Derivation of Laplacian

1D function $f(x)$

$$[f(-1) \quad f(0) \quad f(1)]$$

$$f(0)-f(-1) \qquad f(1)-f(0)$$

$$(f(1)-f(0)) - (f(0)-f(-1))$$

$$f(-1) - 2f(0) + f(1)$$

$$[\,1 \quad -2 \quad 1\,]$$

2D function $f(x,y)$

	$f(0,1)$	
$f(-1,0)$	$f(0,0)$	$f(1,0)$
	$f(0,-1)$	

$$\frac{\partial f}{\partial x^2} = \frac{\partial f}{\partial x}\left(\tfrac{1}{2}\right) - \frac{\partial f}{\partial x}\left(-\tfrac{1}{2}\right)$$

$$\frac{\partial f}{\partial x}\left(\tfrac{1}{2}\right) = f(1,0) - f(0,0)$$

$$\frac{\partial f}{\partial x}\left(-\tfrac{1}{2}\right) = f(0,0) - f(-1,0)$$

So $\partial f/\partial x^2 = f(1,0) - 2f(0,0) + f(-1,0) \qquad [1 \ -2 \ 1]_x$

and $\partial f/\partial y^2 = f(0,1) - 2f(0,0) + f(0,-1) \qquad [1 \ -2 \ 1]_y$

$$\nabla^2 f = \frac{\partial f}{\partial x^2} + \frac{\partial f}{\partial y^2} = f(1,0) - 4f(0,0) + f(-1,0) + f(0,1) + f(0,-1)$$

or $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ yay!