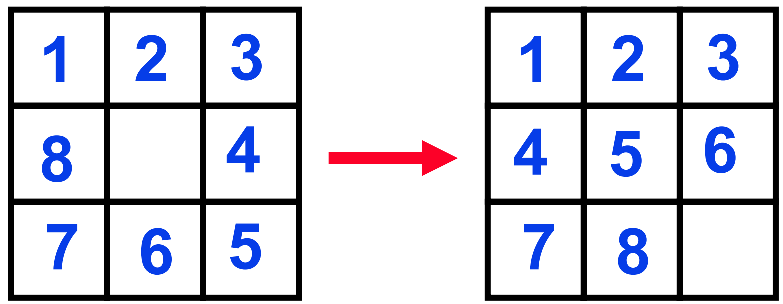
CSEP 573

Chapters 3-5

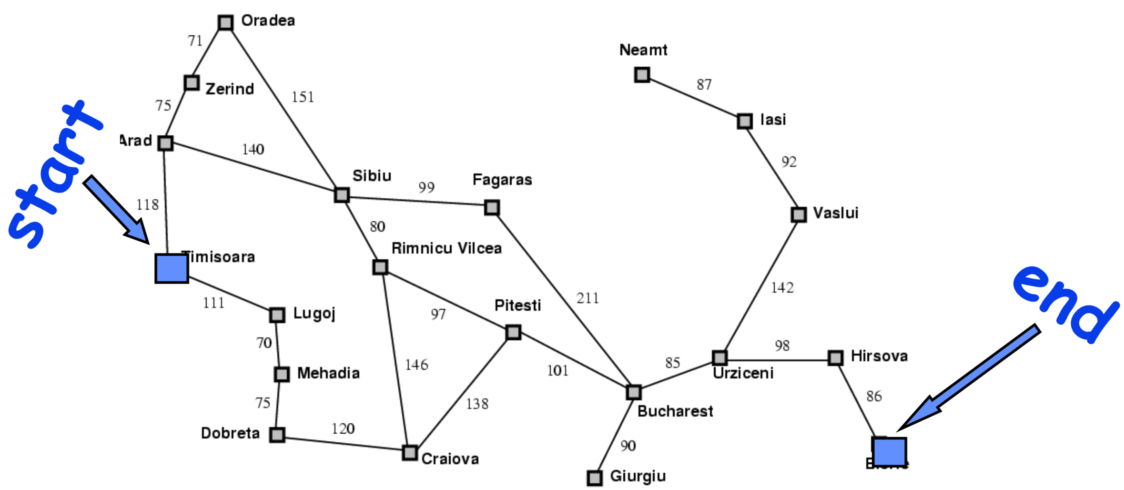
Problem Solving using Search

"First, they do an on-line search"

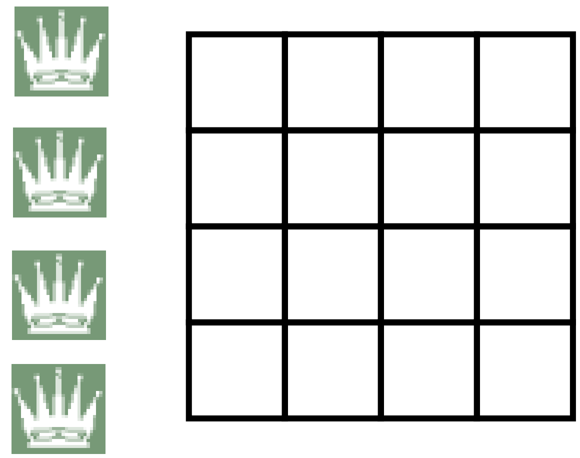
Example: The 8-puzzle



Example: Route Planning

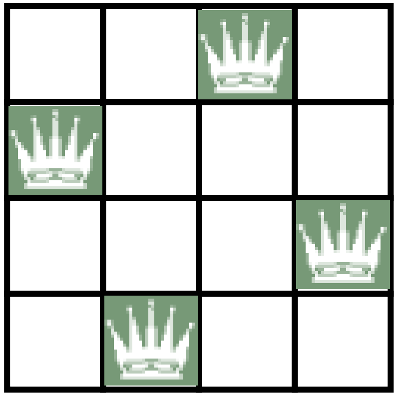


Example: N Queens



4 Queens

Example: N Queens



4 Queens

State-Space Search Problems

General problem:

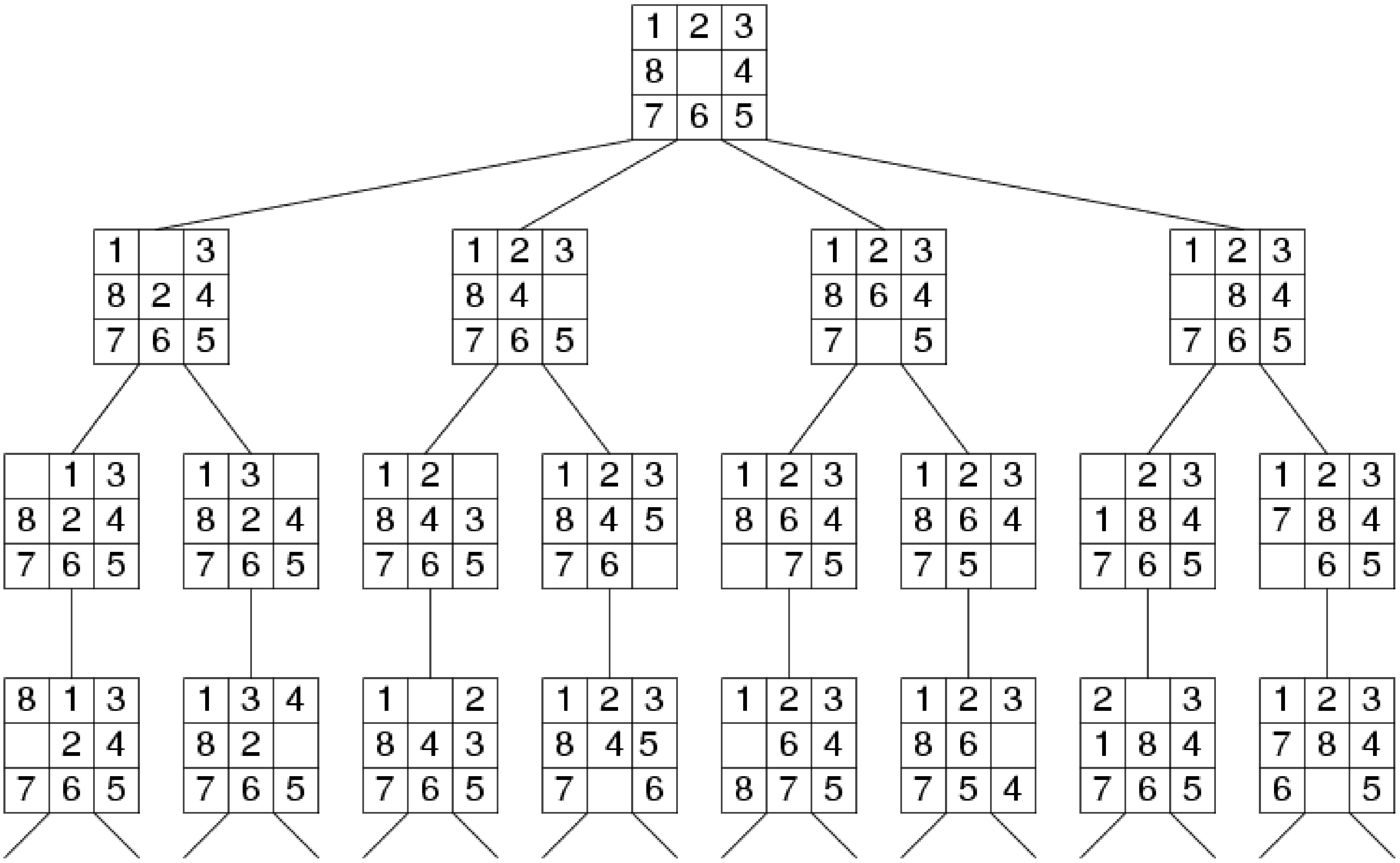
Given a *start state*, find a path to a *goal state*

- **Can test if a state is a goal**
- **Given a state, can generate its *successor* states**

Variants:

- **Find any path *vs.* a least-cost path**
- **Goal is completely specified, task is just to find the path**
  - **Route planning**
- **Path doesn't matter, only finding the goal state**
  - **8 puzzle, N queens**

Tree Representation of 8-Puzzle Problem Space



Implementation: general tree search

function TREE-SEARCH(*problem*, *fringe*) **returns** a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
    loop do
        if *fringe* is empty **then return** failure
        node ← REMOVE-FRONT(*fringe*)
        if GOAL-TEST[*problem*] applied to STATE(*node*) succeeds **return** *node*
        fringe ← INSERTALL(EXPAND(*node*, *problem*), *fringe*)

fringe (= frontier in the textbook*)* is the set of all leaf nodes available for expansion

Implementation: general tree search

function TREE-SEARCH(*problem, fringe*) **returns** a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
  loop do
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    node ← REMOVE-FRONT(*fringe*)
    if GOAL-TEST[*problem*] applied to STATE(*node*) succeeds **return** *node*
    fringe ← INSERTALL(EXPAND(*node, problem*), *fringe*)

---

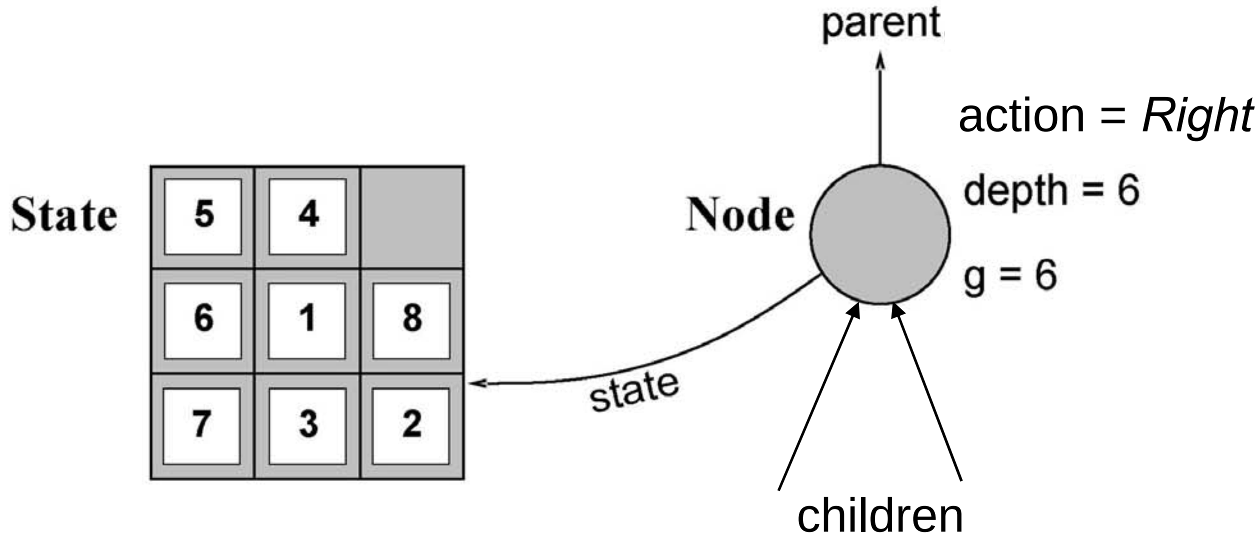
function EXPAND(*node, problem*) **returns** a set of nodes
  successors ← the empty set
  for each *action, result* **in** SUCCESSOR-FN[*problem*](STATE[*node*]) **do**
    s ← a new NODE
    PARENT-NODE[*s*] ← *node*; ACTION[*s*] ← *action*; STATE[*s*] ← *result*
    PATH-COST[*s*] ← PATH-COST[*node*] + STEP-COST(*node, action, s*)
    DEPTH[*s*] ← DEPTH[*node*] + 1
    add *s* to *successors*
  return *successors*

Implementation: states vs. nodes

A *state* is a (representation of) a physical configuration

A *node* is a data structure constituting part of a search tree
includes *parent*, *children*, *depth*, *path cost* $g(x)$

States do not have parents, children, depth, or path cost!



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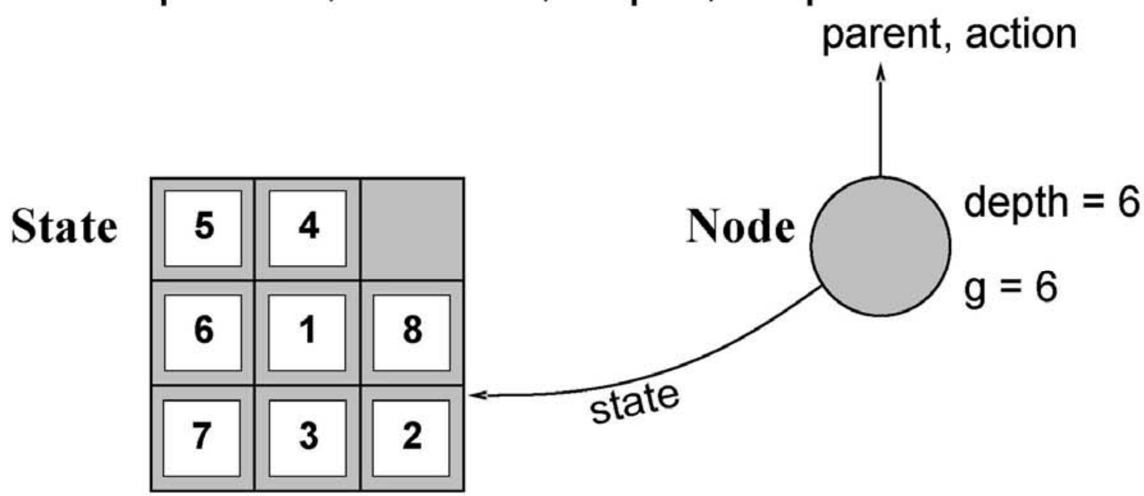


The EXPAND function creates new nodes, filling in the various fields and using the SUCCESSORFN of the problem to create the corresponding states.

Search strategies

A strategy is defined by picking the *order of node expansion*

Strategies are evaluated along the following dimensions:

completeness—does it always find a solution if one exists?
time complexity—number of nodes generated/expanded
space complexity—maximum number of nodes in memory
optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of

b—maximum branching factor of the search tree
d—depth of the least-cost solution
m—maximum depth of the state space (may be ∞)

Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

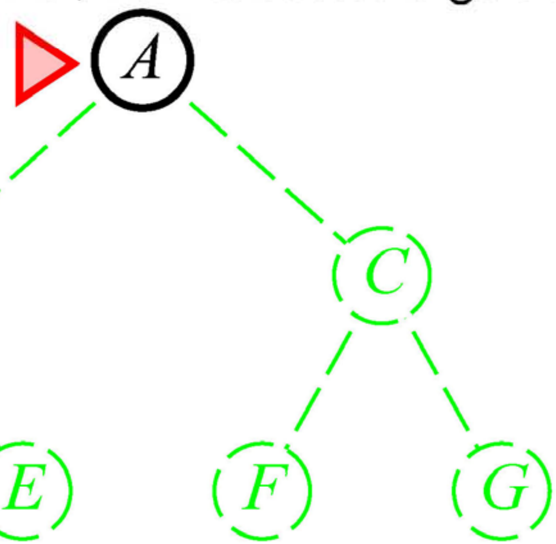
Iterative deepening search

Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end



Breadth-first search

Expand shallowest unexpanded node

Implementation:

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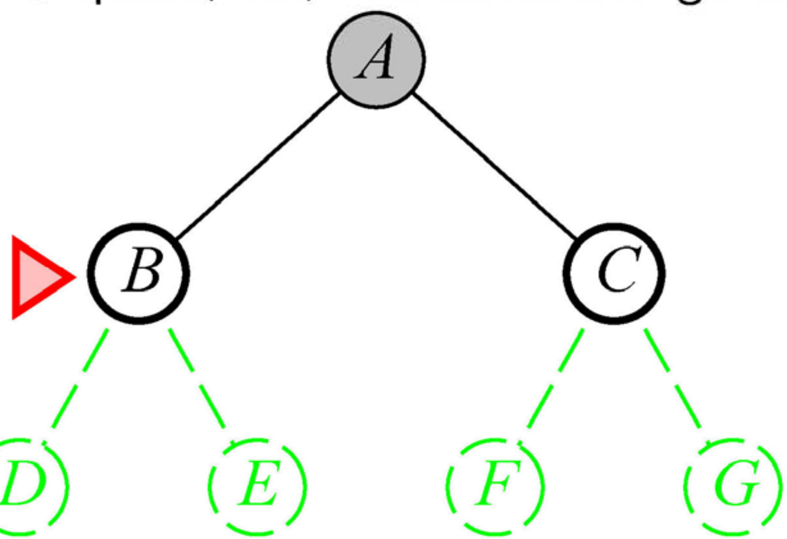

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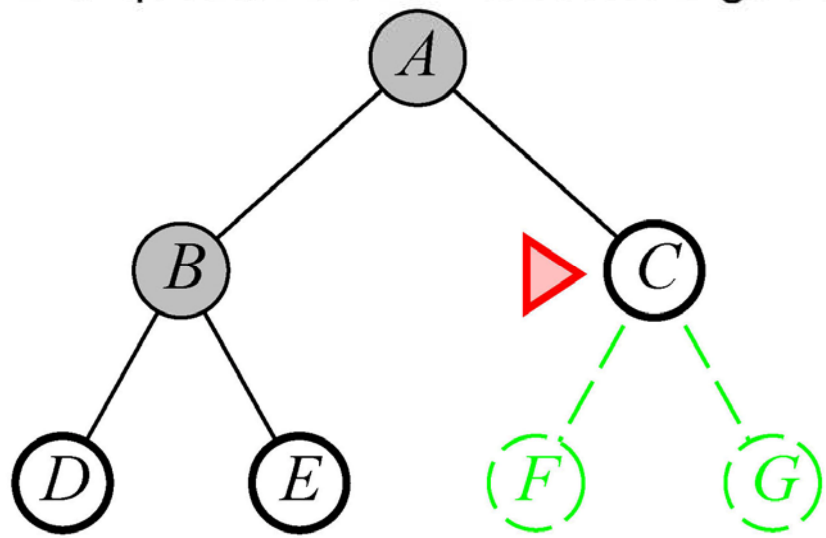

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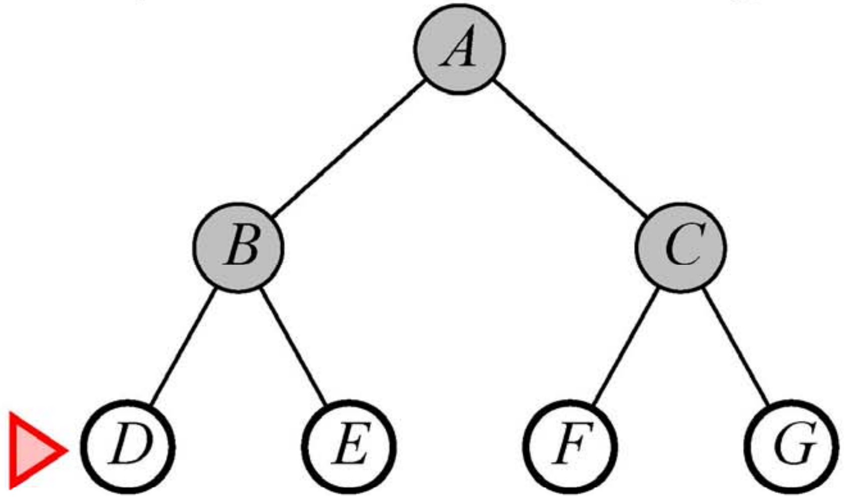


Properties of breadth-first search

Complete??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $b + b^2 + b^3 + \ldots + b^d$ = $O(b^d)$, i.e., exponential in d

Space??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $b + b^2 + b^3 + \ldots + b^d = O(b^d)$, i.e., exponential in d

Space?? $O(b^d)$ (keeps every node in memory)

Optimal??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $b + b^2 + b^3 + \ldots + b^d = O(b^d)$, i.e., exponential in d

Space?? $O(b^d)$ (keeps every node in memory)

Optimal?? Yes (if cost $= 1$ per step); not optimal in general

Space is the big problem for BFS.

