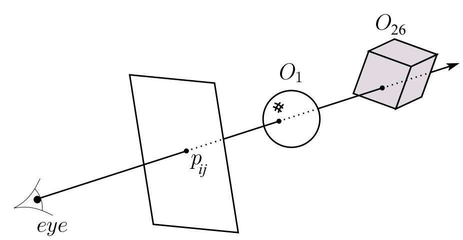


# Z-buffer: Analysis

- Classification?
- Easy to implement?
- Easy to implement in hardware?
- Incremental drawing calculations (uses coherence)?
- Pre-processing required?
- On-line (doesn't need all objects before drawing begins)?
- If objects move, does it take extra work than normal to draw the frame?
- If the viewer moves, does it take extra work than normal to draw the frame?
- Typically polygon-based?
- Efficient shading (doesn't compute colors of hidden surfaces)?
- Handles transparency?
- Handles refraction?



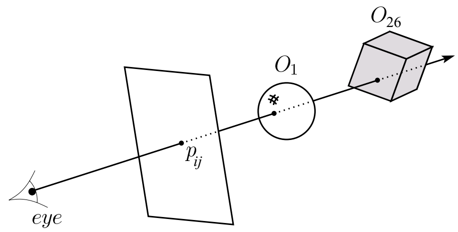
# Ray casting




<u>Idea:</u> For each pixel center $\boldsymbol{P}_{ij}$

- Send ray from eye point (COP), **c**, through $\boldsymbol{P}_{ij}$ into scene.
- Intersect ray with each object.
- Select nearest intersection.

# Ray casting, cont.




<u>Implementation:</u>

- Might parameterize each ray:

  $\mathbf{r}(t) = \mathbf{c} + t\,(\boldsymbol{P}_{ij} - \mathbf{c})$

- Each object $O_k$ returns $t_k > 1$ such that first intersection with $O_k$ occurs at $\mathbf{r}(t_k)$.

**Q**: Given the set $\{t_k\}$ what is the first intersection point?

Note: these calculations generally happen in <u>world</u> coordinates.



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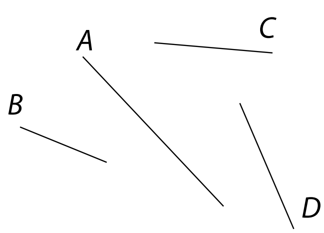
# Binary-space partitioning (BSP) trees




<u>Idea:</u>

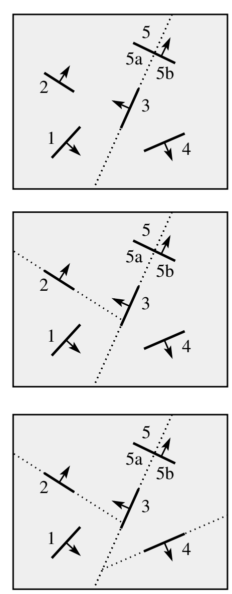
- Do extra preprocessing to allow quick display from <u>any</u> viewpoint.

<u>Key observation:</u> A polygon *A* is painted in correct order if

- Polygons on far side of *A* are painted first
- P is painted next
- Polygons in front of *A* are painted last.



# BSP tree creation



## BSP tree creation (cont'd)

**procedure** *MakeBSPTree*:
**takes** *PolygonList L*
**returns** *BSPTree*
  Choose polygon *A* from *L* to serve as root
  Split all polygons in *L* according to *A*
  node ← *A*
  *node.neg* ← *MakeBSPTree*(Polygon on neg. side of A)
  *node.pos* ← *MakeBSPTree*(Polygon on pos. side of A)
  **return** node
**end** procedure

Note: Performance is improved when fewer polygons are split --- in practice, best of ~ 5 random splitting polygons are chosen.

Note: BSP is created in *world* coordinates.



## BSP tree display

**procedure** *DisplayBSPTree:*
**Takes** *BSPTree T*
  **if** *T* is empty **then return**
  **if** viewer is in front (on pos. side) of *T.node*
    *DisplayBSPTree(T.______ )*
    *Draw T.node*
    *DisplayBSPTree|(T.______)*
  **else**
    *DisplayBSPTree(T.______)*
    *Draw T.node*
    *DisplayBSPTree(T.______)*
  **end if**
**end procedure**



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## Visibility tricks for Z-buffers

Z-buffering is ***the*** algorithm of choice for hardware rendering, so let's think about how to make it run as fast as possible…

What is the cost of the Z-buffer algorithm?

What can we do to decrease the constants?

## Summary

What to take home from this lecture:

- Classification of hidden surface algorithms
- Understanding of Z-buffer, ray casting, and BSP tree hidden surface algorithms
- Familiarity with some Z-buffer acceleration strategies