CSE 591 # Practice problems for Quiz #1 **November 2, 1999**

Problem 1 – Hierarchical Modeling

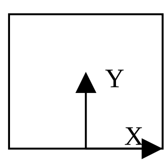
Suppose you want to model a walking table as shown below. There are two primitives available to you: *square* and *foot.* The local coordinates for each primitive are shown.

The following transformation expressions are available to you:

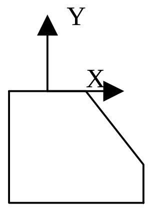
- **R(θ) –** rotate by θ degrees (counter clockwise).
- **T(t_x, t_y) –** translate by t_x, t_y
- **S(s_x, s_y) –** scale s_x and s_y in the x and y directions
- **R_Y** – reflect through the y-axis

Primitives

square

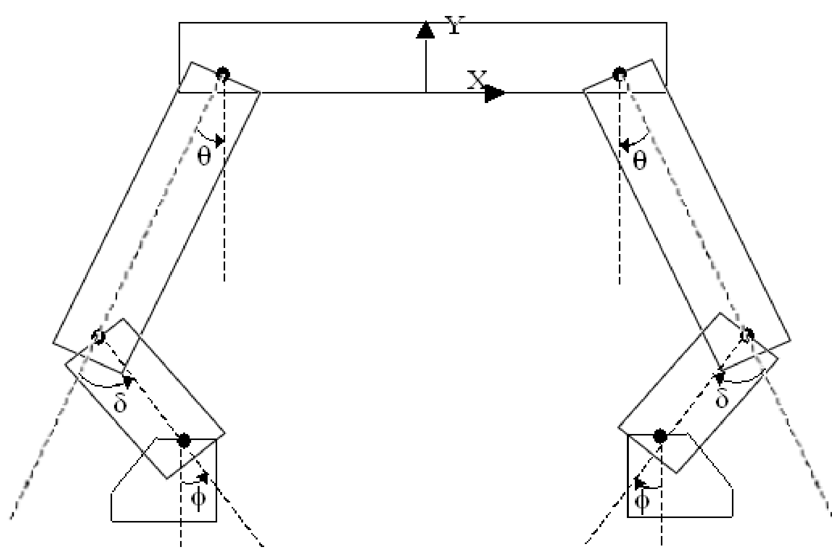


foot



Walking Table Diagram

Problem 1 (continued)



1a) On the diagram on the previous page, label the different pieces of the walking table. Now, draw a tree hierarchy to specify the walking table. Use your naming convention for the nodes and label the transformations along the edges of the tree.

1b) Now write expressions for each of these edge transformations using only the transformation expressions given on the previous page. Leave θ, ϕ, and δ as symbols; these are the parameters that you intend to animate. All other parameters should be definite numbers that you can estimate from the figure. Assume the *square* is one unit in size.

Problem 1 (continued)

1c) What is the entire sequence of transformations applied to the foot on the left hand side of the figure?

1d) Redraw your hierarchy using a directed acyclic graph (DAG) to simplify the specification.

Problem 2 – 2D Transformations

As discussed in class, any two-dimensional affine transformation can be represented as a 3X3 matrix. Here are some useful matrices:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B(a,b) = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \qquad C(a,b) = \begin{pmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$D(a,b) = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad E = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad F(a) = \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2a) Which of the matrices above implements each of the following transformations?

Rotation about the origin

Reflection through the line y=x

Translation

Shearing

Differential (Non-Uniform) Scaling

Reflection through the x-axis

2b) Show that the transformation matrix for reflecting about the line y = -x is equivalent to a reflection relative to the y-axis followed by a counter-clockwise rotation of 90º.

Problem 3- Projections

Projections in computer graphics can be broken down into two major types: “perspective projections” and “parallel projections”.

a) Complete the following table, summarizing properties of these two types of projections.

PROPERTY	PERSPECTIVE	PARALLEL
Parallel lines remain parallel [Y/N]:	___________	___________
Angles are preserved [Y/N]:	___________	___________
Lengths vary with distance to eye [Y/N]:	___________	___________

Under perspective projections, any set of parallel lines that are not parallel to the projection plane will converge to a “vanishing point”. Vanishing points of lines parallel to a principal axis *x*, *y*, or *z* are called “principal vanishing points”.

b) How many different vanishing points can a perspective drawing have?

c) How many different <u>principal</u> vanishing points can a perspective drawing have?

Problem 4 - Hidden Surface Algorithms

The Z-buffer algorithm can be improved by using an image space "Z-pyramid." The basic idea of the Z-pyramid is to use the original Z-buffer as the finest level in the pyramid, and then combine four Z-values at each level into one Z-value at the next coarser level by choosing the farthest (largest) Z from the observer. Every entry in the pyramid therefore represents the farthest (largest) Z for a square area of the Z-buffer.

a) At the coarsest level of the pyramid there is just a single Z value. What does that Z value represent?

Suppose we wish to test the visibility of a polygon P. Let Z_P be the nearest (smallest) Z value of polygon P. Let R be the smallest region in the Z-pyramid that completely covers polygon P, and let Z_R be the Z value that is associated with region R in the Z-pyramid.

b) What can we conclude if $Z_R < Z_P$?

c) What can we conclude if $Z_P < Z_R$?

d) In class, we asked a series of questions about the Z-buffer algorithm. Answer those same questions for the ray casting and BSP tree algorithms.