Example: b = 10, 10,000 nodes/sec, 1KB/node

d = 3 → 1000 nodes, 0.1 sec, 1MB

d = 5 → 100,000 nodes, 10 secs, 100 MB

d = 9 → 10^9 nodes, 31 hours, 1 TB

Uniform-cost search

Expand least-cost unexpanded node (used when step costs are unequal)

Implementation:

$fringe$ = queue ordered by path cost (use priority queue)

Equivalent to breadth-first if step costs all equal

Complete?? Yes, if step cost $\geq \epsilon$ (small positive constant; 0 cost may cause infinite loop)

Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
where C^* is the cost of the optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$

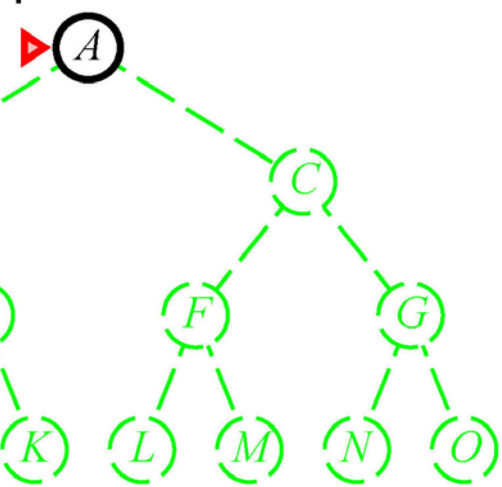
Optimal?? Yes—nodes expanded in increasing order of $g(n)$

Depth-first search

Expand deepest unexpanded node

Implementation:

$fringe$ = LIFO queue, i.e., put successors at front



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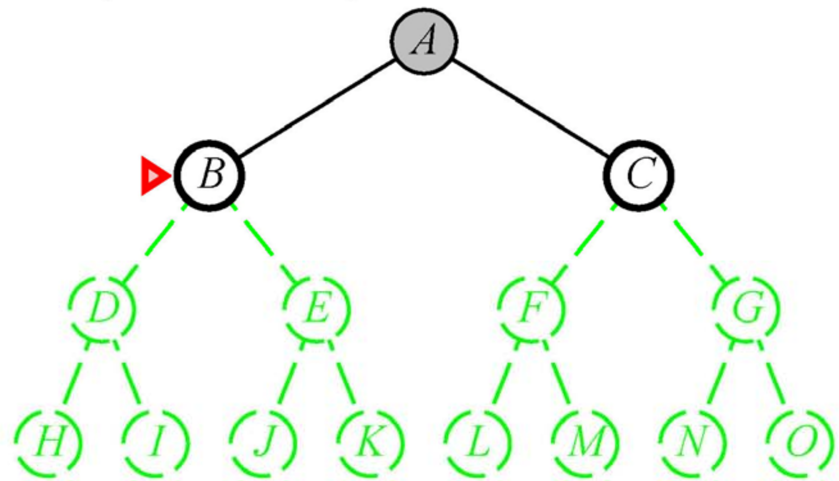

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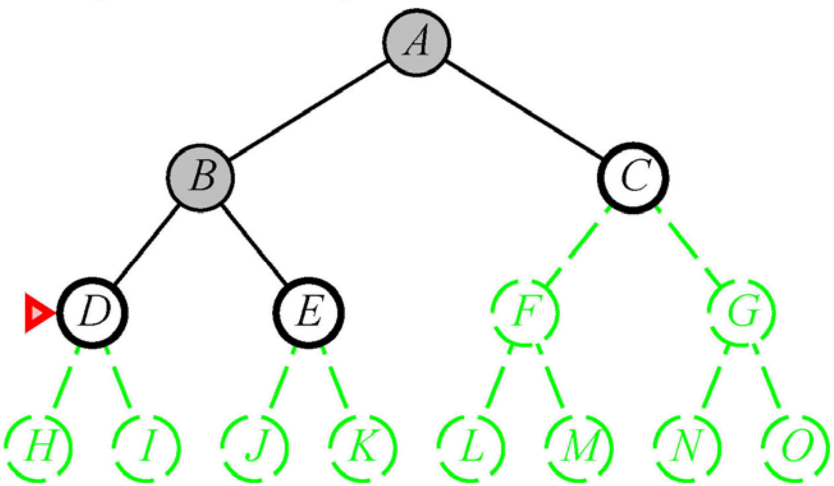

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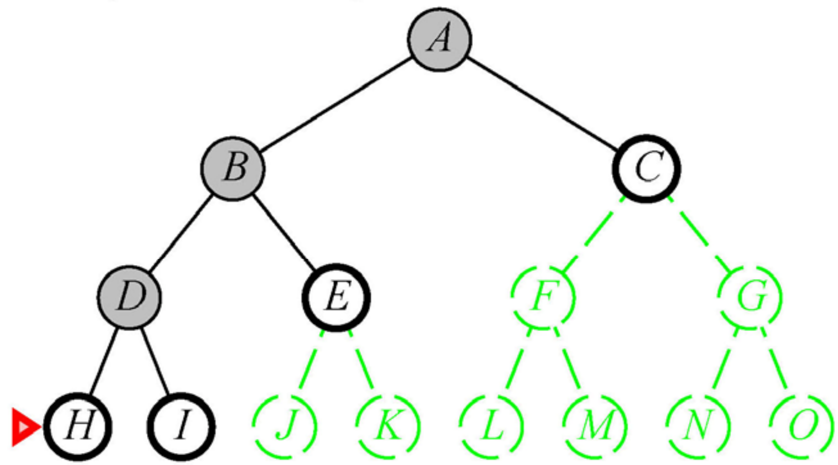

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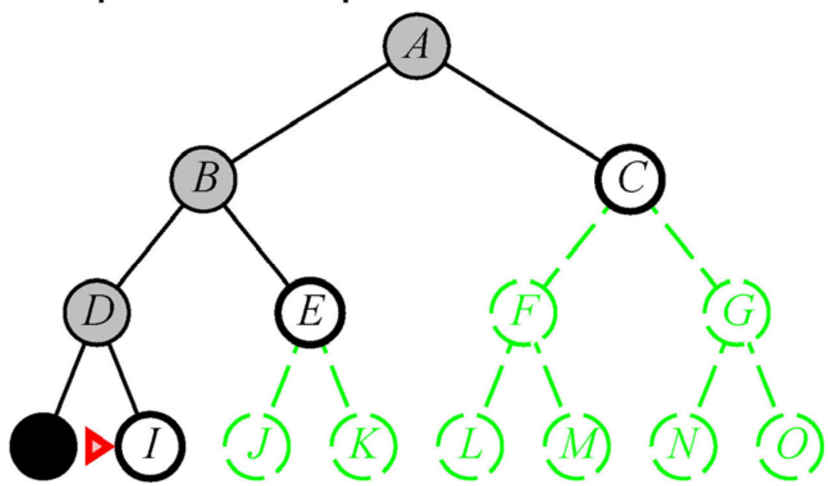

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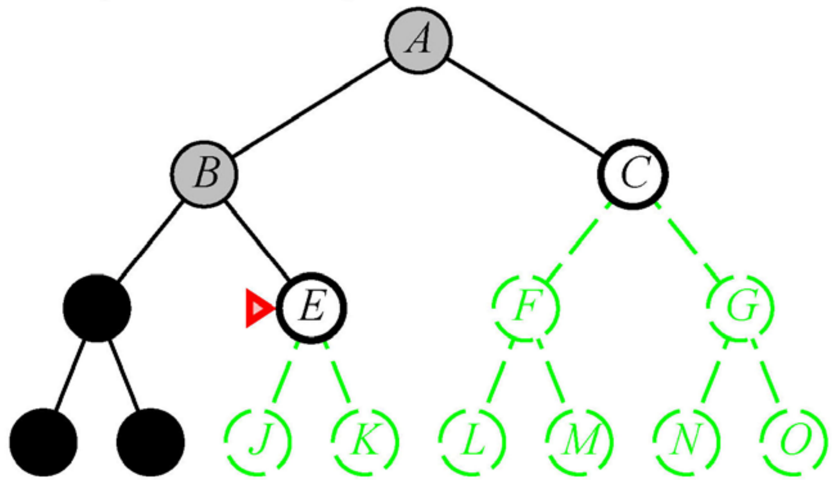

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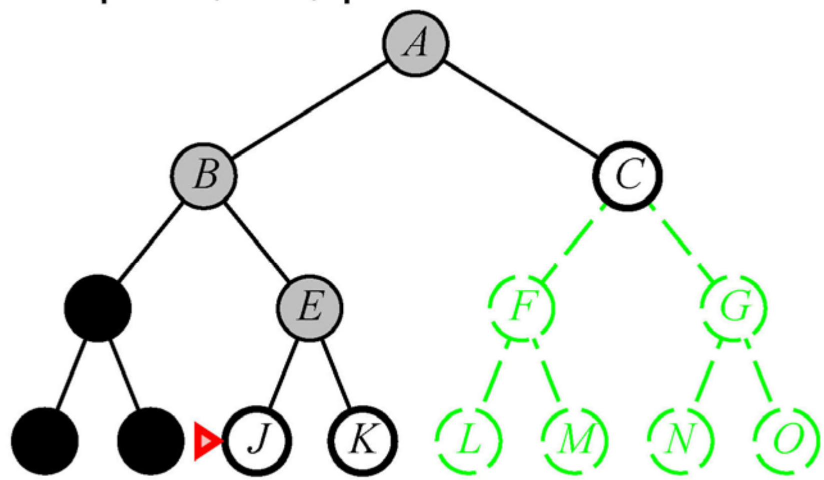

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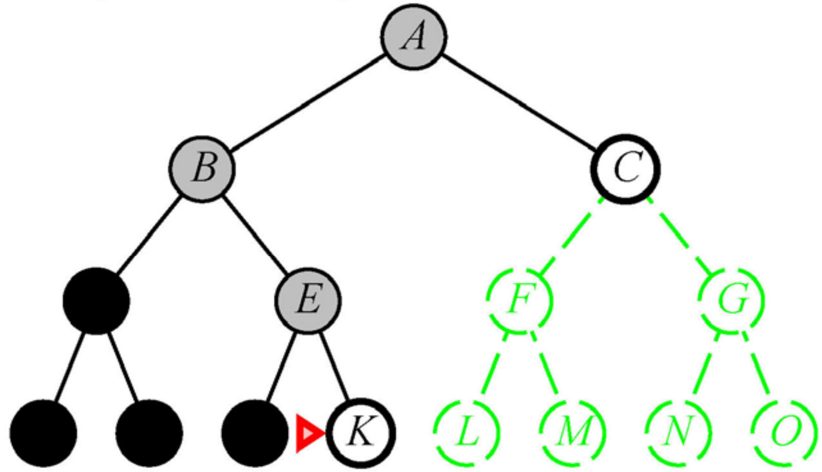

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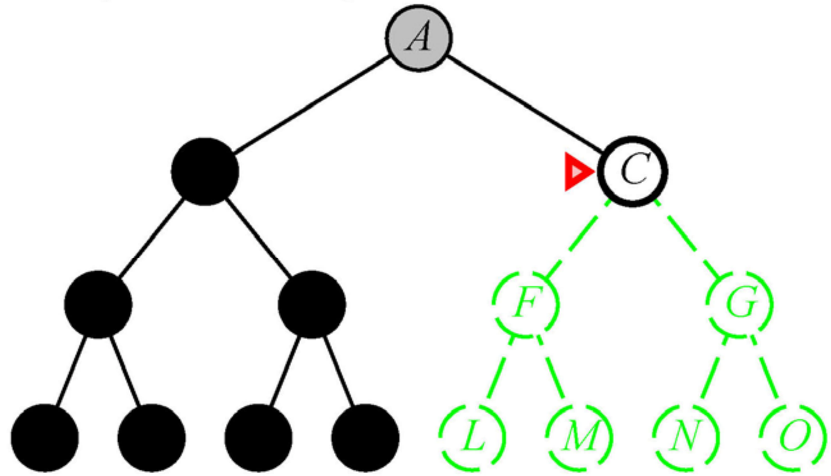

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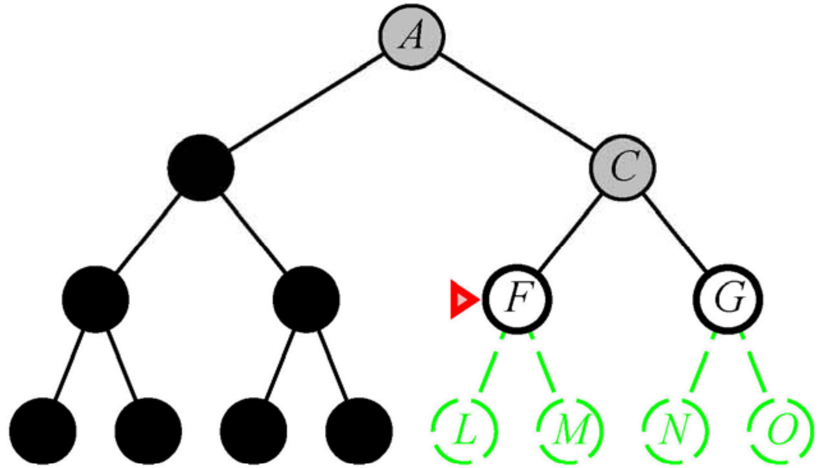

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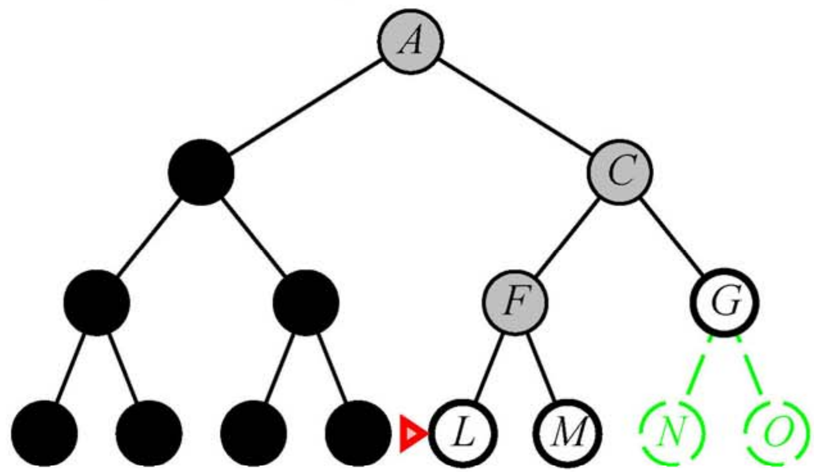

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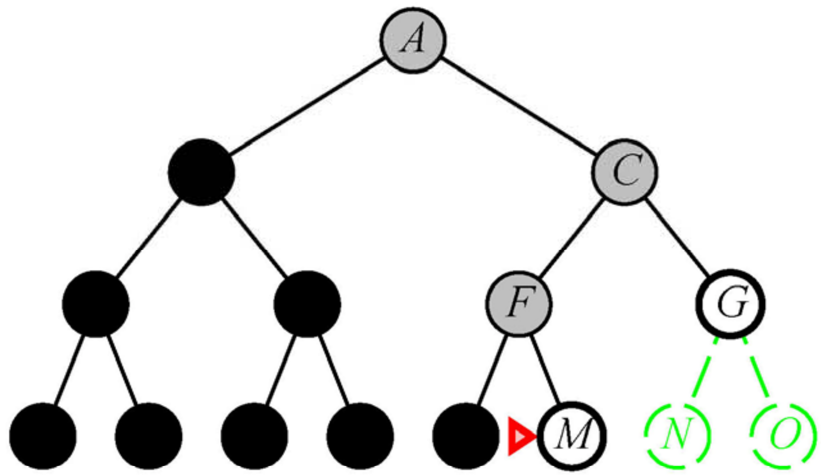


Properties of depth-first search

Complete??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path ("GRAPH-SEARCH" in textbook)
$\Rightarrow$ complete in finite spaces

Time??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path ("GRAPH-SEARCH" in textbook)
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d (m = maximum depth)
but if solutions are dense, may be much faster than breadth-first

Space??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d

but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal?? No (may find a solution but least cost solution may be on a different branch)

Depth-limited search

= depth-first search with depth limit l,
i.e., nodes at depth l have no successors

Recursive implementation:

