

2.1

$$I_{\text{corr}}(x, y) = \int_t \int_s I(x+s, y+t) k(s, t) ds dt$$

$$1D \quad I_{\text{corr}}(x) = \int_s I(x+s) k(s) ds$$

$$I = \quad 9 \quad 5 \quad 2 \quad 1 \quad 3 \quad 4 \quad 6 \quad 2 \quad 4$$

$$k = \quad 1 \quad 2 \quad 1$$

↓

$$21 \quad 10 \quad 7 \quad \dots$$

2.2 let  $s = -a, t = -b$

$$\text{corr}(I, k) = \int_b \int_a I(x-a, y-b) k(-a, -b) |J| da db = \text{conv}(I, k')$$

where  $k'(a, b) = k(-a, -b)$  flip in  $x, y \Rightarrow$  rotate  $180^\circ$

2.3

$$\frac{1}{4} [1 \ 2 \ 1] * [72 \ 88 \ 62 \ 52 \ 37]$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 72 \\ 88 \\ 62 \\ 52 \\ 37 \\ 0 \end{bmatrix}$$

linear,

associative  $a * (b * c) = (a * b) * c$

$$I' = \int_s \int_t I(x-s, y-t) k(s, t) ds dt \quad \begin{array}{l} a = x-s \\ b = y-t \end{array}$$

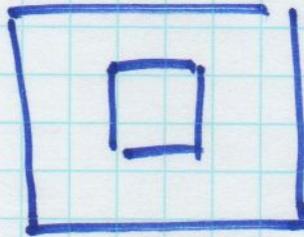
$$\int_a \int_b I(a, b) k(x-a, y-b) db da$$

$$I * k = k * I \rightarrow \text{commutative}$$

2.4



$2D * / +$



$n^2 * / +$

$$I(x, y) * g(x, y) = f(x, y) * g(x) * g(y)$$

$$g(x)g(y) = g(x) * g(y)$$

$$g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$$

2.5

$$SSD = \sum_R |I(x+\Delta x) - I(x)|^2$$

$$I(x+\Delta x) = I(x) + \nabla I^T \Delta x + \dots$$

$$SSD \approx \sum_R |\nabla I^T \Delta x|^2 = \Delta x^T \left( \sum_R \nabla I \nabla I^T \right) \Delta x$$

$$H = \sum_R \nabla I \nabla I^T \text{ corner if 2 large eigval.}$$

$$\nabla I = \begin{bmatrix} I_x \\ I_y \end{bmatrix}$$

$$H = \sum_R \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$$

$$\chi_{Harris} = \det(H) - k \text{Tr}(H)^2$$

derivative scale  $\sigma_D$  used for  $I_x I_y$

integration scale  $\sigma_I$  used for conv of  $I_x^2, I_x I_y \dots$

2.6

Normalized correlation

$$\frac{I^T J}{\|I\| \|J\|}$$

$$\begin{aligned} \text{SSD} &= \|I - J\|^2 = (I - J)^T (I - J) \\ &= \|I\|^2 + \|J\|^2 - 2I^T J \end{aligned}$$

so if  $\|I\| = \|J\| = 1$

$$\text{SSD} = 2 - 2 \text{CORR}$$