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Study sheet 2: curves

Problem 1 A Bézier curve of degree n, which (for the purposes of this problem) we'll donate by $Q^n(u)$, can be defined in terms of the locations of its $n+1$ control points $\{V_0, \ldots, V_n\}$:

$$Q^n(u) \quad = \quad \sum_{i=0}^{n} V_i \begin{pmatrix} n \\ i \end{pmatrix} u^i (1-u)^{n-i}$$

a) Use de Casteljau's algorithm to find the (approximate) position of the Bézier curves $Q^3(u)$ and $Q^4(u)$ defined by the two control polygons below at $u = 1/3$:

<u>True or false:</u>

b) Every Bézier curve $Q^1(u)$ is a line segment (assuming no repeated control points).

c) Every Bézier curve $Q^2(u)$ lies in a plane.

d) Moving one control point on a Bézier curve generally changes the whole curve.

Problem 2 More complex curves can be designed by piecing together different Bézier curves to make mathematical "splines." Two popular splines are the B-spline and the Catmull-Rom spline. If $\{B_0, B_1, B_2, B_3\}$ and $\{C_0, C_1, C_2, C_3\}$ are cubic B-spline and Catmull-Rom spline control points, respectively, then the corresponding Bézier control points $\{V_0, V_1, V_2, V_3\}$ can be constructed by the following identity:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 0 & 6 & 0 & 0 \\ -1 & 6 & 1 & 0 \\ 0 & 1 & 6 & -1 \\ 0 & 0 & 6 & 0 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

True or false:

a) B-splines and Catmull-Rom splines both have C^2 continuity.

b) Neither B-splines nor Catmull-Rom splines interpolate their control points.

c) B-splines and Catmull-Rom splines both provide local control.

Problem 2 (continued)

d) The points $\{B_0, B_1, B_2, B_3\}$ below are control points for a cubic B-spline. Construct, as carefully as you can on the diagram below, the Bézier control points $\{V_0, V_1, V_2, V_3\}$ corresponding to the same curve.

e) The points $\{C_0, C_1, C_2, C_3\}$ below are control points for a cubic Catmull-Rom spline. Construct, as carefully as you can on the diagram below, the Bézier control points $\{V_0, V_1, V_2, V_3\}$ corresponding to the same curve.

Problem 3

In class, we described the process of creating subdivision curves by starting with a sequence of splitting and averaging steps, followed by an evaluation mask that sends points to their limit positions. Let's assume that we have the following averaging mask:

$$(r_{-1}, r_0, r_1) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$$

Starting with the control polygon below:

1. Insert the vertices that correspond to the splitting step and label each with an **S**.
2. Apply the averaging mask, indicate the new vertex positions, and label each with an **A**.
3. Indicate the resulting polygon with solid lines.