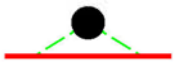
```
function DEPTH-LIMITED-SEARCH(problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST[problem](STATE[node]) then return node
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
       result ← RECURSIVE-DLS(successor, problem, limit)
       if result = cutoff then cutoff-occurred? ← true
       else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

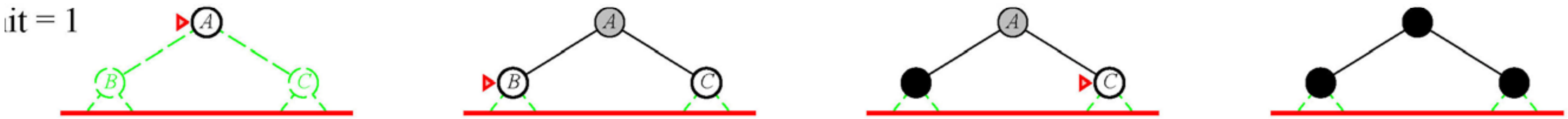
Iterative deepening search

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

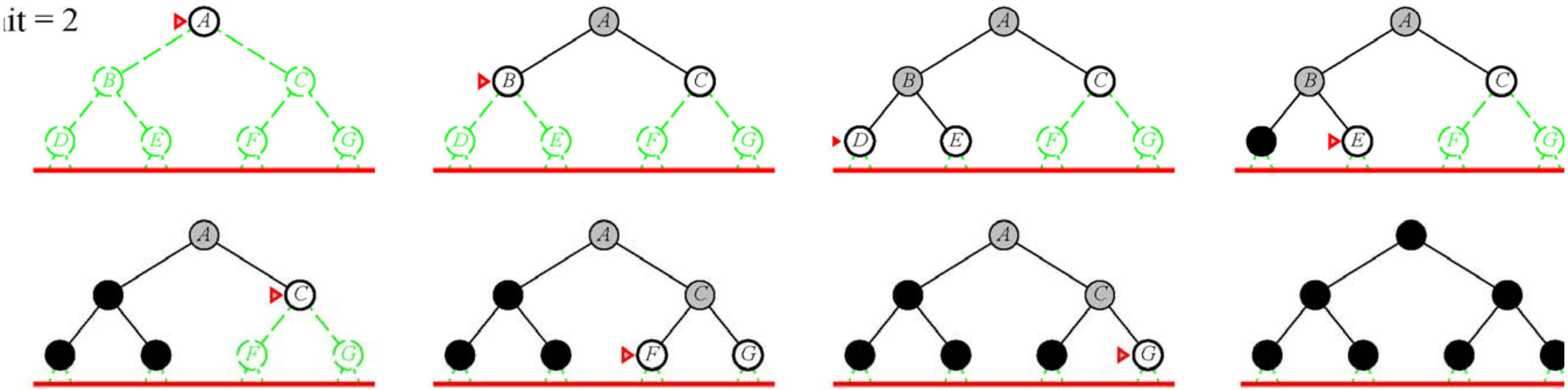
Iterative deepening search $l = 0$



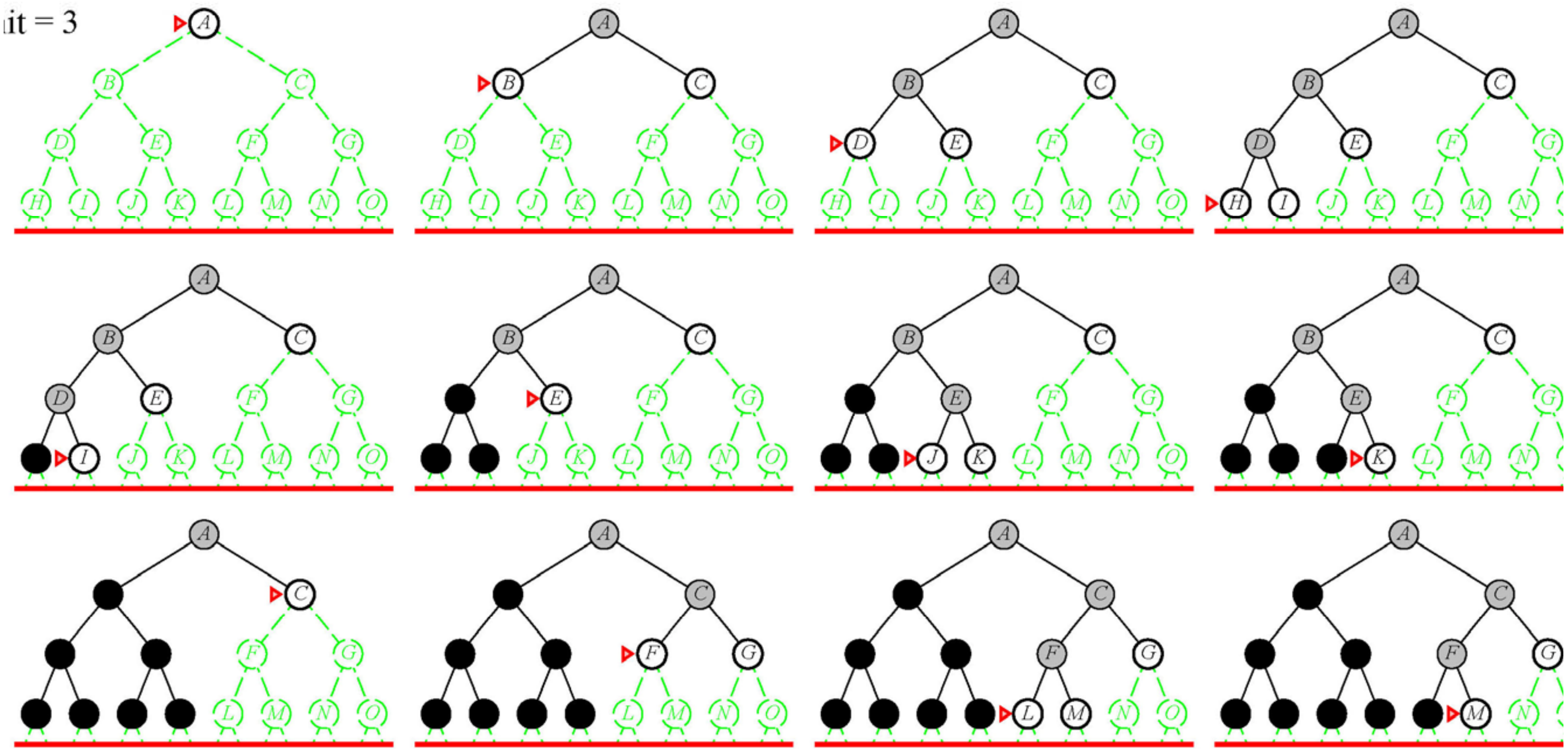
Iterative deepening search $l = 1$



Iterative deepening search $l = 2$



Iterative deepening search $l = 3$



Properties of iterative deepening search

Complete??

Properties of iterative deepening search

Complete?? Yes

Time??

Properties of iterative deepening search

Complete?? Yes

Time?? $$db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$$

Space??

Properties of iterative deepening search

Complete?? Yes

Time?? $db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal??

Properties of iterative deepening search

Complete?? Yes

Time?? $db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? Yes, if step cost $= 1$

Can be modified to explore uniform-cost tree

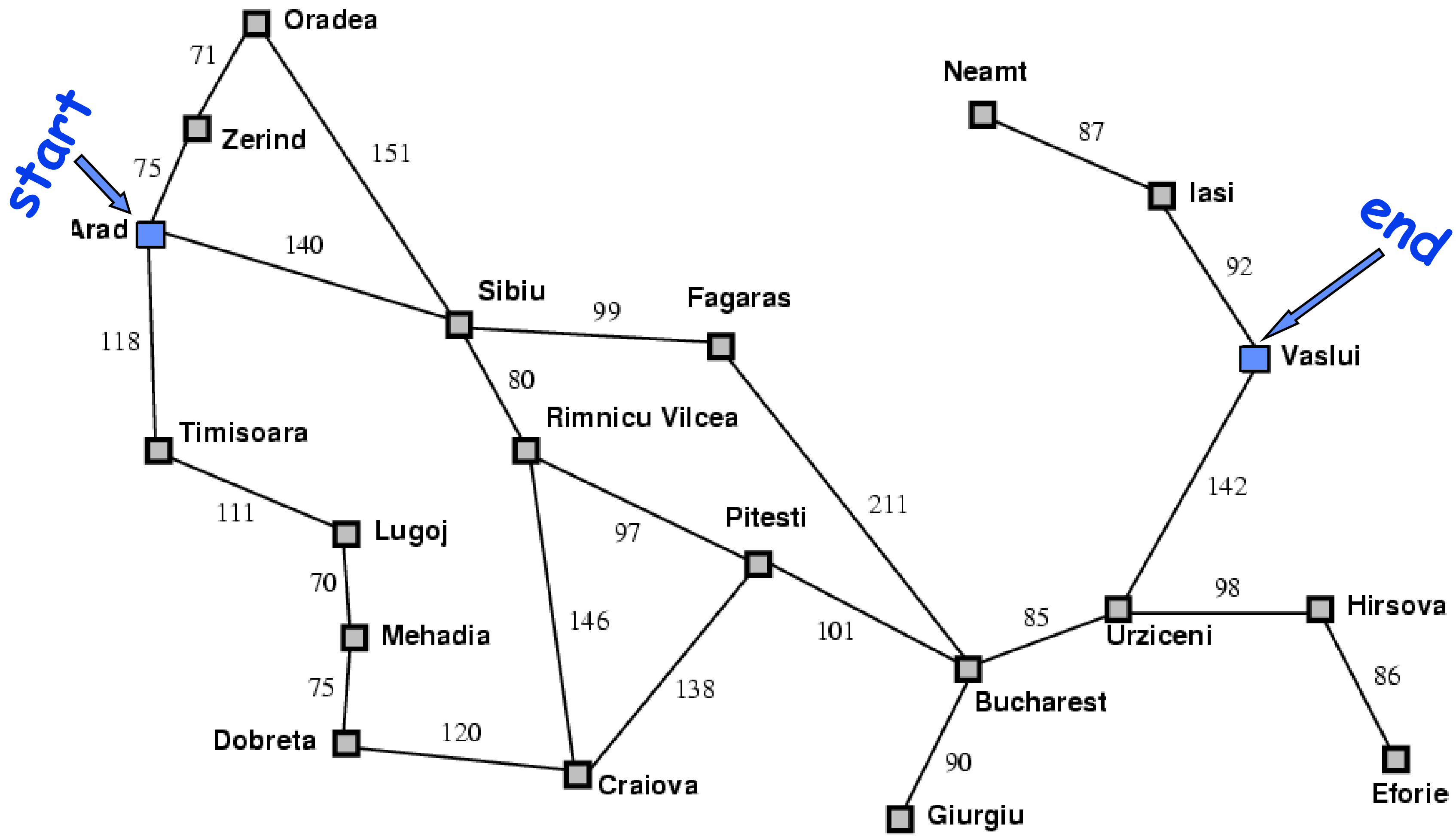
Increasing path-cost limits instead of depth limits

This is called Iterative lengthening search (exercise 3.17)

Summary of algorithms

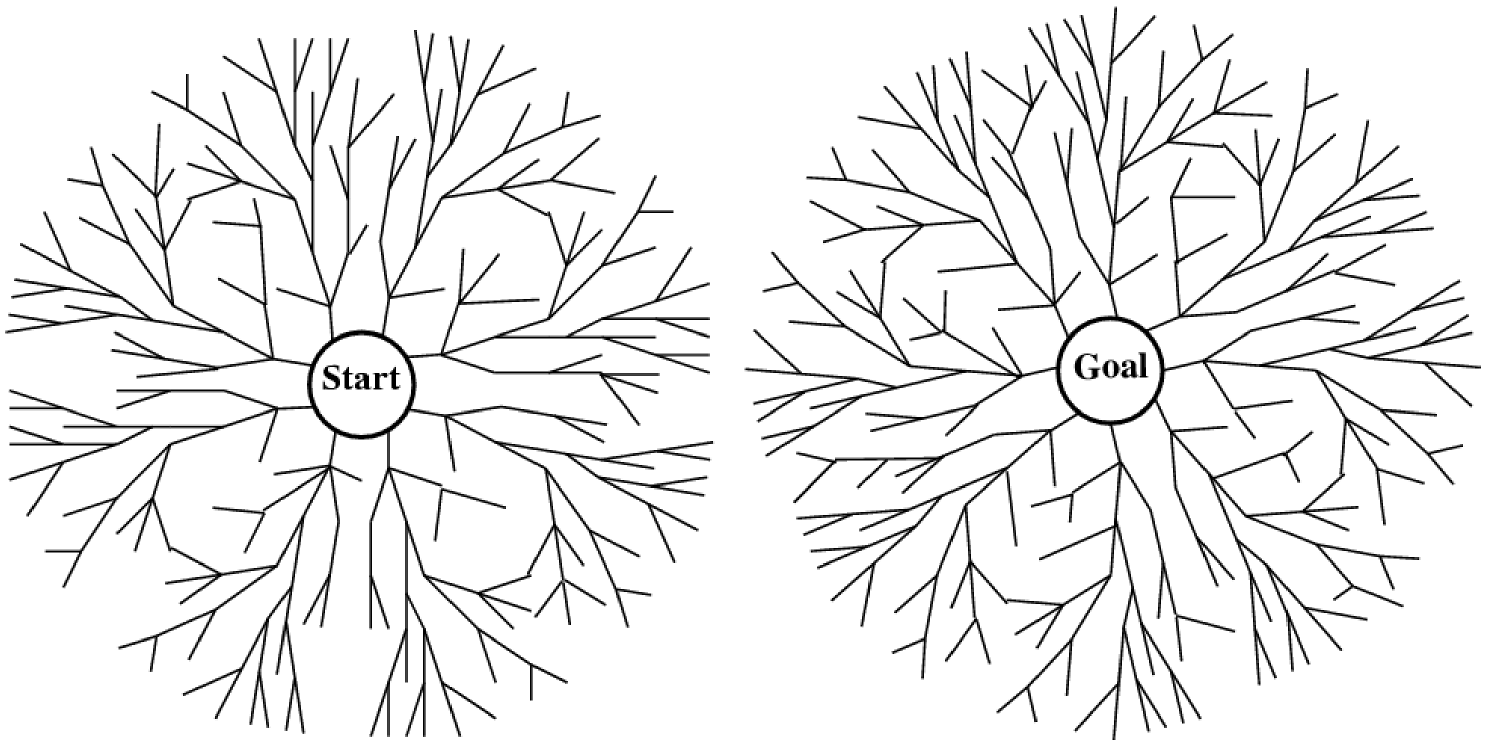
Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^d	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^d	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes*	No	No	Yes

Forwards *vs.* Backwards



Problem: Find the shortest route

Bidirectional Search



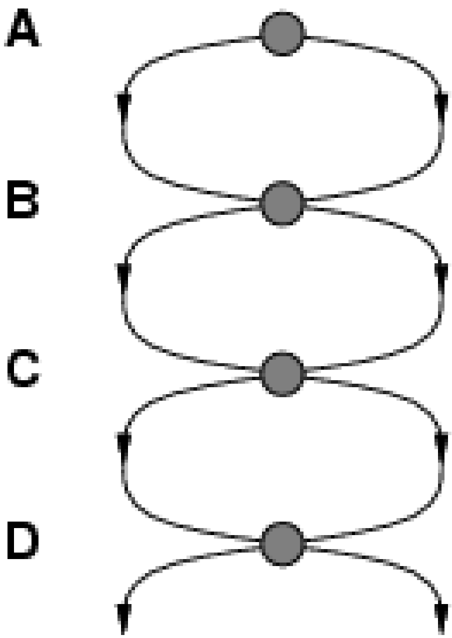
Motivation: $b^{d/2} + b^{d/2} << b^d$

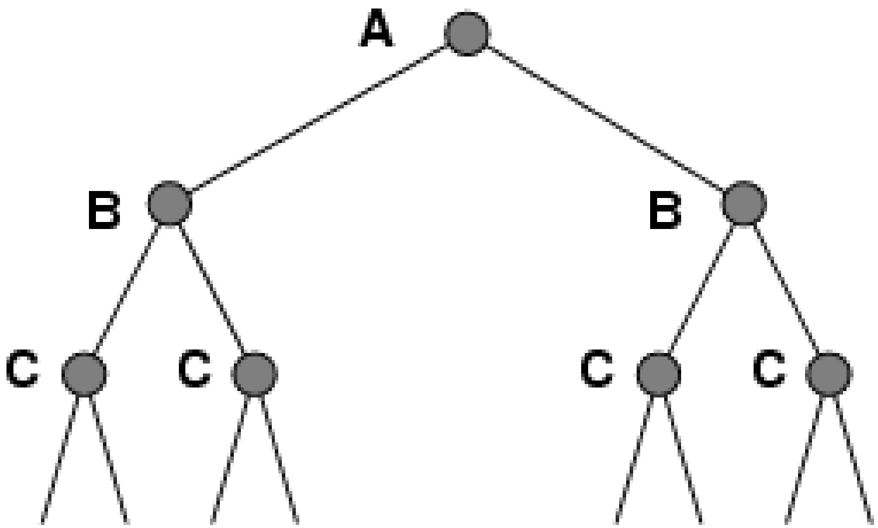
Can use breadth-first search or uniform-cost search

Hard for implicit goals e.g., goal = "checkmate" in chess

Repeated States

Failure to detect repeated states can turn a linear problem into an exponential one! (e.g., repeated states in 8 puzzle)





Graph search algorithm: Store expanded nodes in a set called *closed* (or *explored*) and only add new nodes to the fringe

Graph Search

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure

    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

All these methods are slow (blind)

Can we do better?

Informed Search

Use problem-specific knowledge to guide search (use "heuristic function")

Best-first Search

Generalization of breadth first search

Priority queue of nodes to be explored

<u>Evaluation function f(n)</u> used for each node

Insert initial state into priority queue
While queue not empty
    Node = head(queue)
    If goal(node) then return node
    Insert children of node into pr. queue

Who’s on (best) first?

Breadth first search is special case of best first

- **with f(n) = depth(n)**

Dijkstra’s Algorithm is best first

- **with f(n) = g(n)**

  where g(n) = sum of edge costs from start to n

Greedy best-first search

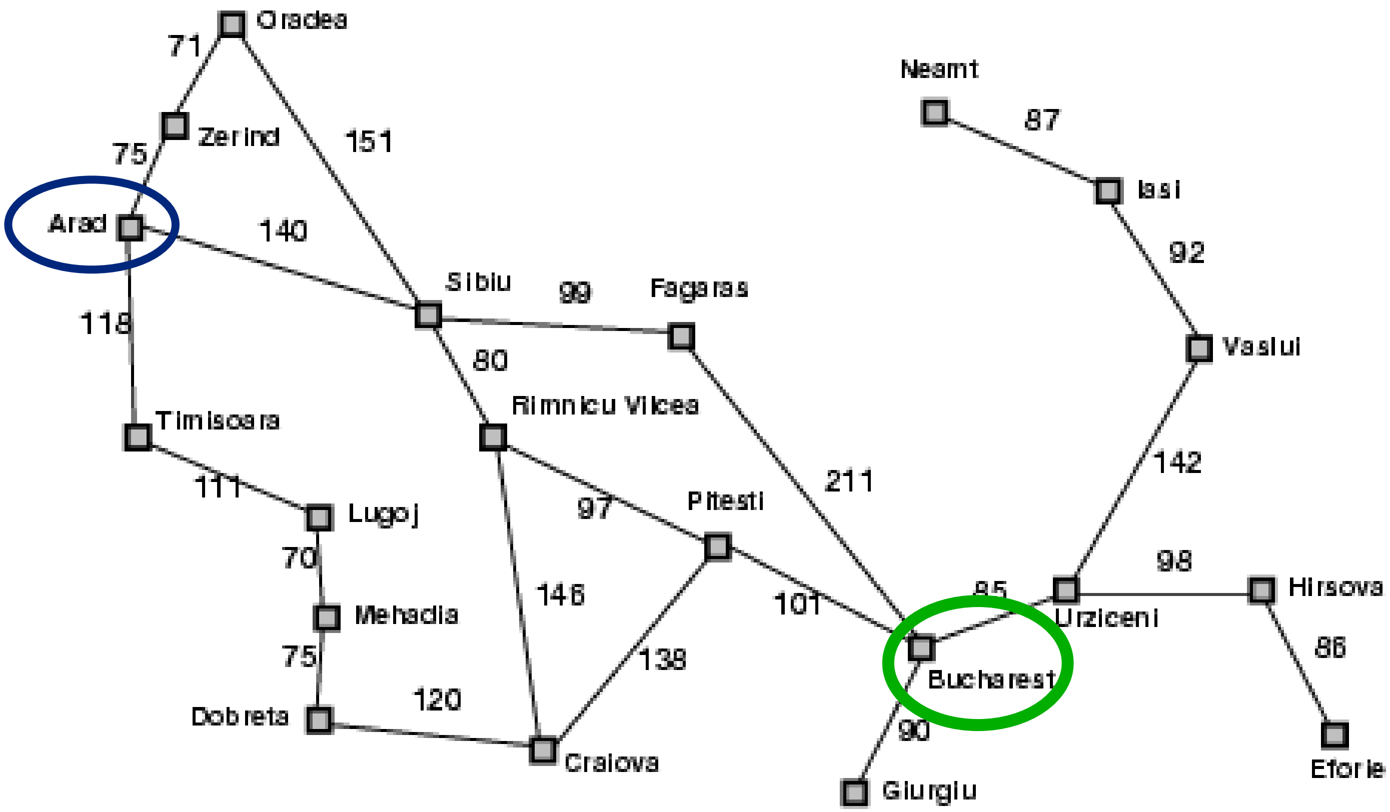
Evaluation function $f(n) = h(n)$ (**h**euristic) = estimate of cost from n to *goal*

e.g., Route finding problems: $h_{SLD}(n)$ = straight-line distance from n to destination

Greedy best-first search expands the node that **appears** to be closest to goal

Example: Lost in Romania

Need: Shortest path from Arad to Bucharest

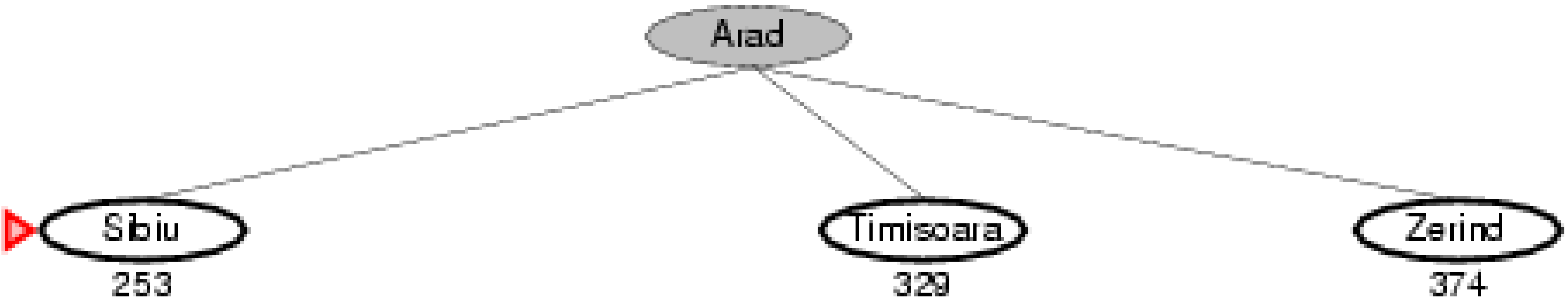


Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Example: Greedily Searching for Bucharest

$h_{SLD}(Arad)$

Example: Greedily Searching for Bucharest



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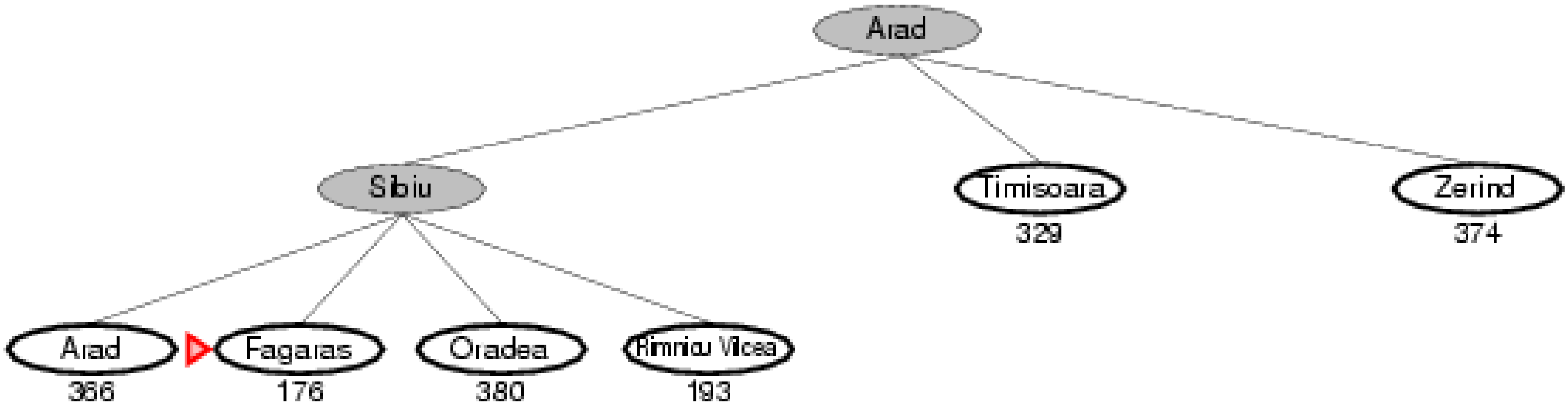

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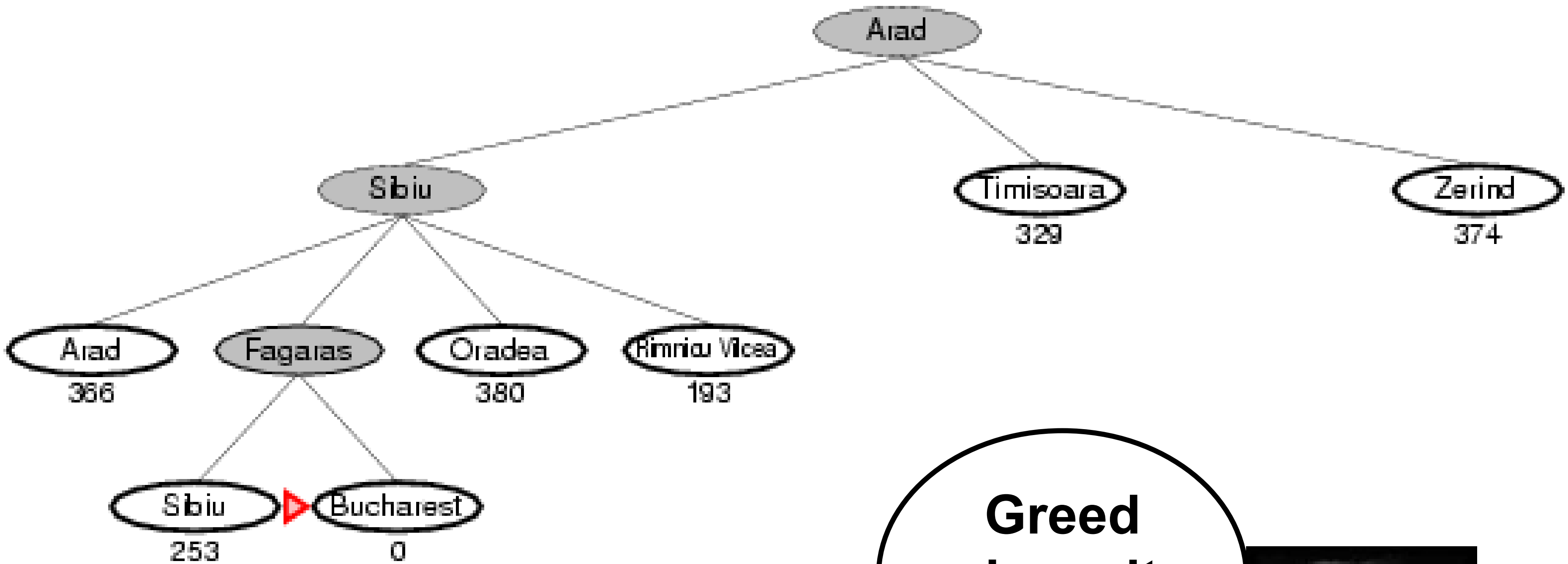


Not optimal!
Arad, Sibiu, Rimnicu Vilcea, Pitesti, Bucharest shorter

Properties of Greedy Best-First Search

Complete? **No – can get stuck in loops (unless *closed* list is used)**

Time? ***$O(b^m)$*, but a good heuristic can give dramatic improvement**

Space? ***$O(b^m)$* -- keeps all nodes in memory *a la* breadth first search**

Optimal? **No, as our example illustrated**

A* Search
(Hart, Nilsson & Rafael 1968)

- Best first search with f(n) = g(n) + h(n)

g(n) = sum of edge costs from start to n

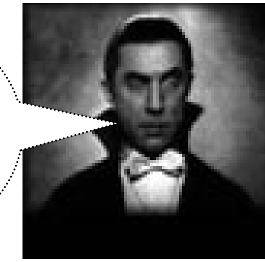
h(n) = **heuristic function** = estimate of lowest cost path from n to goal

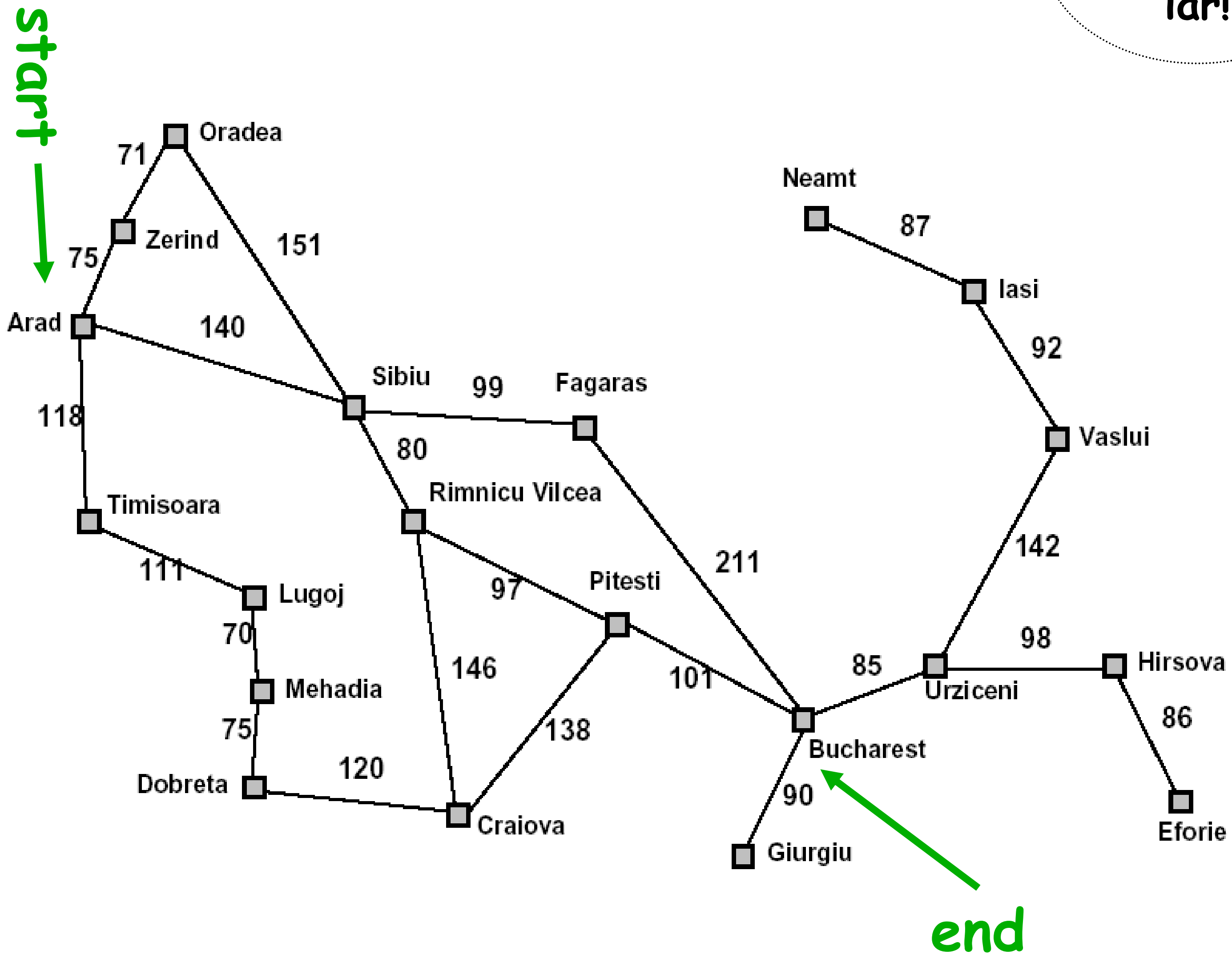
- If h(n) is "admissible" then search will be optimal

Underestimates cost
of any solution which
