Edge Detection

From Sandlot Science

Today's reading

- Cipolla & Gee on edge detection (available online)
- Szelisi 3.4.1-3.4.2

Announcements

Mailing list on catalyst

- you should have received a message

Office hours by appointment

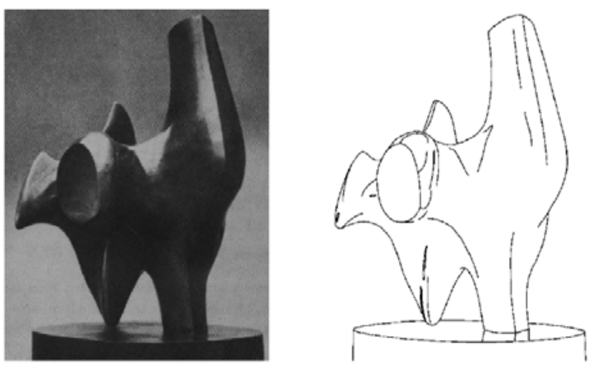
Project 1 out today (due in two weeks)

  – posted on course web page

New late day policy: 3 free late days

- use any time over the quarter
- e.g., if you use them all in project 1, you would need to turn it in by Saturday at 6:30.

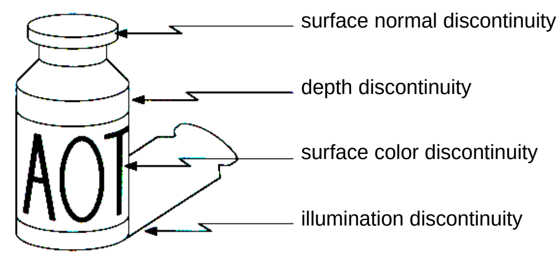
Edge detection



Convert a 2D image into a set of curves

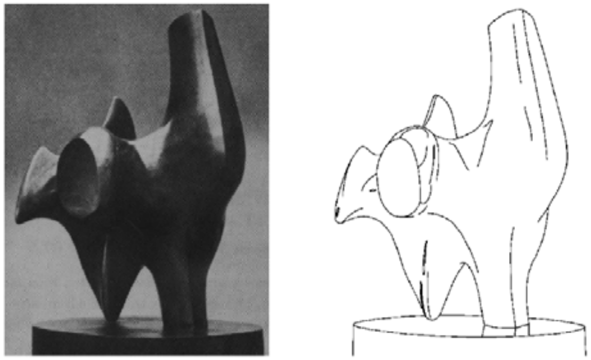
- Extracts salient features of the scene
- More compact than pixels

Origin of Edges



Edges are caused by a variety of factors

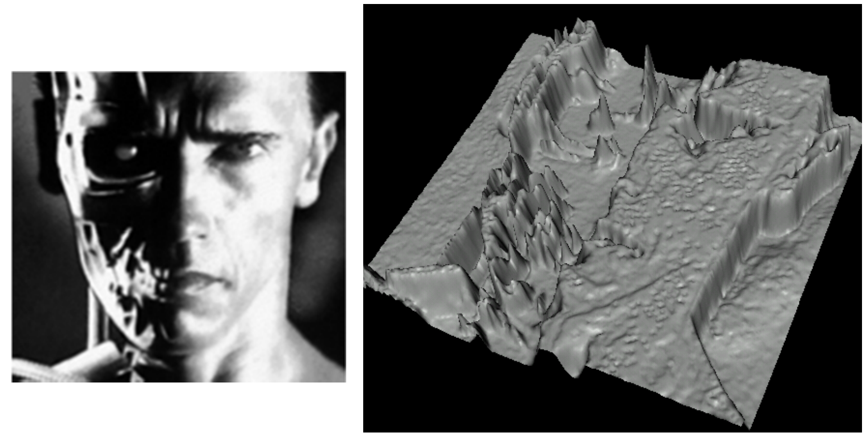
Edge detection



How can you tell that a pixel is on an edge?

snoop demo

Images as functions…

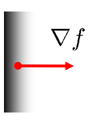


Edges look like steep cliffs

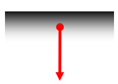
Image gradient

The gradient of an image:

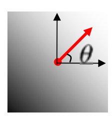
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The gradient points in the direction of most rapid increase in intensity

The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge? perpendicular

The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The discrete gradient

How can we differentiate a *digital* image F[x,y]?

The discrete gradient

How can we differentiate a *digital* image F[x,y]?

- Option 1: reconstruct a continuous image, then take gradient
- Option 2: take discrete derivative ("finite difference")

$$\frac{\partial f}{\partial x}[x,y] \approx F[x+1,y] - F[x,y]$$

How would you implement this as a cross-correlation?