can be reached from node

Back in Romania Again

Aici noi energie iar!





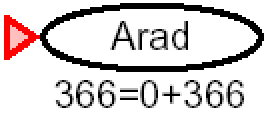
Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A* Example for Romania

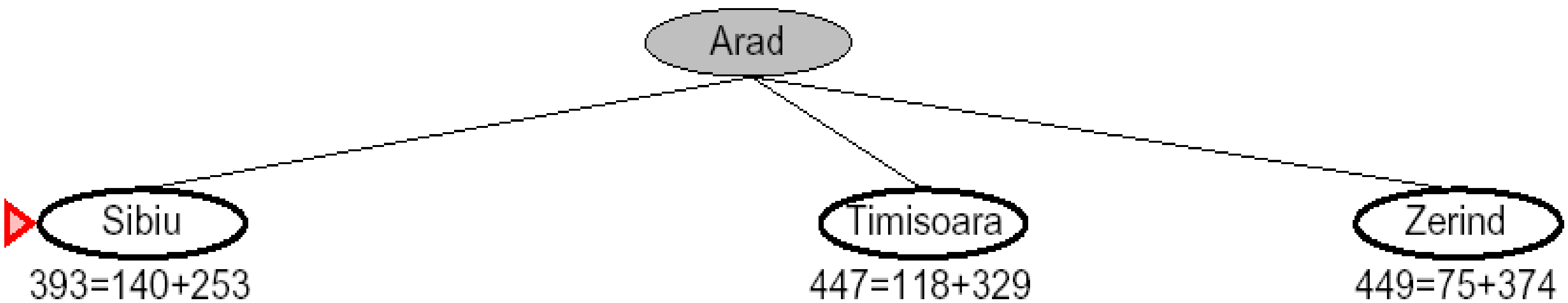
$f(n) = g(n) + h(n)$ where

$g(n)$ = sum of edge costs from start to n

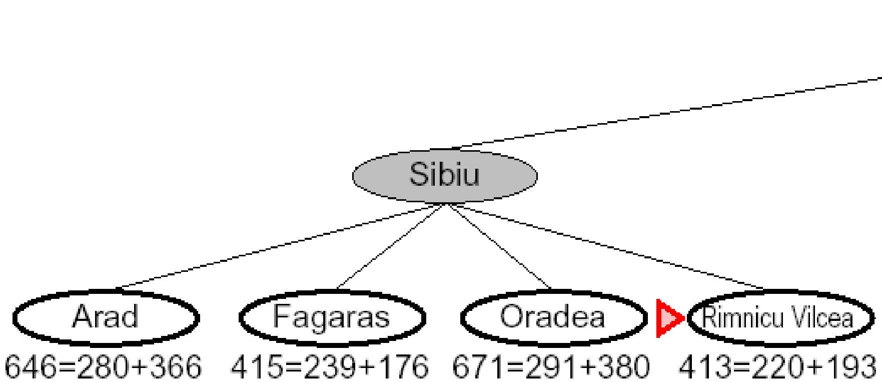
$h(n) = h_{SLD}(n)$ = straight-line distance from n to destination



A* Example

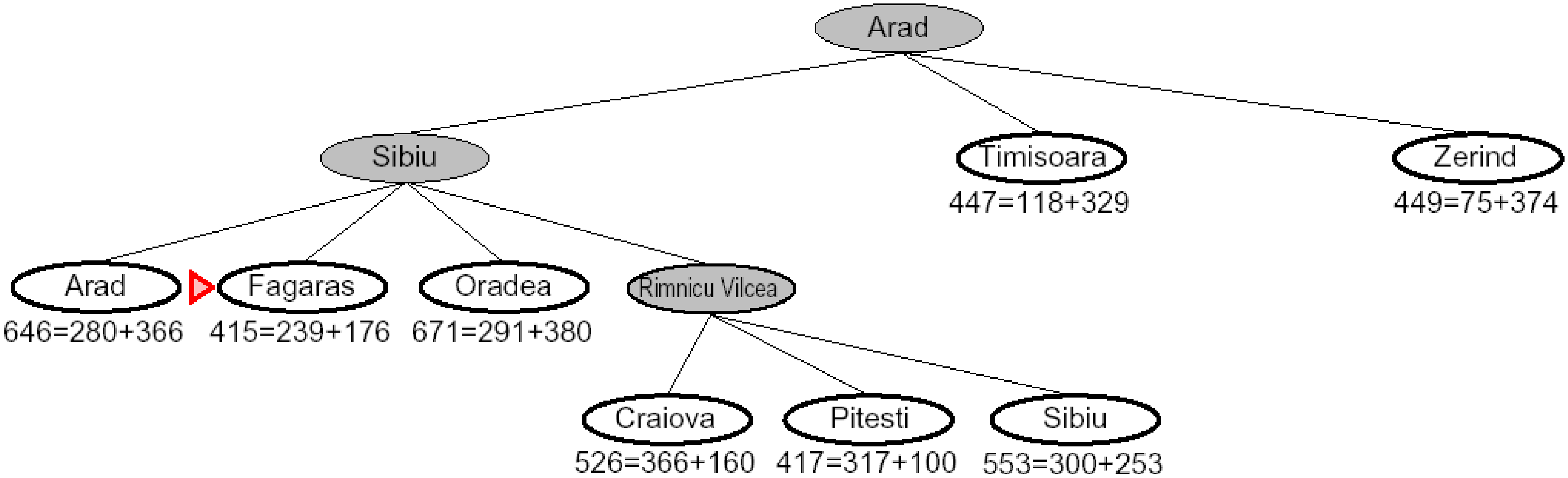

Arad
Sibiu
393=140+253
Timisoara
447=118+329
Zerind
449=75+374

A* Example



Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
413=220+193

A* Example



Arad
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447=118+329
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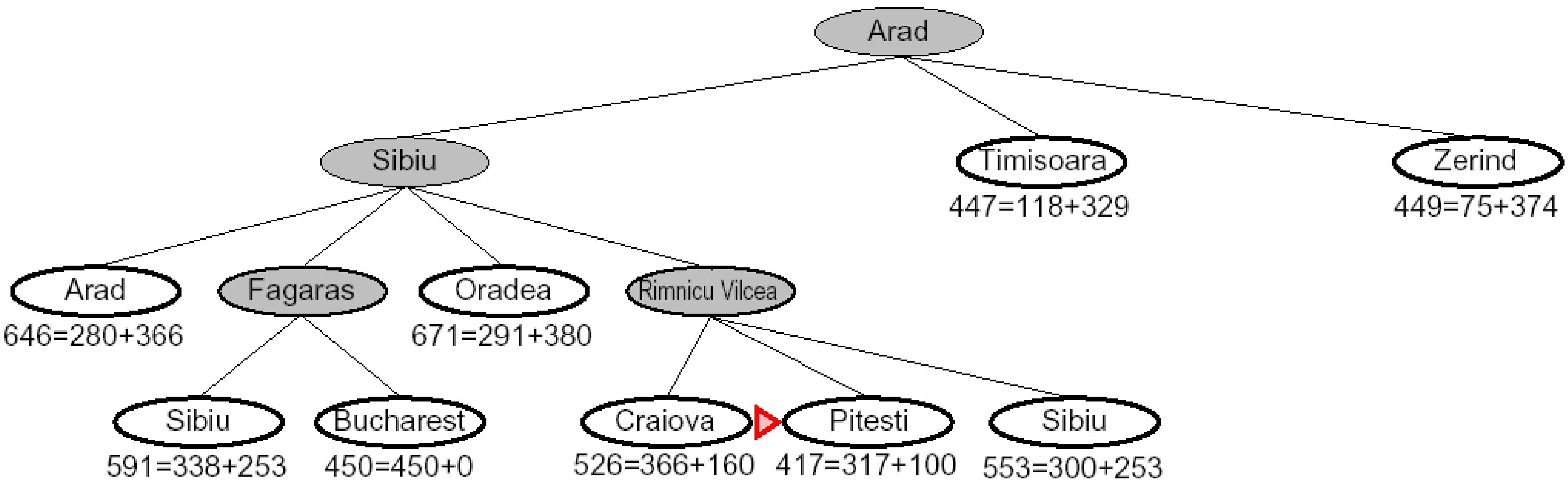
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
Craiova
526=366+160
Pitesti
417=317+100
Sibiu
553=300+253

A* Example


Arad
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646=280+366
Fagaras
Oradea
671=291+380
Rimnicu Vilcea
Sibiu
591=338+253
Bucharest
450=450+0
Craiova
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Pitesti
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Sibiu
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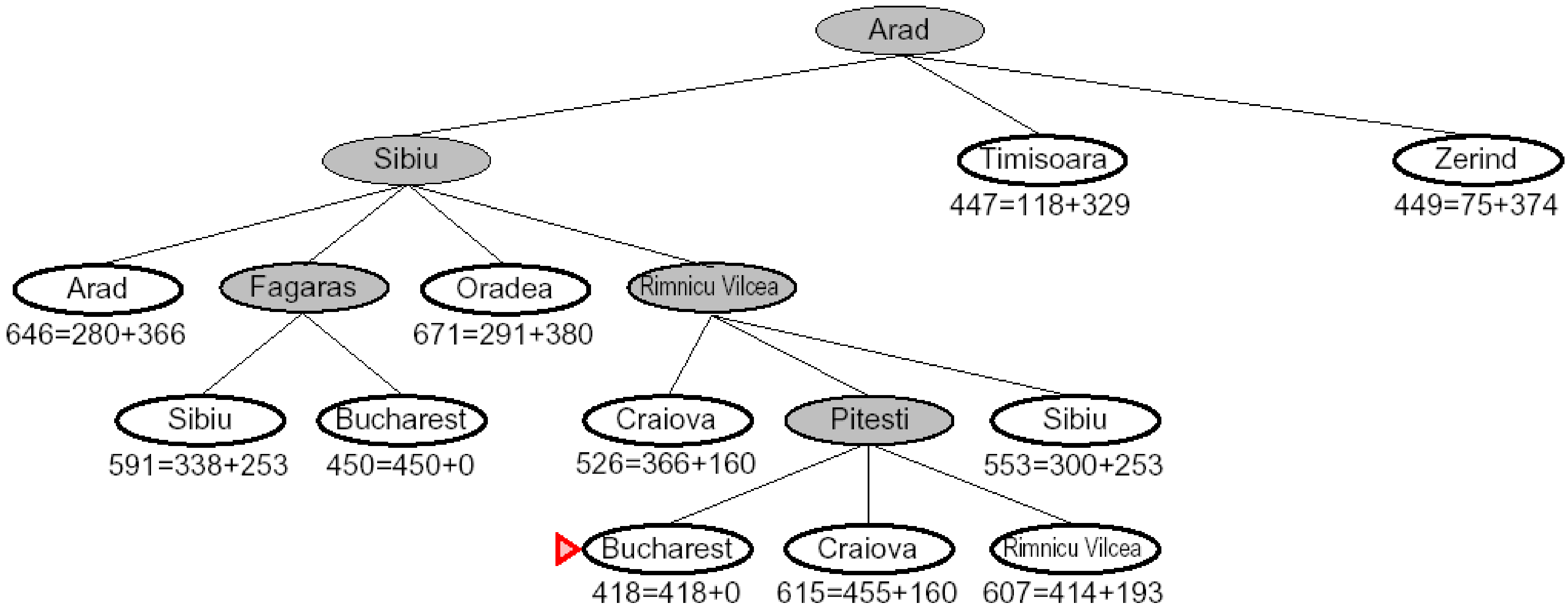
A* Example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450=450+0
Craiova
526=366+160
Pitesti
Sibiu
553=300+253
Bucharest
418=418+0
Craiova
615=455+160
Rimnicu Vilcea
607=414+193

Admissible heuristics

A heuristic $h(n)$ is admissible if

for every node n,

$$h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to reach the goal state from n.

An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic

Admissible Heuristics

Is the Straight Line Distance heuristic $h_{SLD}(n)$ *admissible?*

Admissible Heuristics

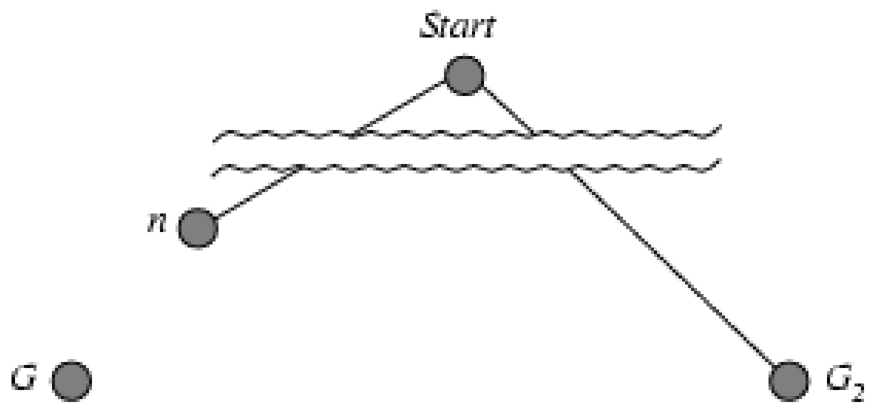
Is the Straight Line Distance heuristic $h_{SLD}(n)$ *admissible*?

Yes, it never overestimates the actual road distance

Theorem: If $h(n)$ is admissible, A* using TREE-SEARCH is optimal.

Optimality of A* (proof)

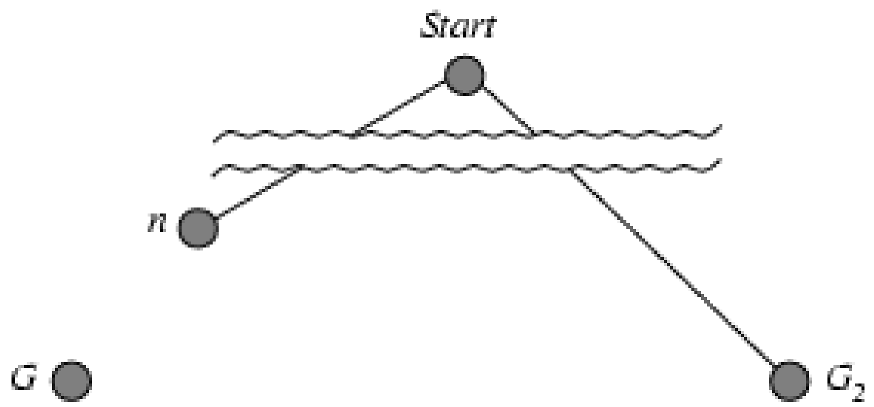
Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n *is on a shortest path to an optimal goal G.*



$f(G_2)$	$= g(G_2)$		since $h(G_2) = 0$
	$> g(G)$		since G_2 is suboptimal
$f(G)$	$= g(G)$		since $h(G) = 0$
$f(G_2)$	$> f(G)$		from above

Optimality of A* (cont.)

Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* *is on a shortest path to an optimal goal G.*



$f(G_2) > f(G)$ from prev slide

$h(n) \leq h^*(n)$ since h is admissible

$g(n) + h(n) \leq g(n) + h^*(n)$

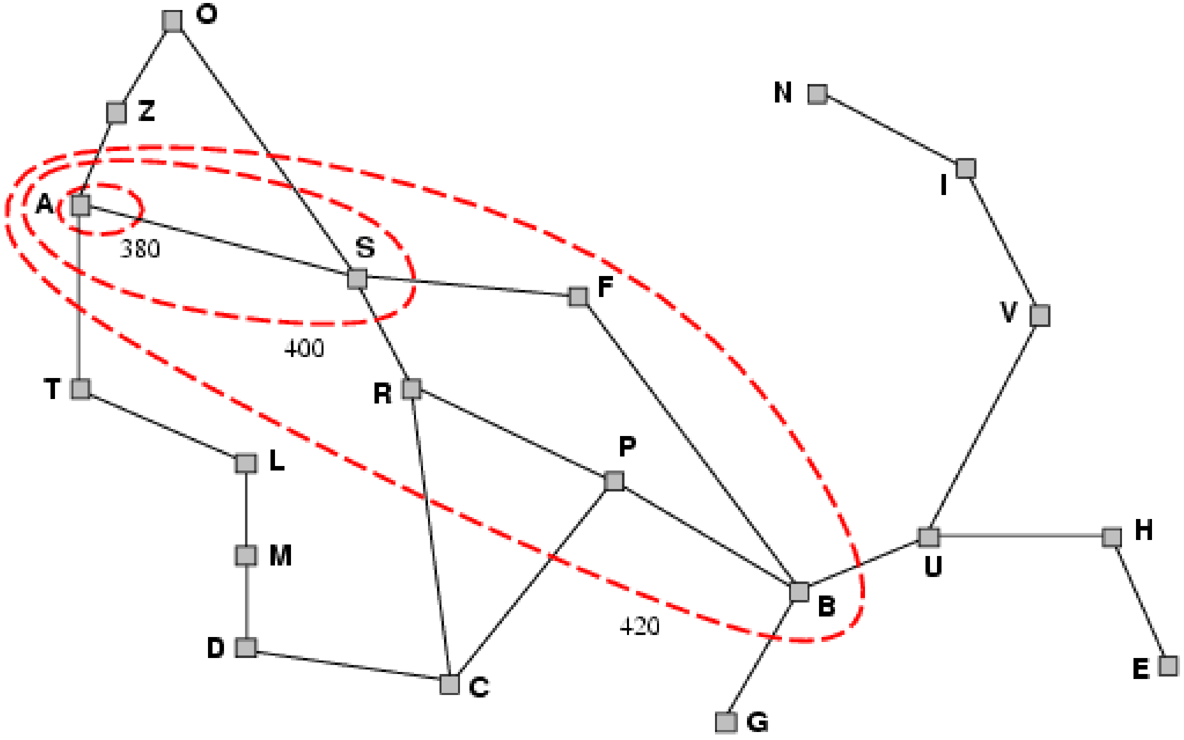
$f(n) \leq f(G) < f(G_2)$

Hence $f(n) < f(G_2) \Rightarrow A^*$ will never select G_2 for expansion.

Optimality of A*

A* expands nodes in order of increasing *f* value

Gradually adds "*f*-contours" of nodes



Okay, proof is done! Time to wake up…

Properties of A*

Complete? Yes (unless there are infinitely many nodes with $f \leq f(G)$)

Time? Exponential (for most heuristic functions in practice)

Space? Keeps all generated nodes in memory (exponential number of nodes)

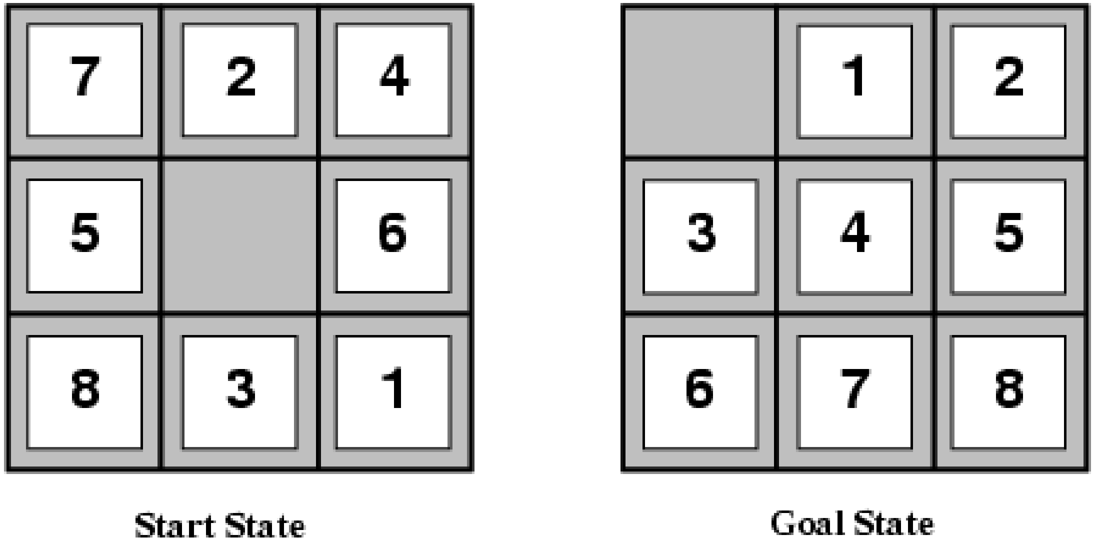
Optimal? Yes

Admissible heuristics

E.g., for the 8-puzzle, what are some admissible heuristic functions? (for # steps to goal state)

$h_1(n) = ?$

$h_2(n) = ?$

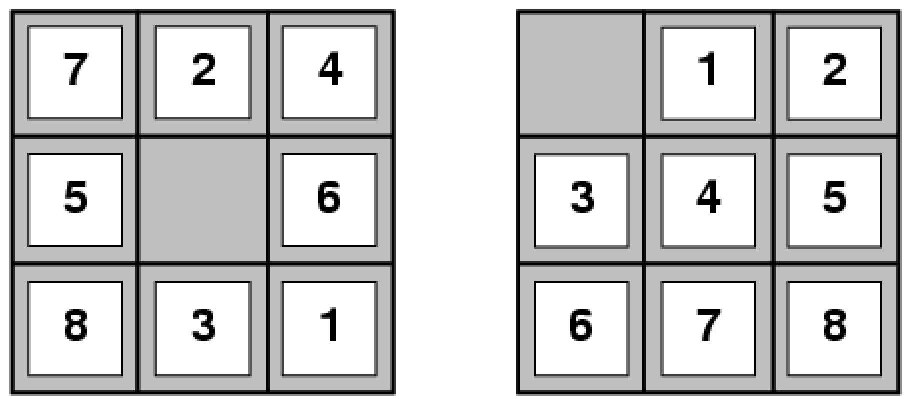


Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total Manhattan distance (no. of squares from desired location of each tile)



Start State

S

Goal State

$h_1(S)$ = ?

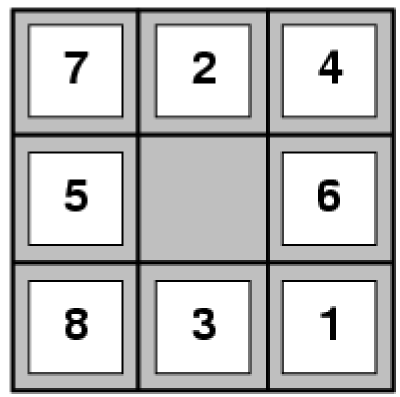
$h_2(S)$ = ?

Admissible heuristics

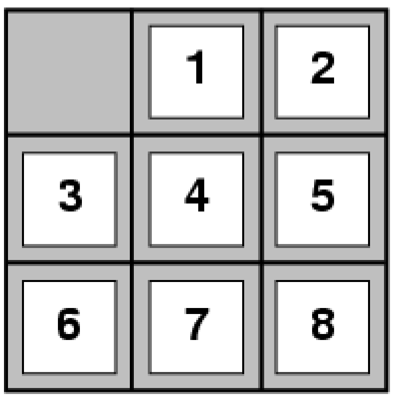
E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total Manhattan distance (no. of squares from desired location of each tile)



Start State



Goal State

$\underline{h_1(S) = ?}$ 8

$\underline{h_2(S) = ?}$ 3+1+2+2+2+3+3+2 = 18

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 **dominates** h_1

h_2 is better for search

Dominance

E.g., for 8-puzzle heuristics h_1 and h_2, typical search costs (average number of nodes expanded for solution depth d):

d=12 IDS = 3,644,035 nodes
$A^*(h_1)$ = 227 nodes
$A^*(h_2)$ = 73 nodes

d=24 IDS = too many nodes
$A^*(h_1)$ = 39,135 nodes
$A^*(h_2)$ = 1,641 nodes

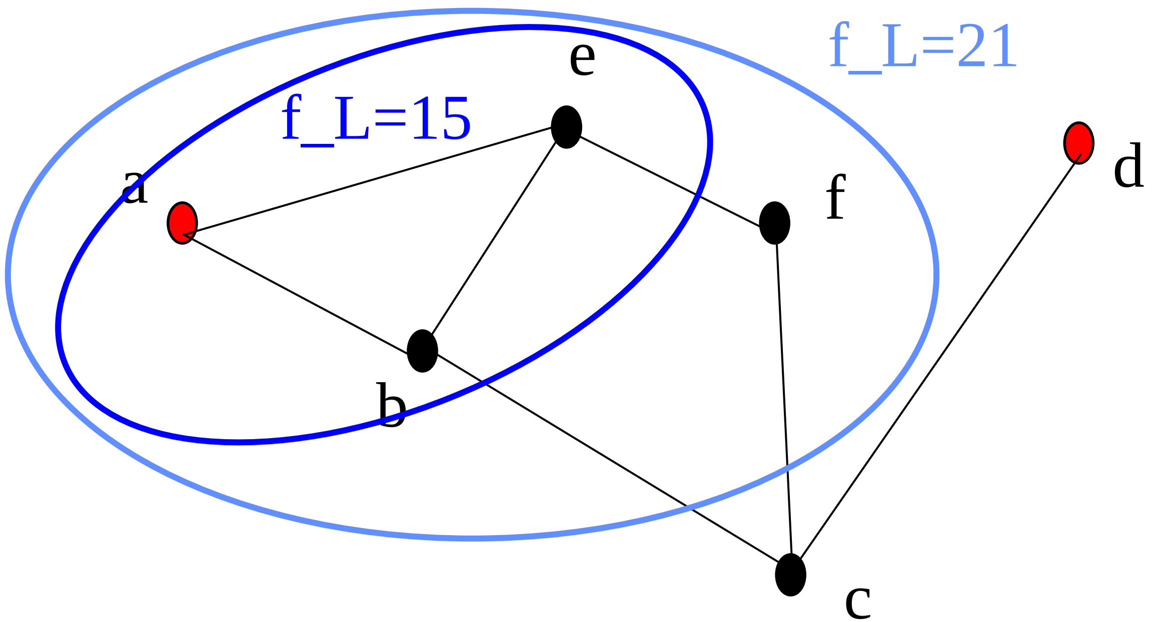
In general, A* not practical for large scale problems due to memory requirements (all generated nodes in memory)

Idea: Use iterative deepening

Iterative-Deepening A*

Like iterative-deepening search, but cutoff is f cost (= g + h) rather than depth

At each iteration, cutoff is smallest f cost among nodes that exceeded cutoff on prev iteration

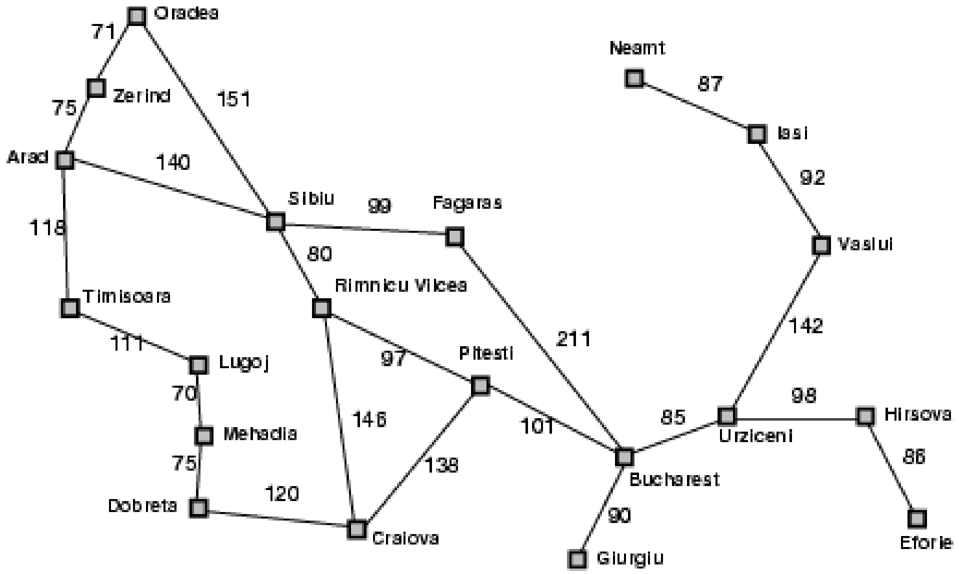


Back to Admissable Heuristics

f(x) = g(x) + h(x)

g: cost so far

h: underestimate of remaining costs



e.g., h_{SLD}

Where do heuristics come from?

Relaxed Problems

Derive admissible heuristic from exact **cost of a solution to a** relaxed **version of problem**

- ***For route planning, what is a relaxed problem?***

Relax requirement that car stay on road → Straight Line Distance becomes optimal cost

Cost of optimal solution to relaxed problem ≤ cost of optimal solution for real problem

Heuristics for eight puzzle

7	2	3
5	1	6
8	4	

→

1	2	3
4	5	6
7	8	

start goal

What can we relax?

Original Problem: Tile can move from location A to B if A is horizontally or vertically next to B *and* B is blank

Heuristics for eight puzzle

7	2	3
5	1	6
8	4	

→

1	2	3
4	5	6
7	8	

Relaxed 1: Tile can move from any location A to any location B

Cost = h_1 = number of misplaced tiles

Relaxed 2: Tile can move from A to B if A is horizontally or vertically next to B (*note*: B does not have to be blank)

Cost = h_2 = total Manhattan distance

You can try other possible heuristics in your HW #1

Need for Better Heuristics

Performance of h_2 (Manhattan Distance Heuristic)

• 8 Puzzle	< 1 second
• 15 Puzzle	1 minute
• 24 Puzzle	65000 years

Can we do better?

Adapted from Richard Korf presentation

Creating New Heuristics

Given admissible heuristics h_1, h_2, …, h_m, none of them dominating any other, how to choose the best?

Answer: No need to choose only one! Use:

$$h(n) = \max \{h_1(n), h_2(n), \ldots, h_n(n)\}$$

h is admissible (why?)

h dominates all h_i (by construction)

Can we do better with:

$$h'(n) = h_1(n) + h_2(n) + \ldots + h_n(n)?$$

Pattern Databases

Idea: Use solution cost of a subproblem as heuristic. For 8-puzzle: pick any subset of tiles

E.g., 3, 7, 11, 12

Precompute a table

- Compute optimal cost of solving just these tiles
  - This is a lower bound on actual cost with all tiles
- For all possible configurations of these tiles
  - Could be several million
- Use breadth first search back from goal state
  - State = position of just these tiles (& blank)
- Admissible heuristic h_{DB} for complete state = cost of corresponding sub-problem state in database

Combining Multiple Databases

Can choose another set of tiles

- Precompute multiple tables

How to combine table values?

- Use the *max* trick!

E.g. Optimal solutions to Rubik's cube

- First found w/ IDA* using pattern DB heuristics
- Multiple DBs were used (diff subsets of cubies)
- Most problems solved optimally in 1 day
- Compare with *574,000 years* for IDS

Adapted from Richard Korf presentation

Drawbacks of Standard Pattern DBs

Since we can only take *max*

- Diminishing returns on additional DBs

Would like to be able to *add* values

- But not exceed the actual solution cost (to ensure admissible heuristic)
- How?

Disjoint Pattern DBs

Partition tiles into disjoint sets

- For each set, precompute table
- Don't count moves of tiles not in set
  - This makes sure costs are disjoint
  - Can be added without overestimating!
  - E.g. For 15 puzzle shown, 8 tile DB has 519 million entries
  - And 7 tile DB has 58 million

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

During search

- Look up costs for each set in DB
- *Add values to get heuristic function value*

- Manhattan distance is a special case of this idea where each set is a single tile

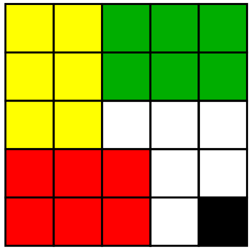
Performance

15 Puzzle: 2000x speedup *vs* Manhattan dist

- IDA* with the two DBs solves 15 Puzzle optimally in 30 milliseconds

24 Puzzle: 12 millionx speedup *vs* Manhattan

- IDA* can solve random instances in 2 days.
- Requires 4 DBs as shown
  - Each DB has 128 million entries
- Without PDBs: 65000 years



Next: Local Search

How to climb hills

How to reach the top by annealing

How to simulate and profit from evolution

Local search algorithms

In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution

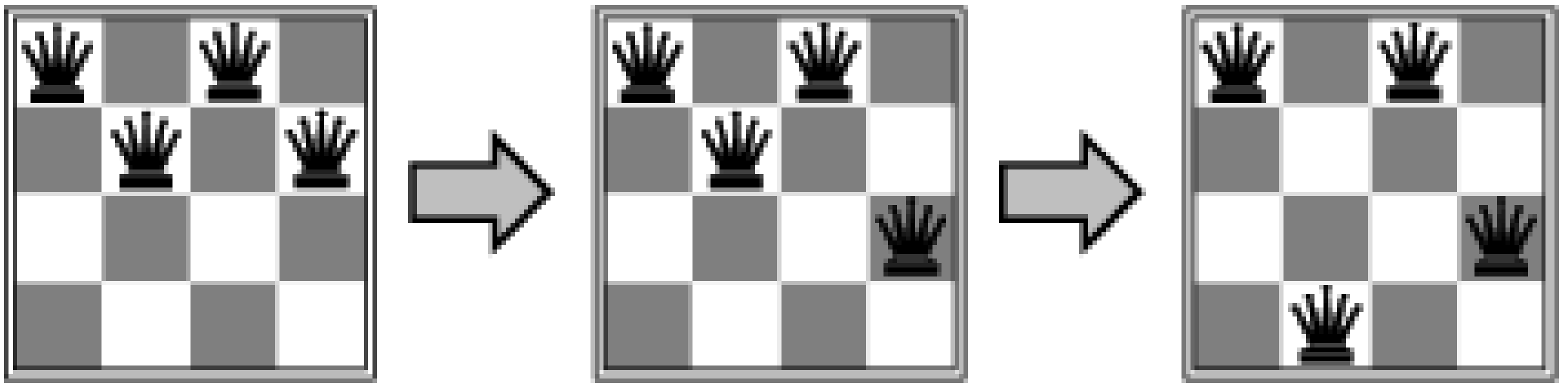
Find configuration satisfying constraints, e.g., n-queens

In such cases, we can use local search algorithms

Keep a single "current" state, try to improve it

Example: *n*-queens

Put *n* queens on an *n* × *n* board with no two queens on the same row, column, or diagonal



Hill-climbing search

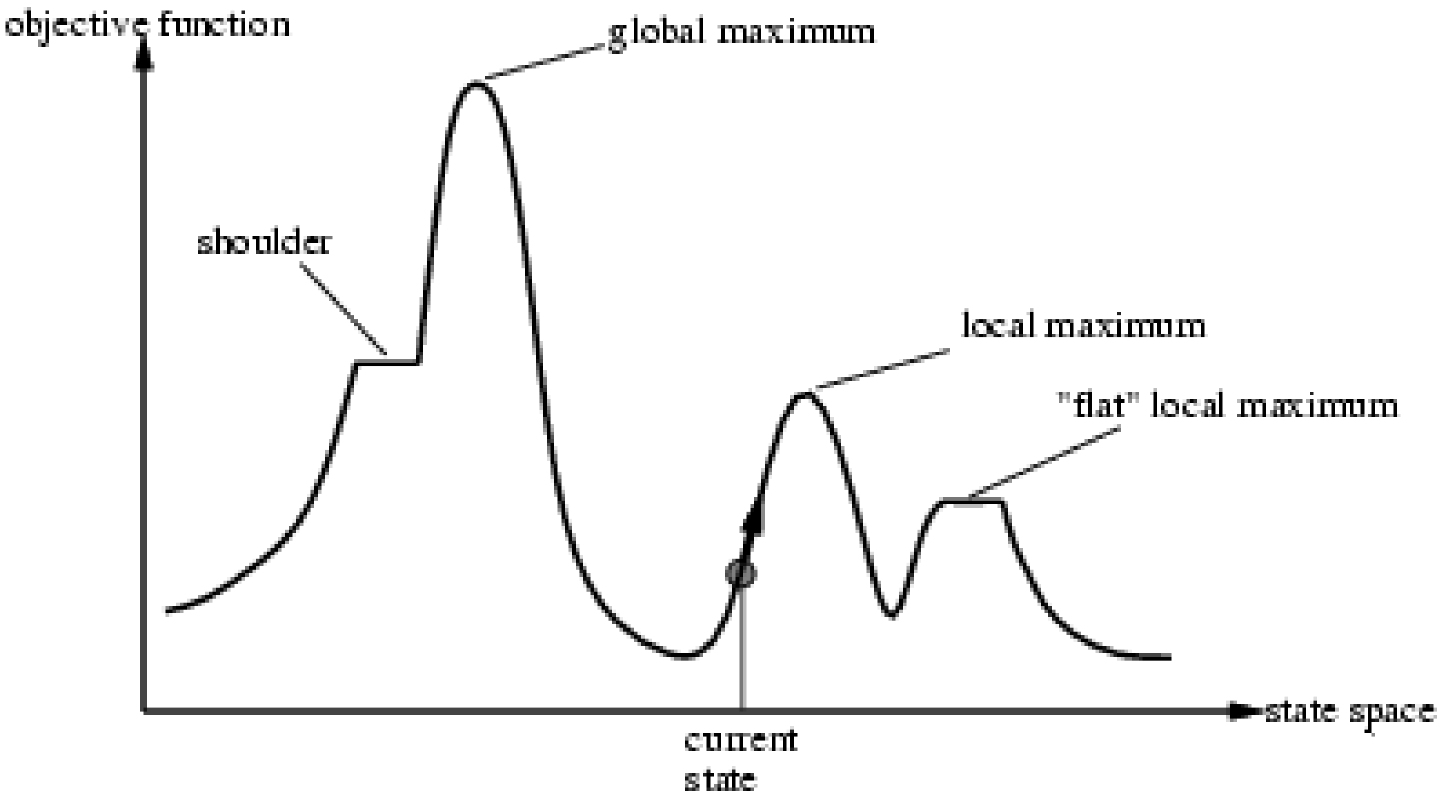
"Like climbing Everest in thick fog with amnesia"

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node

   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
        neighbor ← a highest-valued successor of current
        if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
        current ← neighbor
```