0	0	0
0	-1	1
0	0	0

H

filter demo

The Sobel operator

Better approximations of the derivatives exist

- The *Sobel* operators below are very commonly used

$\frac{1}{8}$

-1	0	1
-2	0	2
-1	0	1

s_x

$\frac{1}{8}$

1	2	1
0	0	0
-1	-2	-1

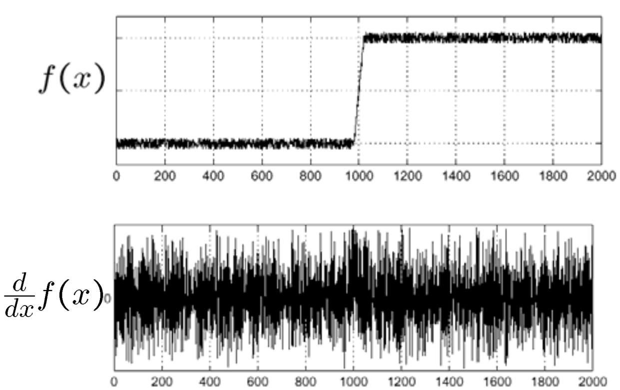
s_y

- The standard defn. of the Sobel operator omits the 1/8 term
  - doesn't make a difference for edge detection
  - the 1/8 term **is** needed to get the right gradient value, however

Effects of noise

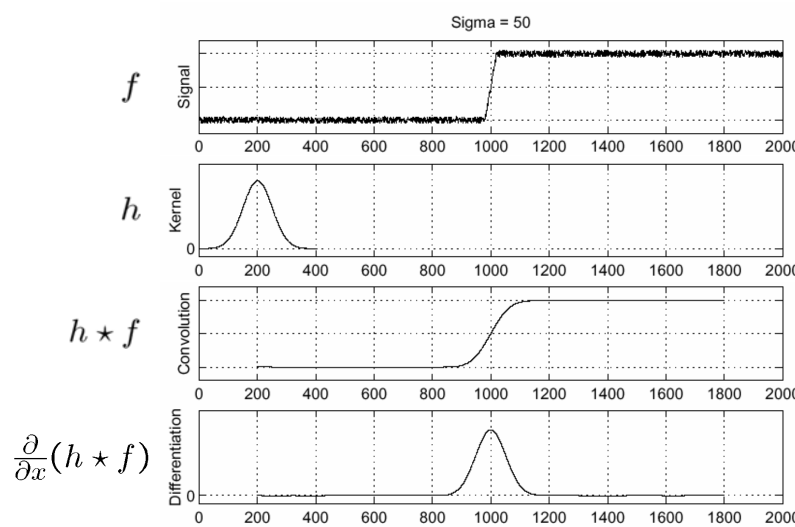
Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal



Where is the edge?

Solution: smooth first



Where is the edge? Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

Derivative theorem of convolution

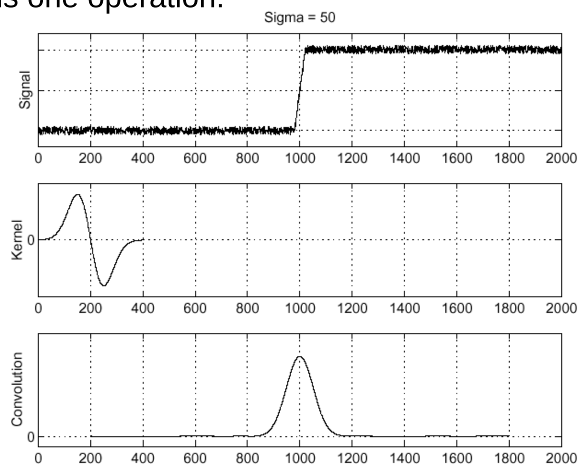
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

This saves us one operation:

f

$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) \star f$



How can we find (local) maxima of a function?

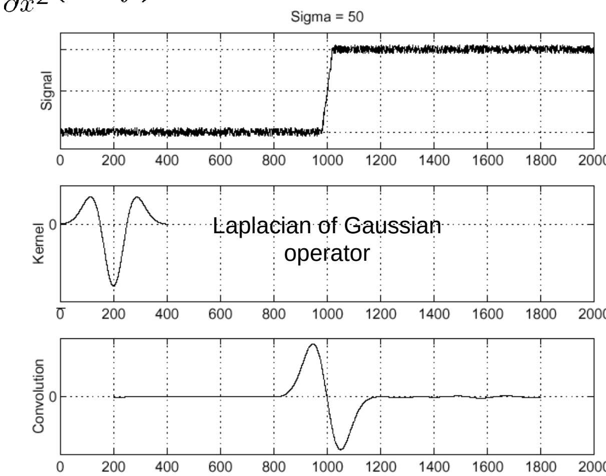
Laplacian of Gaussian

Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

f

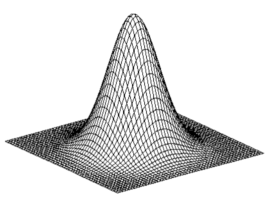
$\frac{\partial^2}{\partial x^2}h$

$(\frac{\partial^2}{\partial x^2}h) \star f$



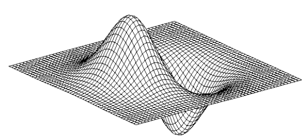
Where is the edge? Zero-crossings of bottom graph

2D edge detection filters



Gaussian

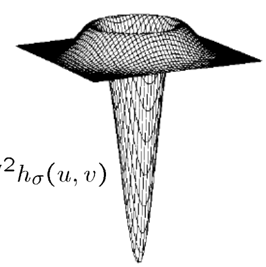
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2}e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x}h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

∇^2 is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

filter demo

Edge detection by subtraction

original

Edge detection by subtraction

smoothed (5x5 Gaussian)

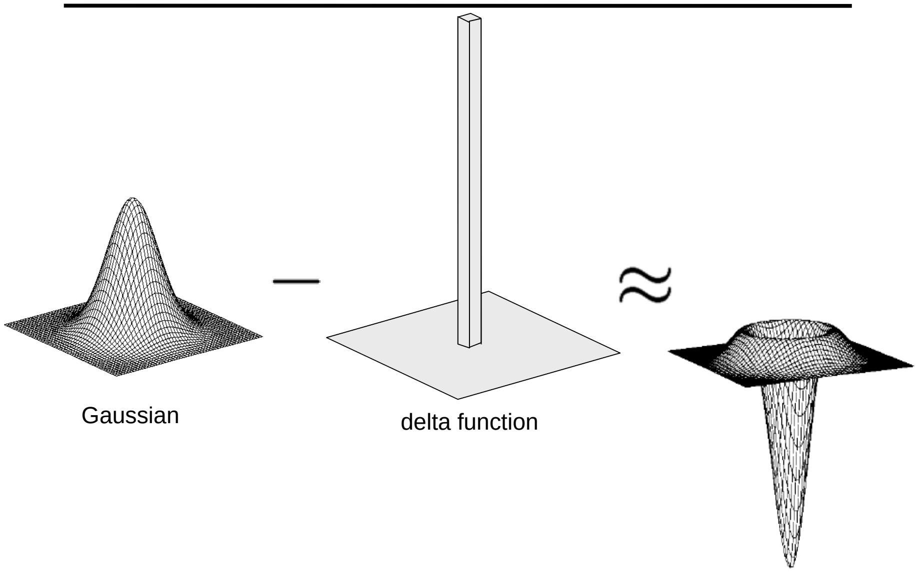
Edge detection by subtraction

smoothed – original
(scaled by 4, offset +128)

Why does this work?

filter demo

Gaussian - image filter



Gaussian − delta function ≈ Laplacian of Gaussian

The Canny edge detector

original image (Lena)

The Canny edge detector

norm of the gradient

The Canny edge detector

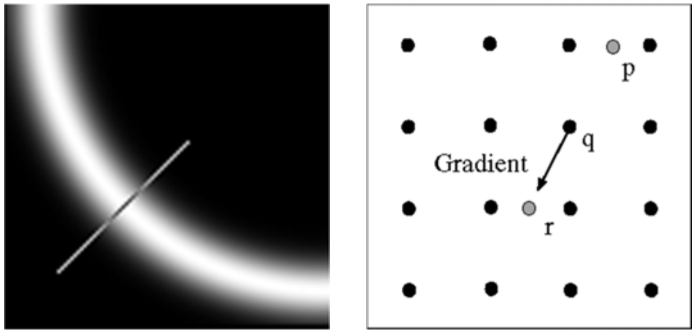
thresholding

The Canny edge detector

thinning

(non-maximum suppression)

Non-maximum suppression



Check if pixel is local maximum along gradient direction

- requires checking interpolated pixels p and r

Effect of σ (Gaussian kernel spread/size)

original

Canny with $\sigma = 1$

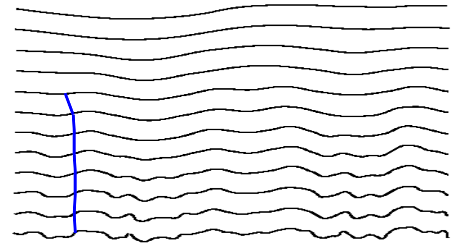
Canny with $\sigma = 2$

The choice of σ depends on desired behavior

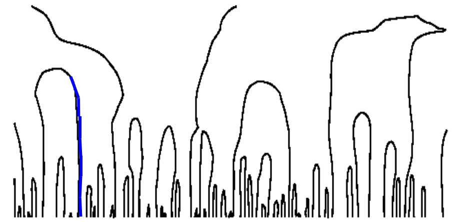
- large σ detects large scale edges
- small σ detects fine features

The effect of scale on edge detection

larger σ



larger σ



Scale space (Witkin 83)