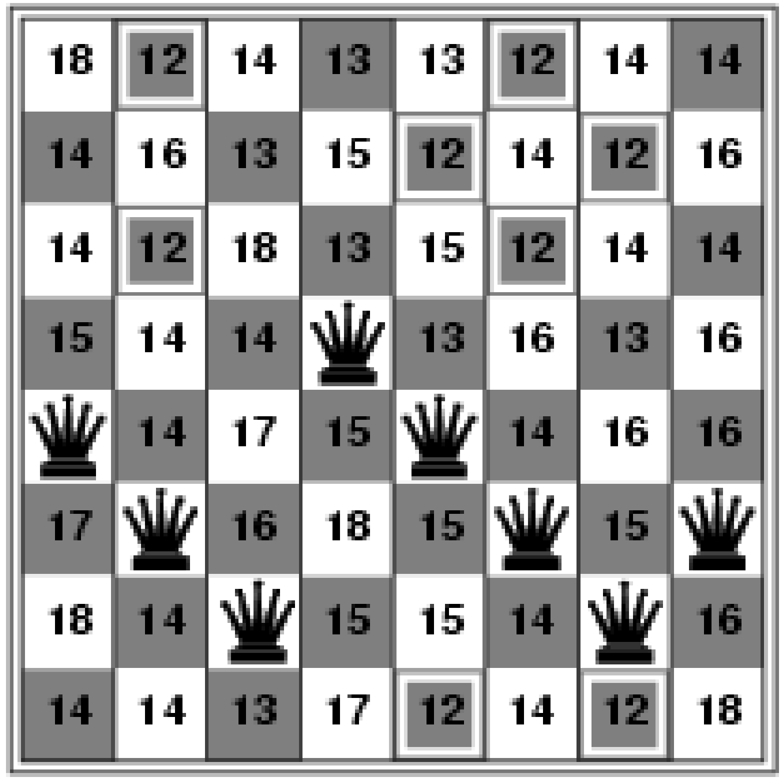
Hill-climbing search

Problem: depending on initial state, can get stuck in local maxima



Example: 8-queens problem

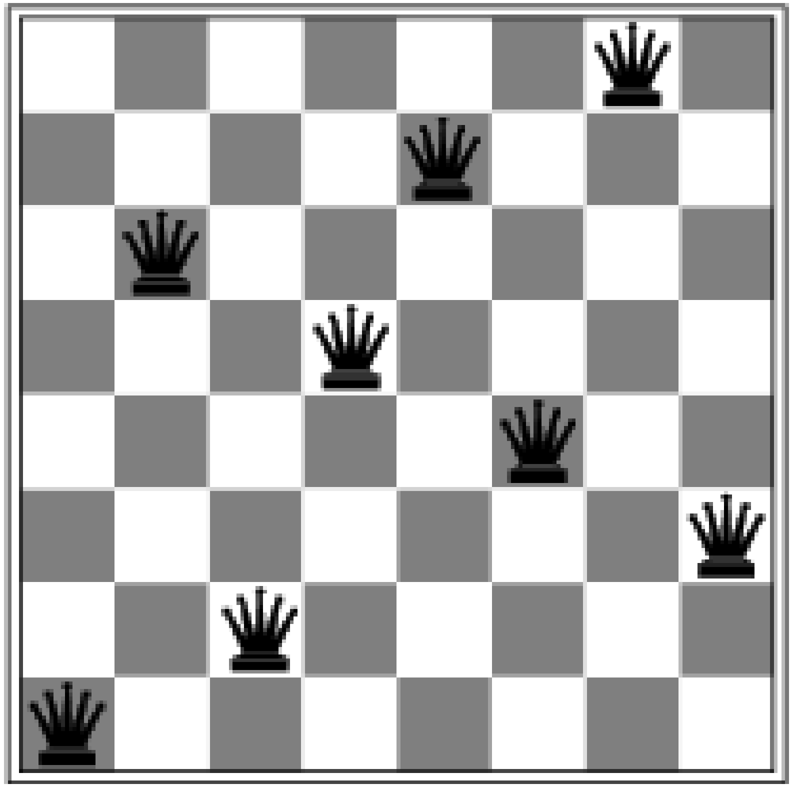
Heuristic?
(Value function)



h = number of pairs of queens that are attacking each other, either directly or indirectly

$h = 17$ for the above state (would like to minimize this)

Example: 8-queens problem



A local minimum with $h = 1$. Need $h = 0$

How to find global minimum (or maximum)?

Simulated Annealing

Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"
  local variables: *current*, a node
    next, a node
    $\boxed{T\text{, a "temperature" controlling prob. of downward steps}}$

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  for $t \leftarrow 1$ **to** ∞ **do**
    $T \leftarrow schedule[t]$
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*]
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$

Properties of simulated annealing

One can prove: If T decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1

Widely used in VLSI layout, airline scheduling, etc

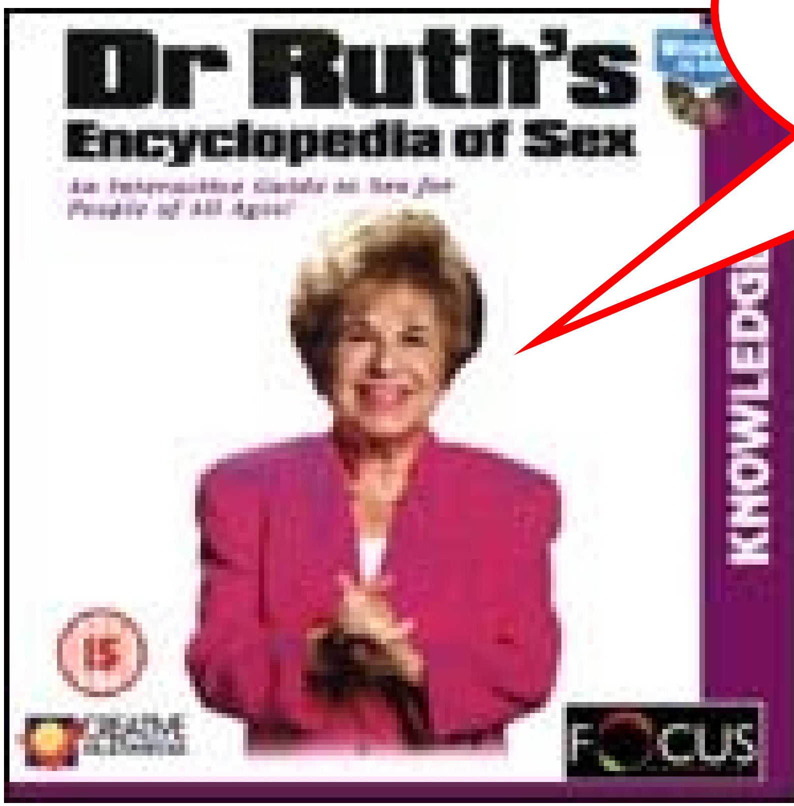
Local Beam Search

Keep track of *k* states rather than just one

Start with *k* randomly generated states

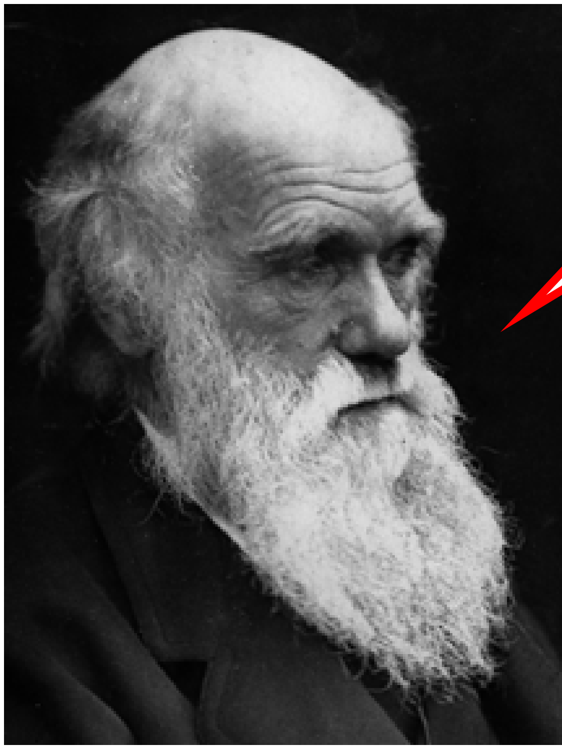
At each iteration, all the successors of all *k* states are generated

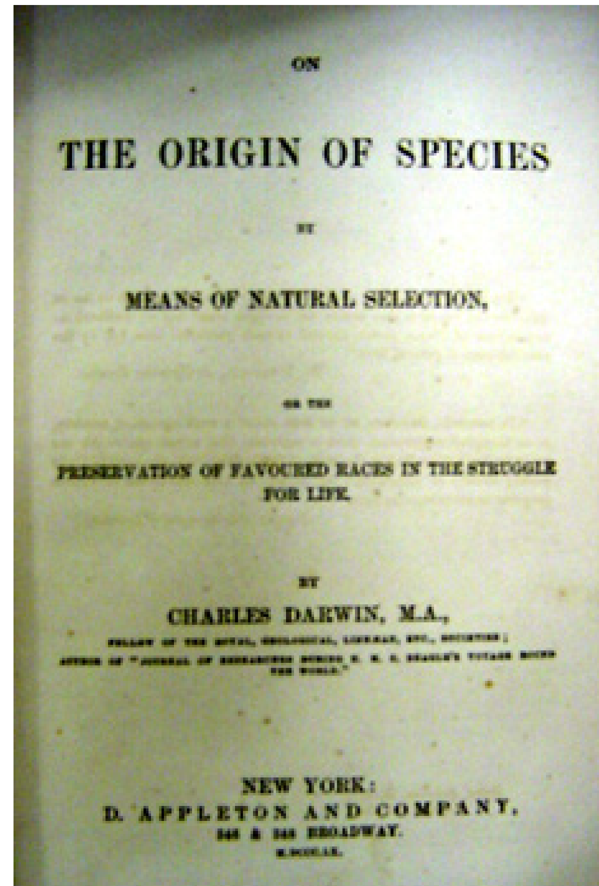
If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat.



Hey, perhaps sex can improve search?

Sure – check out ye book.





ON

THE ORIGIN OF SPECIES

BY

MEANS OF NATURAL SELECTION,

OR THE

PRESERVATION OF FAVOURED RACES IN THE STRUGGLE FOR LIFE.

BY

CHARLES DARWIN, M.A.,

NEW YORK:
D. APPLETON AND COMPANY,

Genetic Algorithms

A successor state is generated by combining two parent states

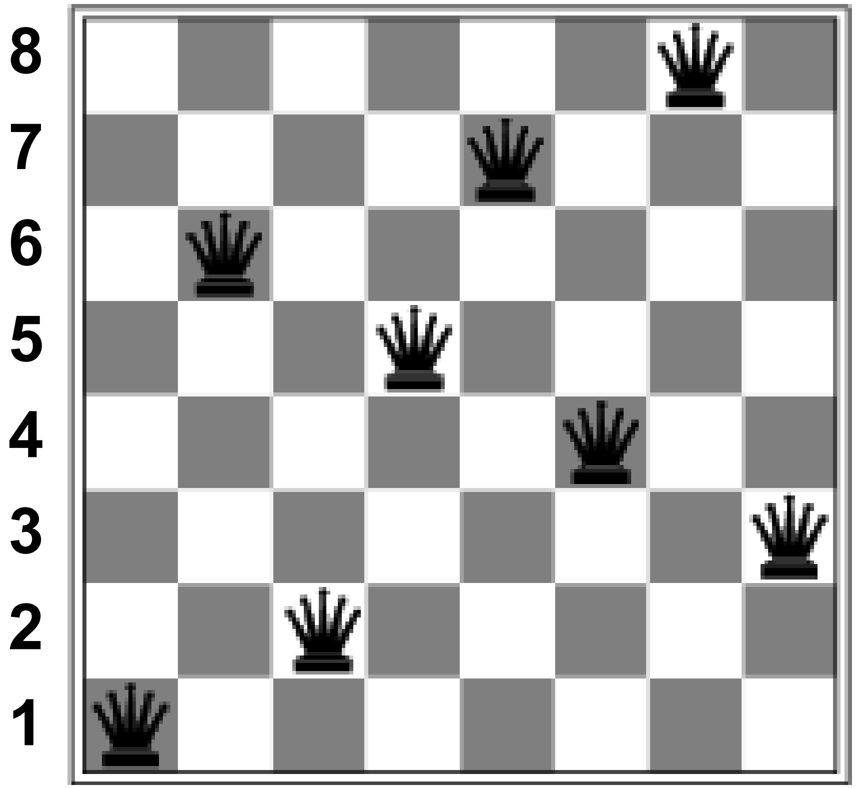
Start with *k* randomly generated states (population)

A state is represented as a string over a finite alphabet (often a string of 0s and 1s)

Evaluation function (fitness function). Higher values for better states.

Produce the next generation of states by selection, crossover, and mutation

Example: 8-queens problem



String Representation: 16257483

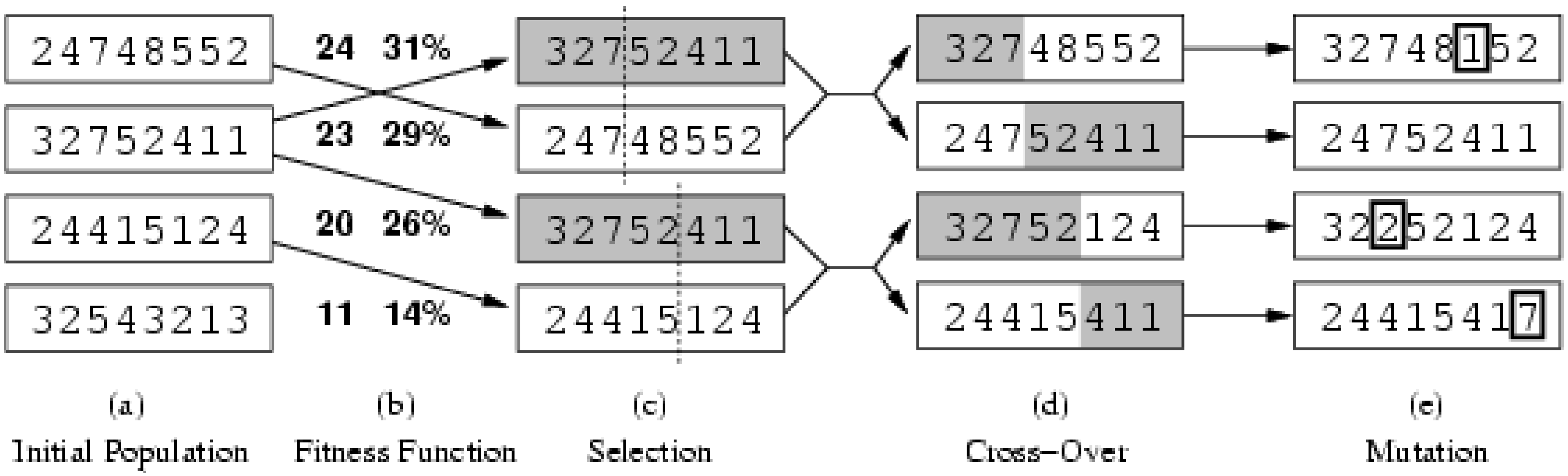
Can we evolve a solution through genetic algorithms?

Example: Evolving 8 Queens

?

Sorry, wrong queens

Example: Evolving 8 Queens

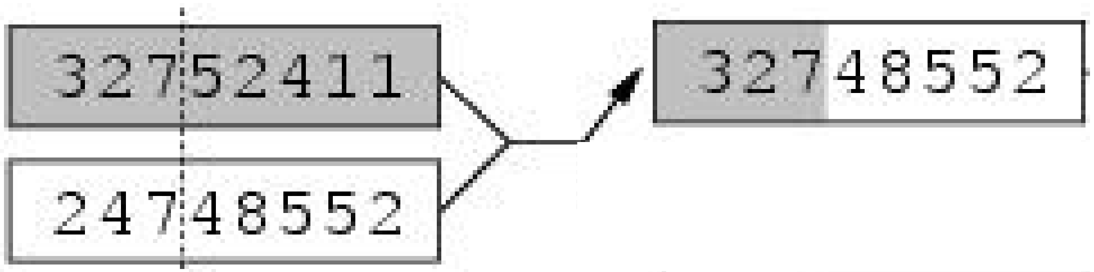


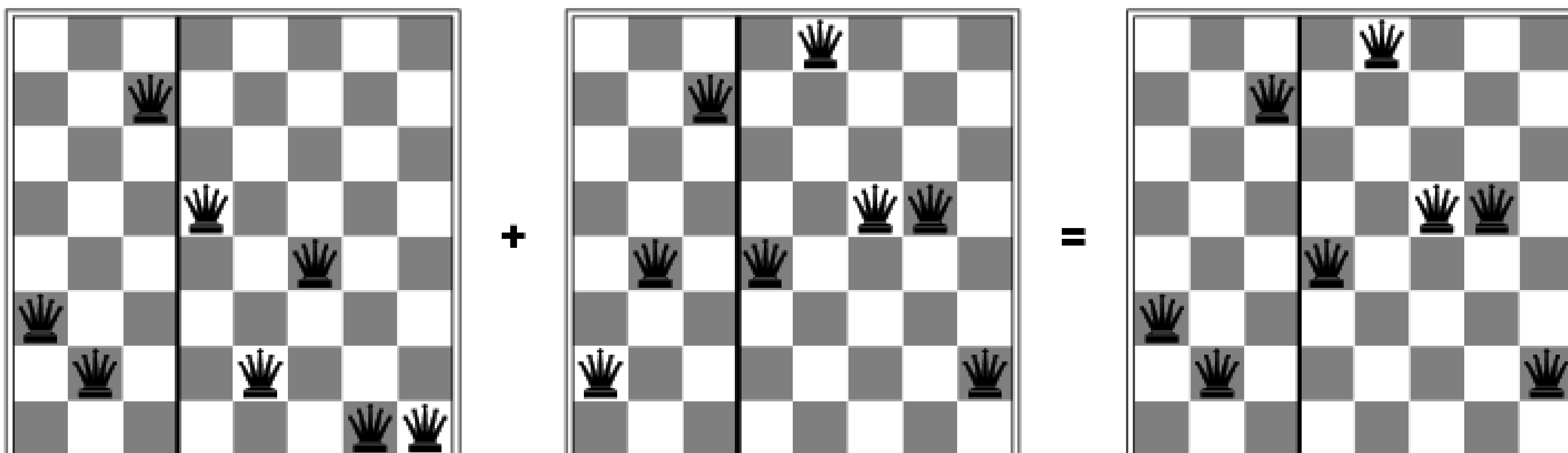
Fitness function: number of <u>non-attacking pairs</u> of queens (min = 0, max = 8 × 7/2 = 28)

24/(24+23+20+11) = 31% probability of selection for reproduction

23/(24+23+20+11) = 29% etc

Queens crossing over






Let's move on to adversarial games

Adversarial Games

Programs that can play competitive board games

Minimax Search

Alpha-Beta Pruning

Games Overview

	deterministic	chance
Perfect information	chess, checkers, go, othello	backgammon, monopoly
Imperfect information		poker, bridge, scrabble

Games & Game Theory

When there is ***more than one agent***, the future is not easily predictable anymore for the agent

In ***competitive*** environments (conflicting goals), adversarial search becomes necessary

In AI, we usually consider special type of games:

- **board games**, which can be characterized as *deterministic, turn-taking, two-player, zero-sum* games with *perfect information*

Games as Search

Components:

- **States:**
- **Initial state:**
- **Successor function:**
- **Terminal test:**
- **Utility function:**

Games as Search

Components:

- **States**: board configurations
- **Initial state**: the board position and which player will move
- **Successor function**: returns list of *(move, state)* pairs, each indicating a legal move and the resulting state
- **Terminal test**: determines when the game is over
- **Utility function**: gives a numeric value in terminal states (e.g., -1, 0, +1 in chess for loss, tie, win)

Games as Search

Convention: first player is called MAX,
2nd player is called MIN

MAX moves first and they take turns until game is over

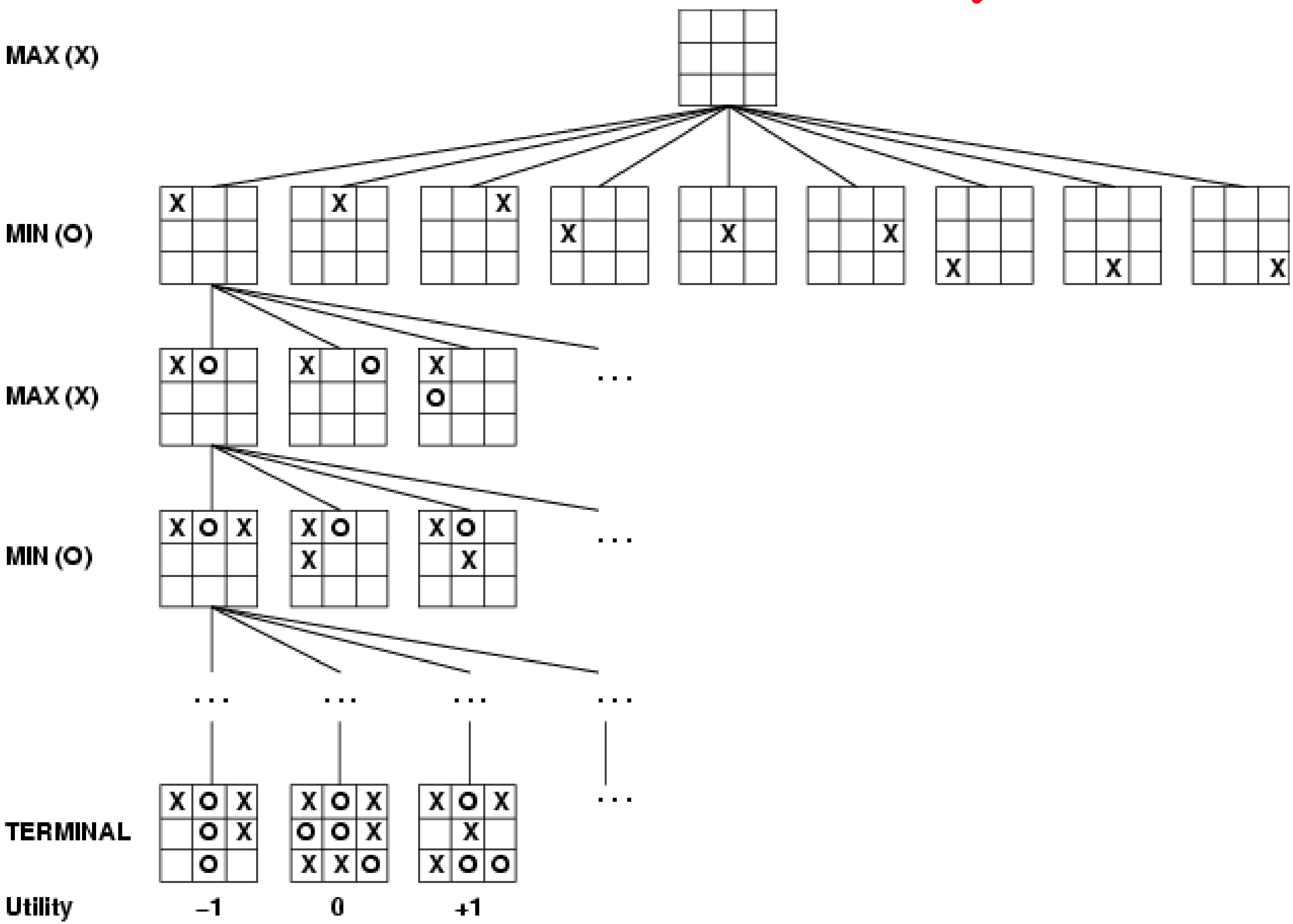
Winner gets reward, loser gets penalty

Utility values stated from MAX's perspective

Initial state and legal moves define the *game tree*

MAX uses game tree to determine next move

Tic-Tac-Toe Example



Optimal Strategy: Minimax Search

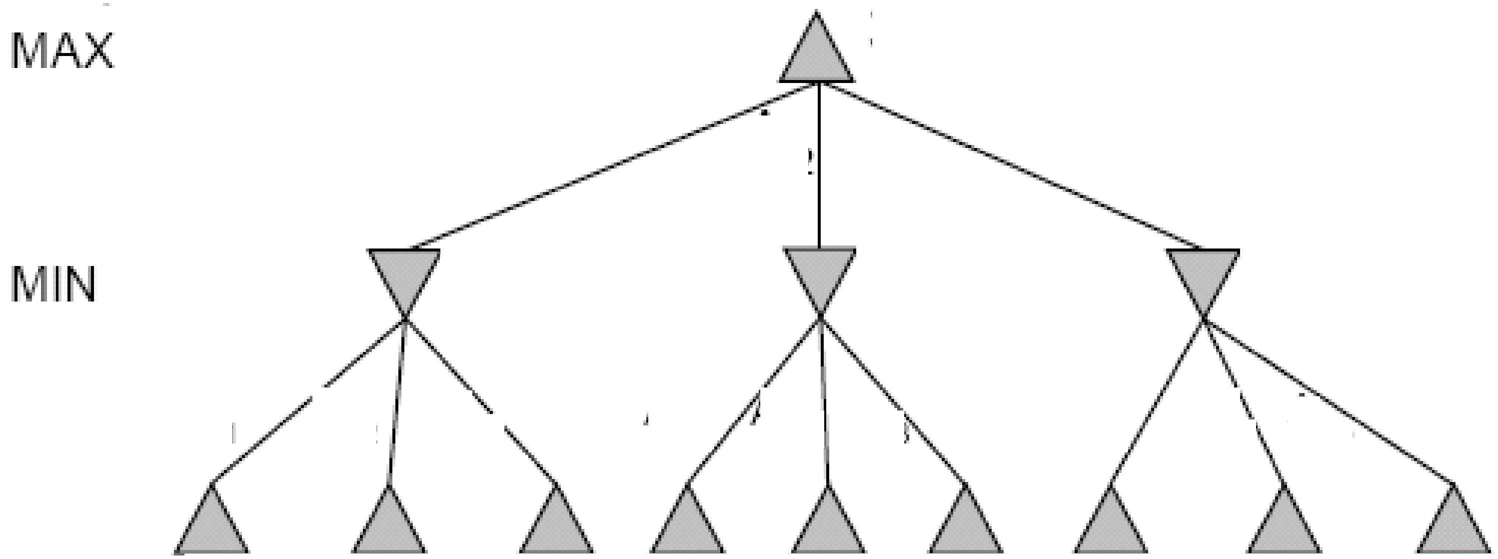
- Find the contingent *strategy* for MAX assuming an infallible MIN opponent
- Assumption: Both players play optimally!
- Given a game tree, the optimal strategy can be determined by using the *minimax* value of each node (defined recursively):

MINIMAX-VALUE(n)=

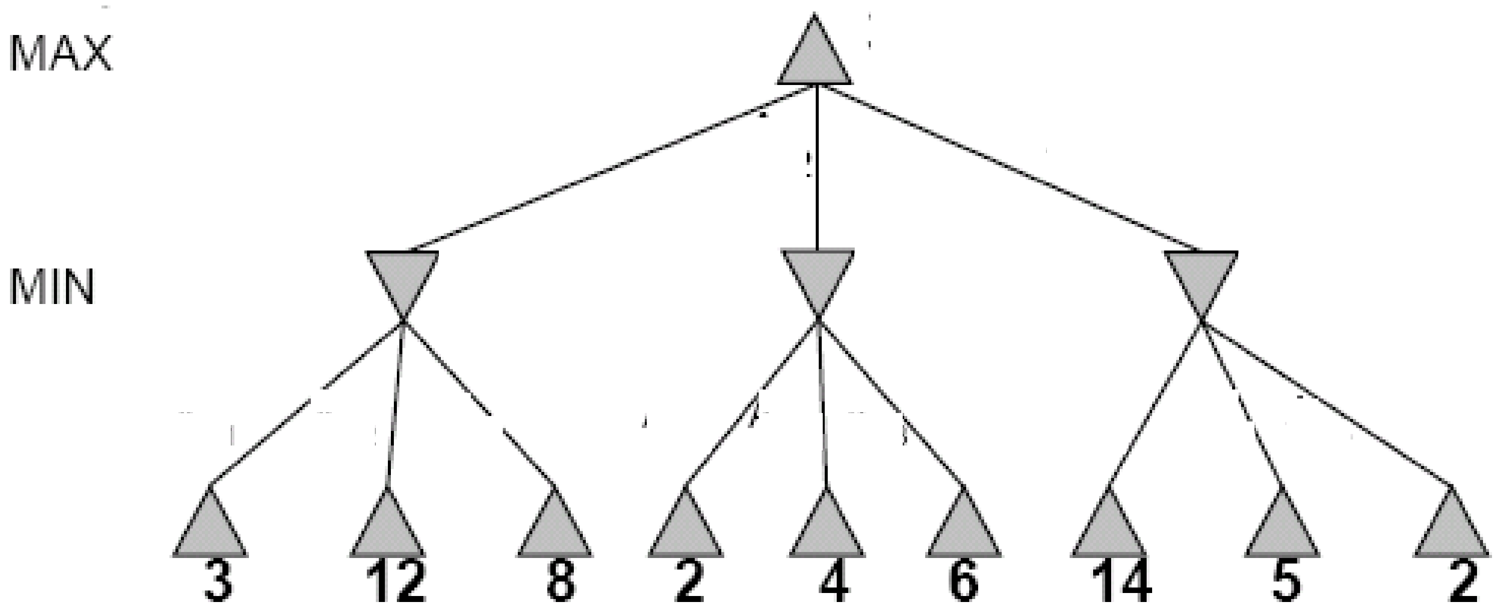
UTILITY(n)	If n is a terminal
$\max_{s \in succ(n)}$ MINIMAX-VALUE(s)	If n is a MAX node
$\min_{s \in succ(n)}$ MINIMAX-VALUE(s)	If n is a MIN node

Two-Ply Game Tree

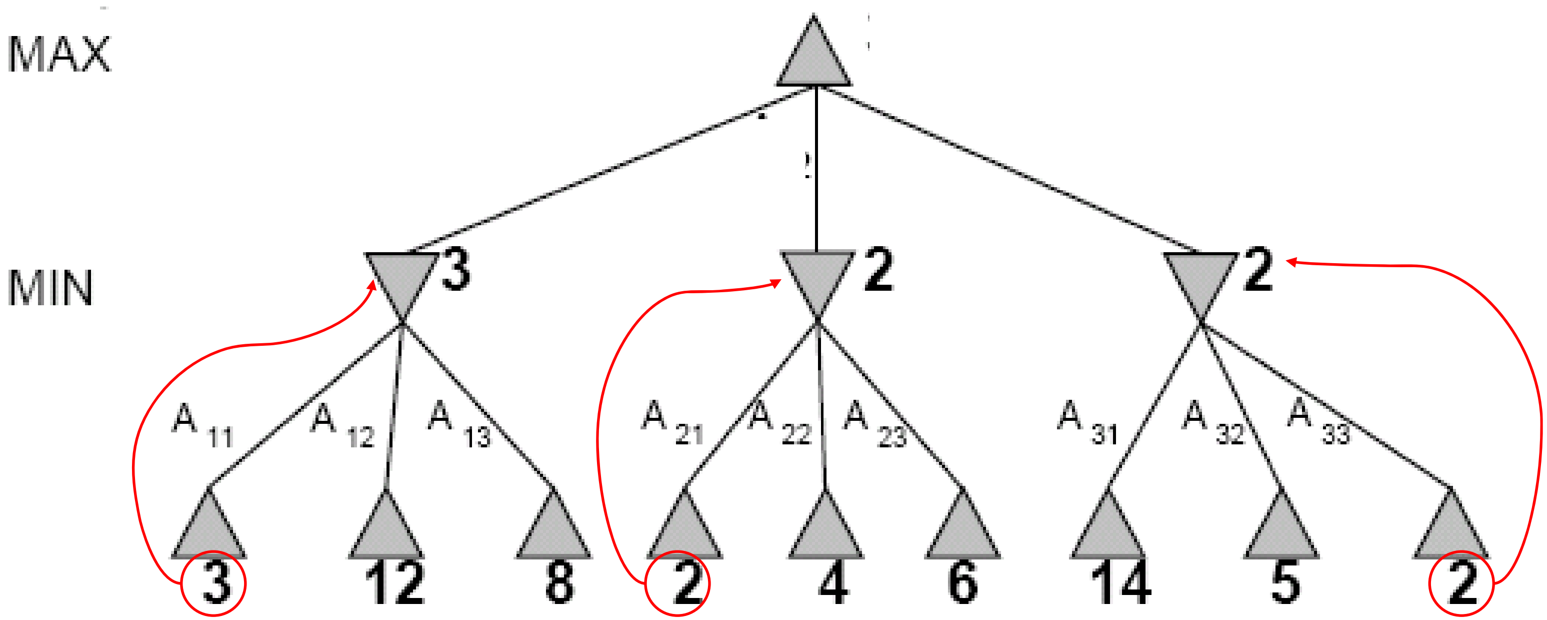
"Ply" = move by 1 player



Two-Ply Game Tree

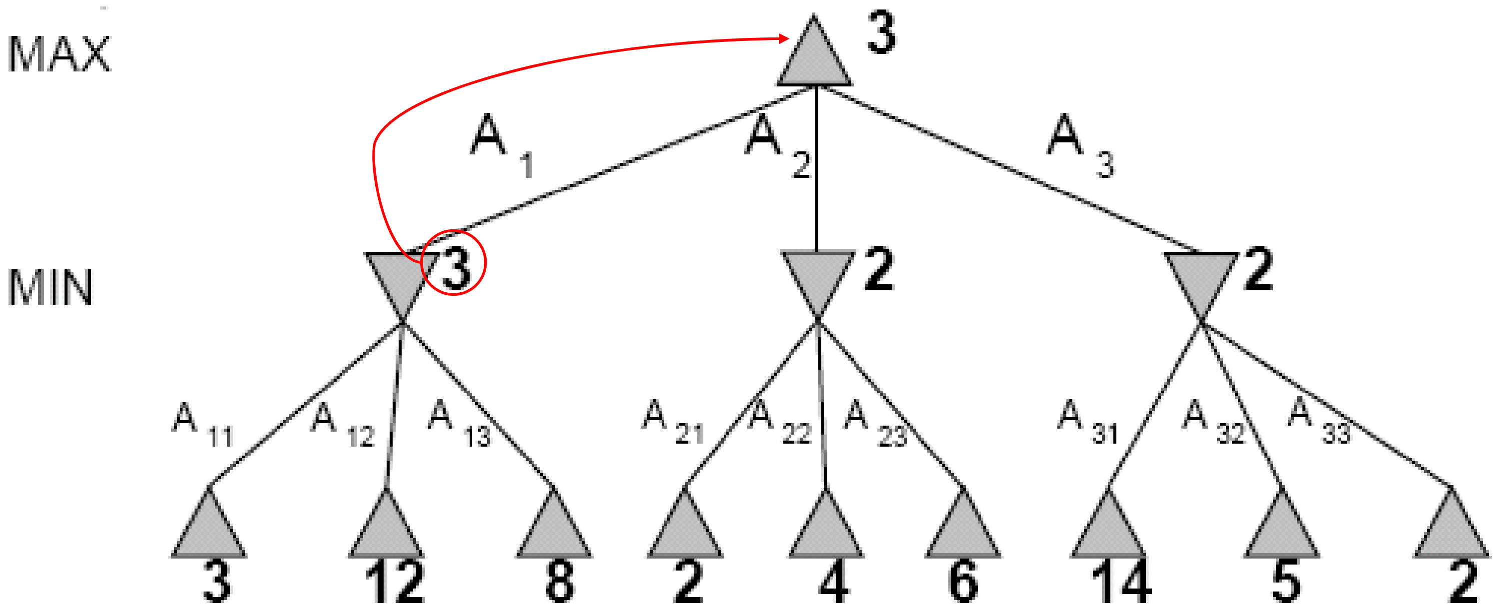


Two-Ply Game Tree



Two-Ply Game Tree

Minimax decision = A_1



Minimax maximizes the worst-case outcome for max

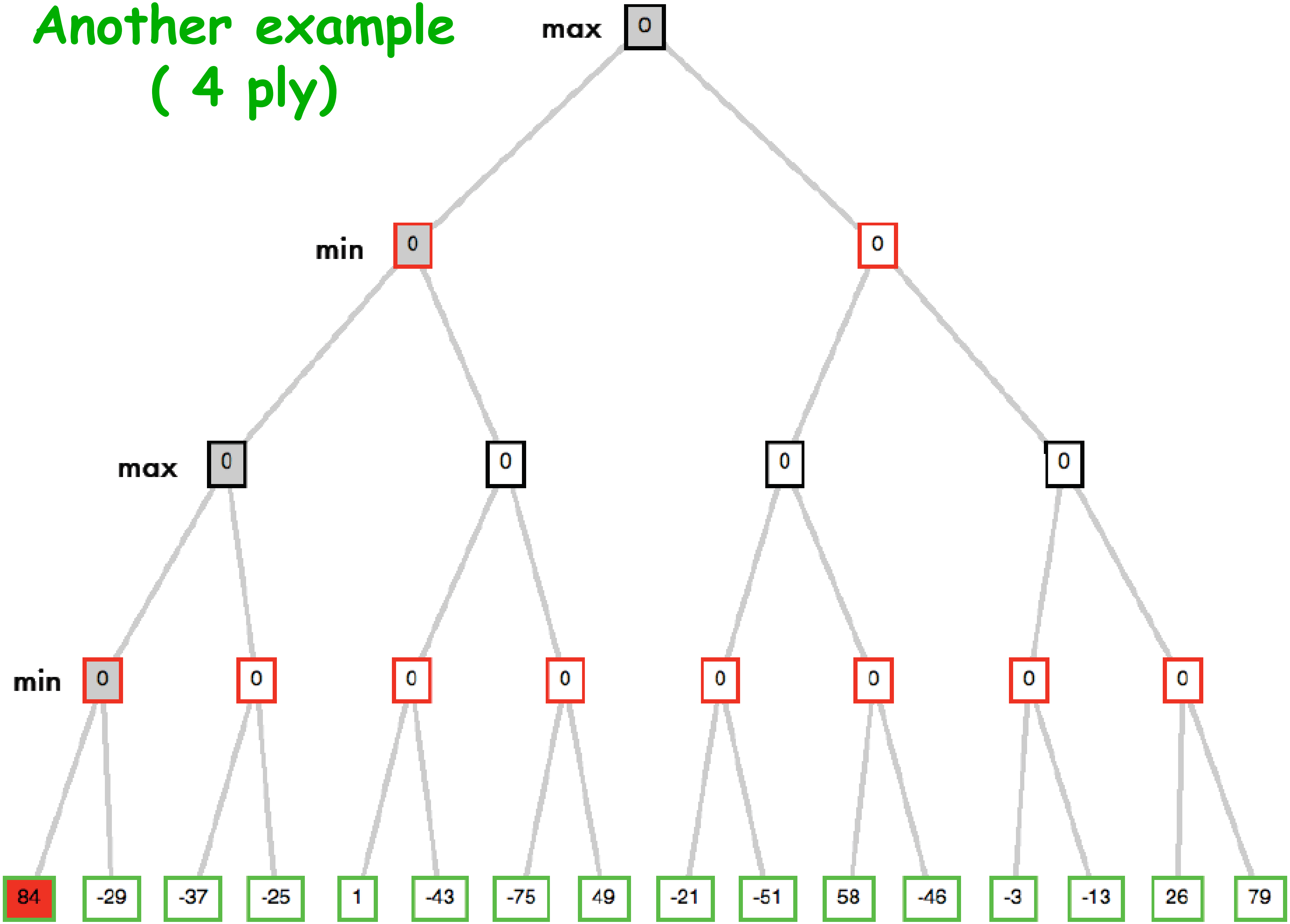
What if MIN does not play optimally?

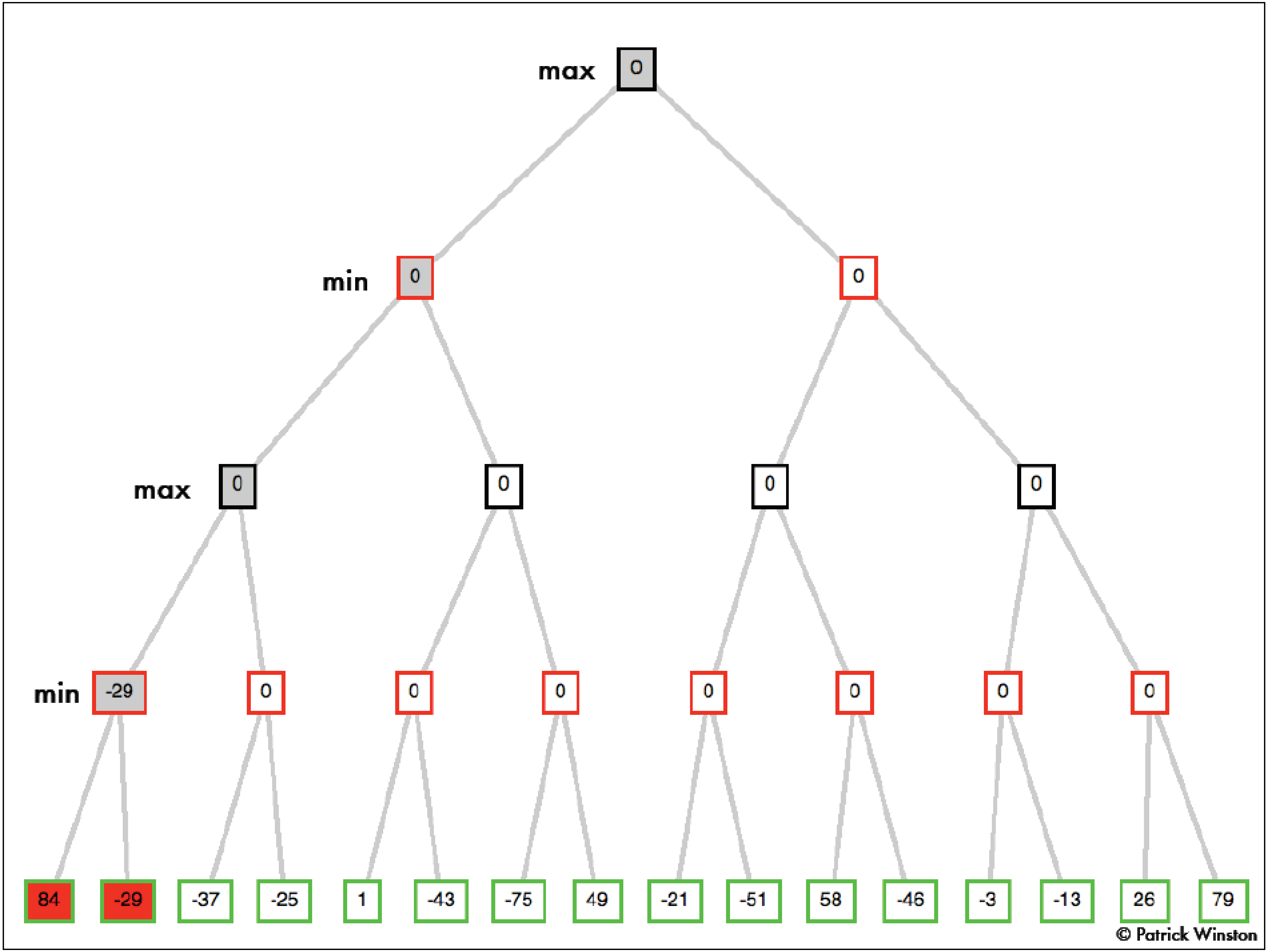
Definition of optimal play for MAX assumes MIN plays optimally

- Maximizes worst-case outcome for MAX

If MIN does not play optimally, MAX will do even better (i.e. at least as much or more utility obtained than if MIN was optimal) [Exercise 5.7 in textbook]

Another example
(4 ply)




max
0
min
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max
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min
-29
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0
84
-29
-37
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-21
-51
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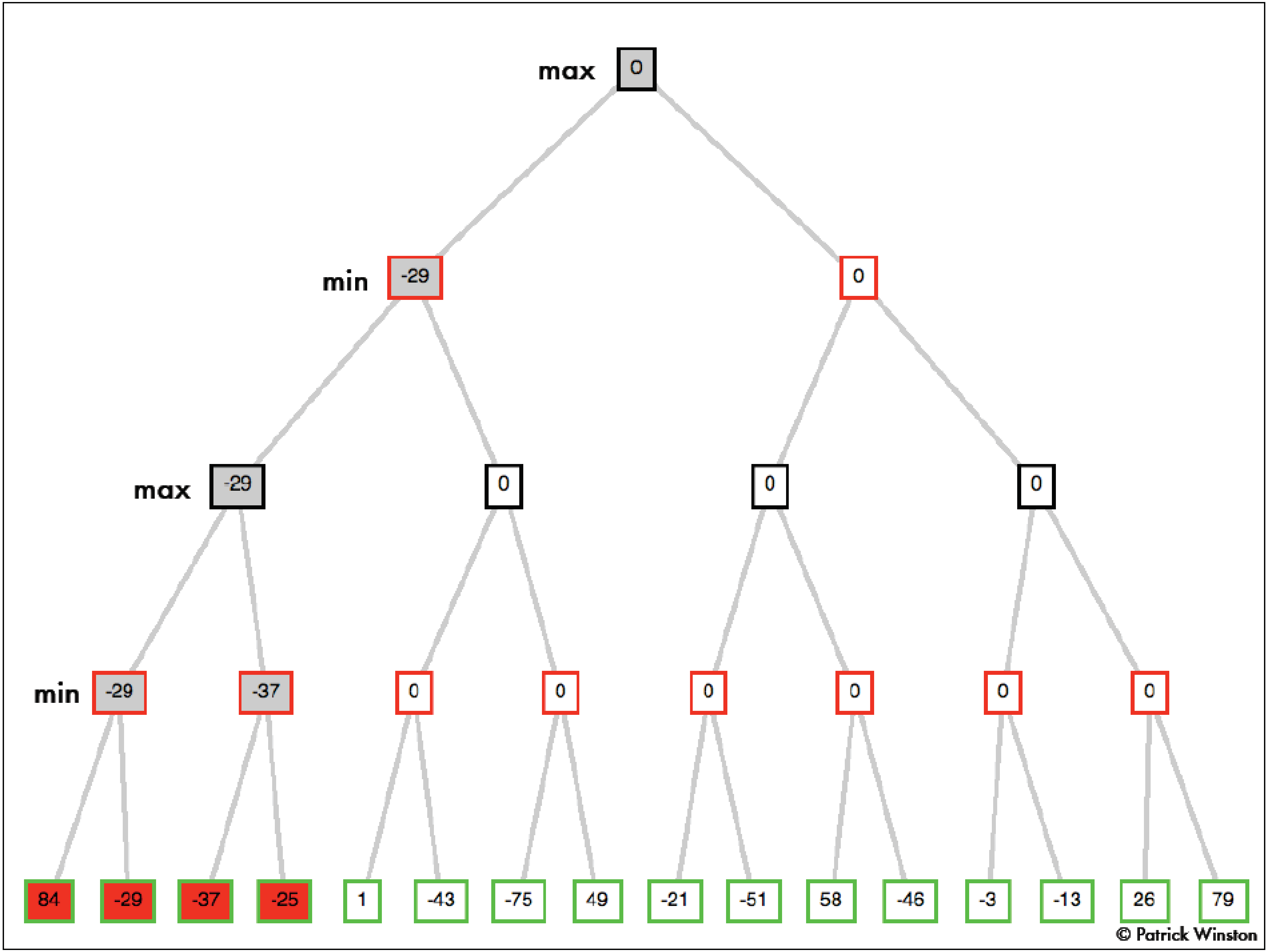
-13
26
79
© Patrick Winston


max
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max
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84
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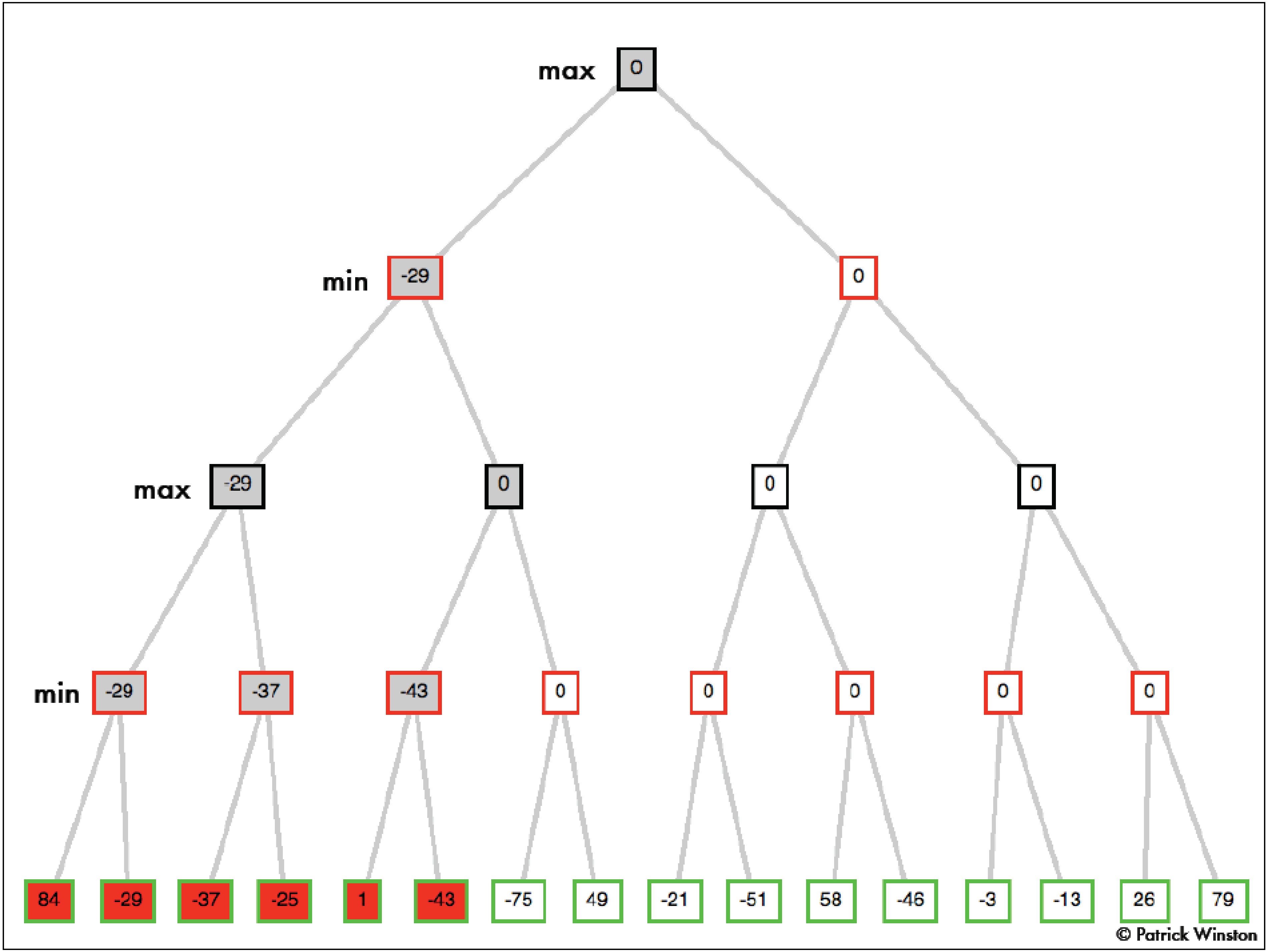
© Patrick Winston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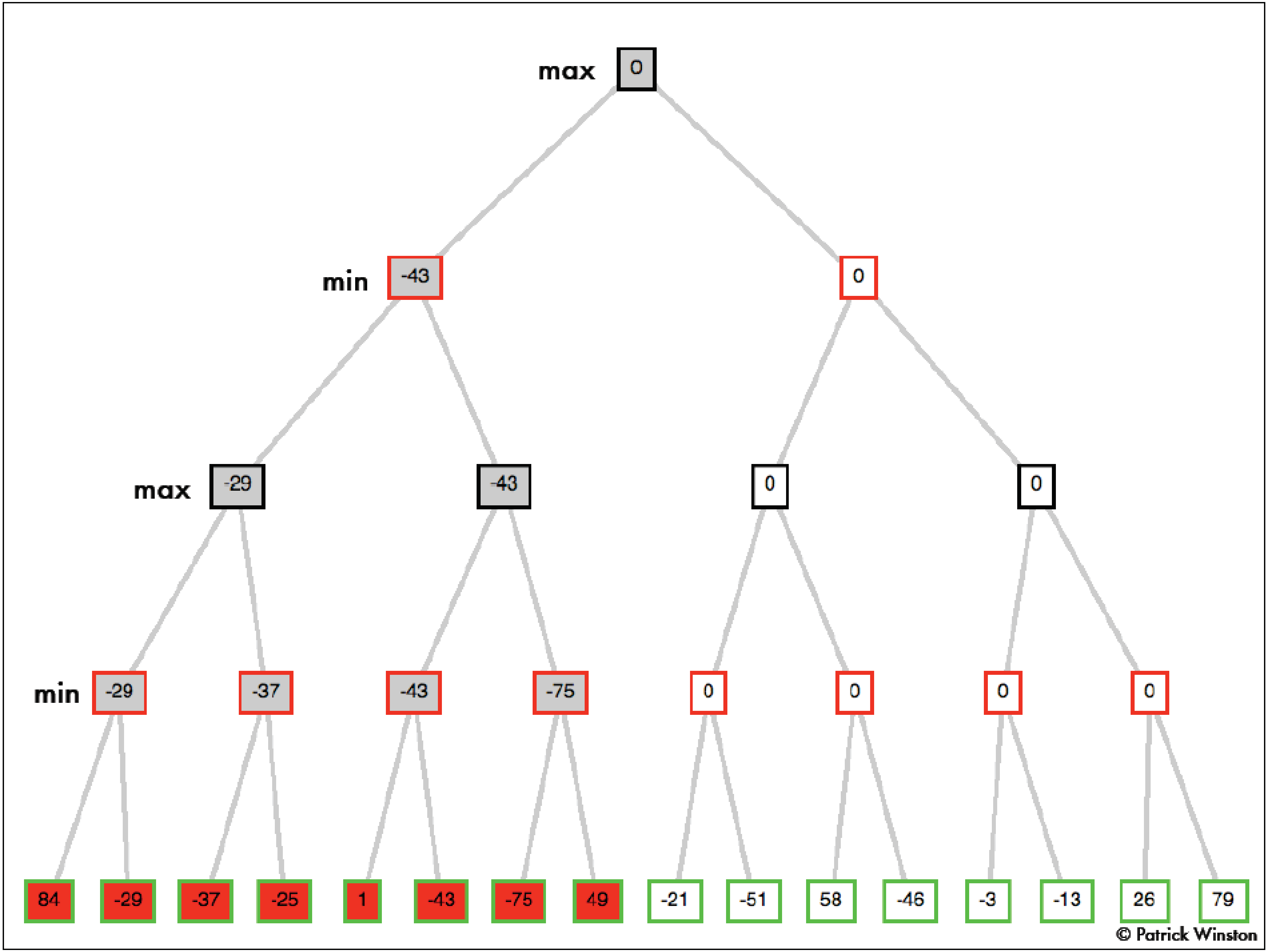
© Patrick Winston



max
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84
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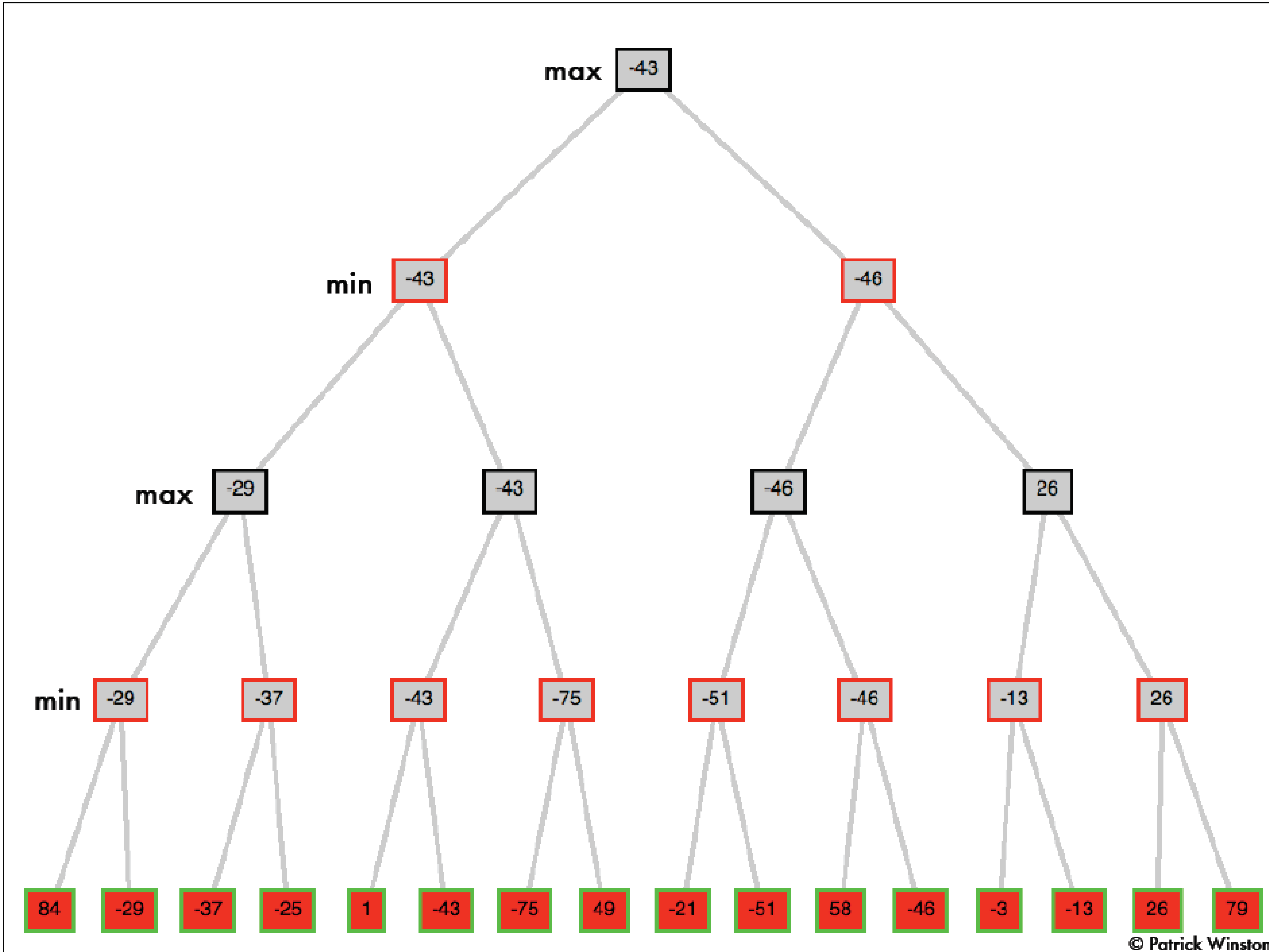
© Patrick Winston


max
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min
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max
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min
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-46
-13
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84
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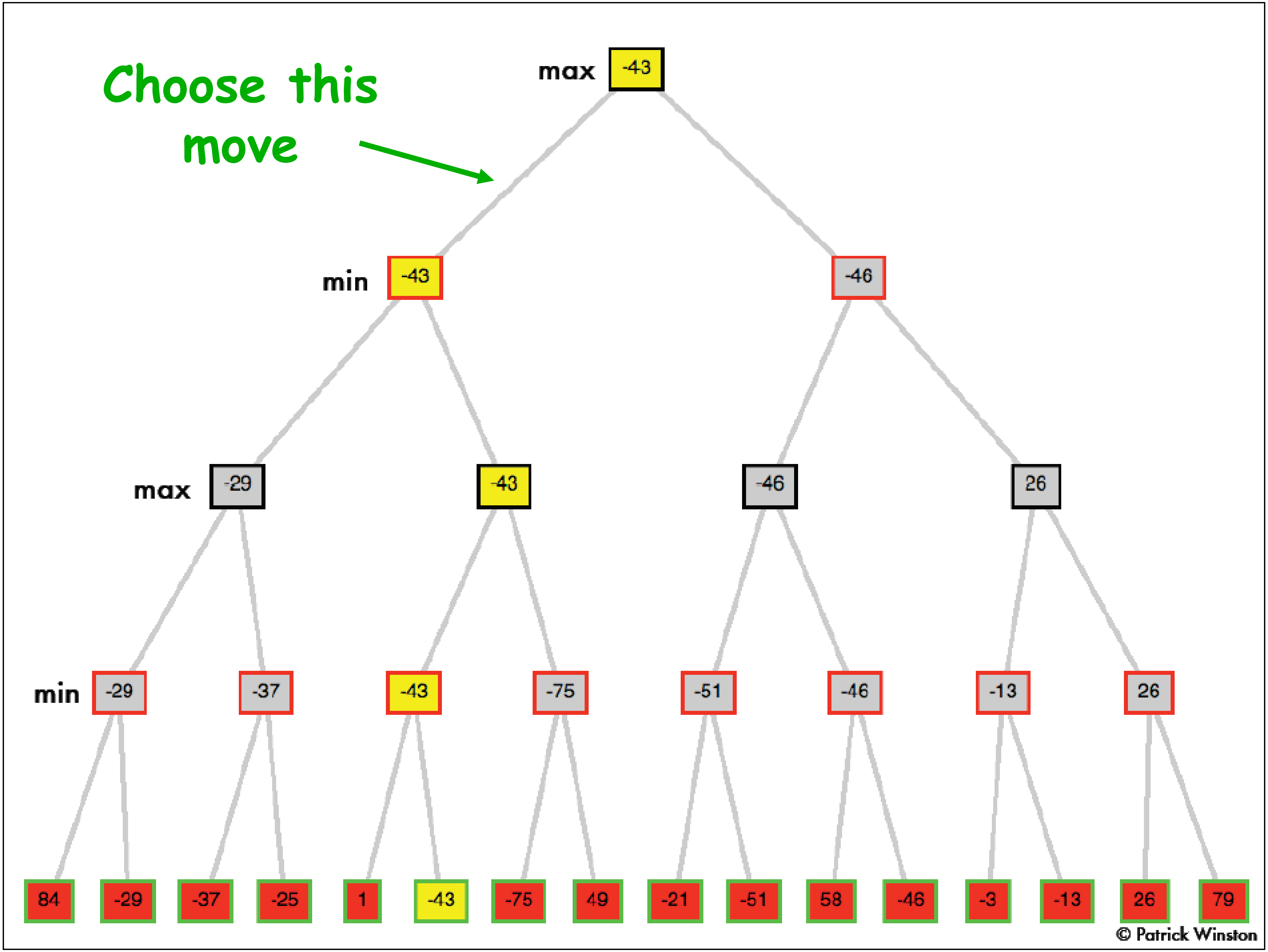
© Patrick Winston


Choose this move
max
-43
min
-43
-46
max
-29
-43
-46
26
min
-29
-37
-43
-75
-51
-46
-13
26
84
-29
-37
-25
1
-43
-75
49
-21
-51
58
-46
-3
-13
26
79
© Patrick Winston

Minimax Algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*

  $v \leftarrow$ MAX-VALUE(*state*)
  return the *action* in SUCCESSORS(*state*) with value v

---

function MAX-VALUE(*state*) **returns** *a utility value*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s))
  return v

---

function MIN-VALUE(*state*) **returns** *a utility value*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow \infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(s))
  return v

Properties of minimax

Complete? Yes (if tree is finite)

Optimal? Yes (against an optimal opponent)

Time complexity? $O(b^m)$

Space complexity? $O(bm)$ (depth-first exploration)

Good enough?

Chess:

- branching factor $b \approx 35$
- game length $m \approx 100$
- search space $b^m \approx 35^{100} \approx 10^{154}$

The Universe:

- number of atoms $\approx 10^{78}$
- age $\approx 10^{21}$ milliseconds

Can we search more efficiently?

Next Class:
Wrap up of search
Logic and Reasoning

To do:

Homework #1

Sign up for class mailing list