Epipolar Geometry

CSE P576

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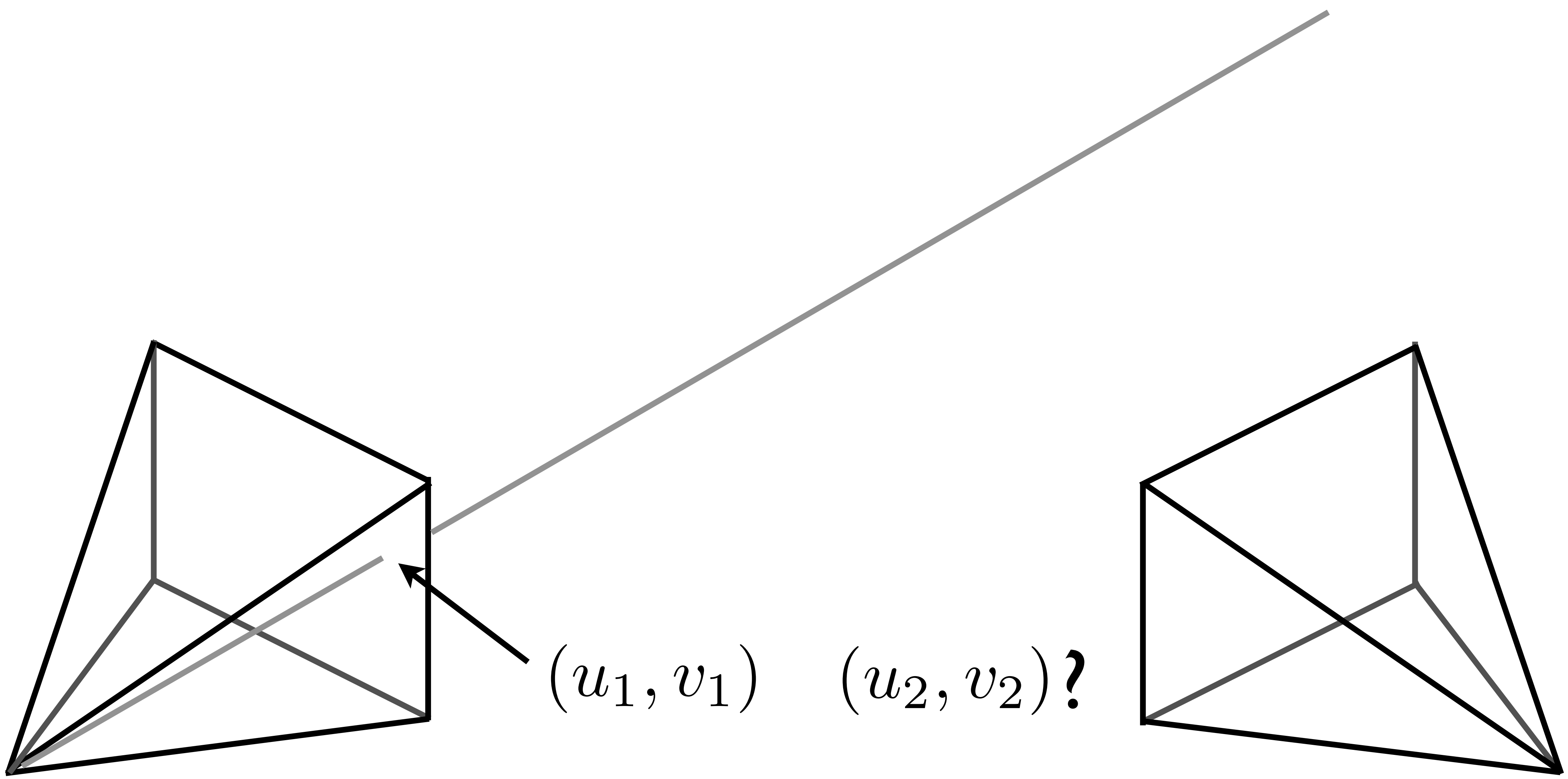
Epipolar Geometry

- Epipolar Lines, Plane Constraint
- Fundamental Matrix, Linear solution + RANSAC
- Applications: Structure from Motion, Stereo

[Szeliski 11]

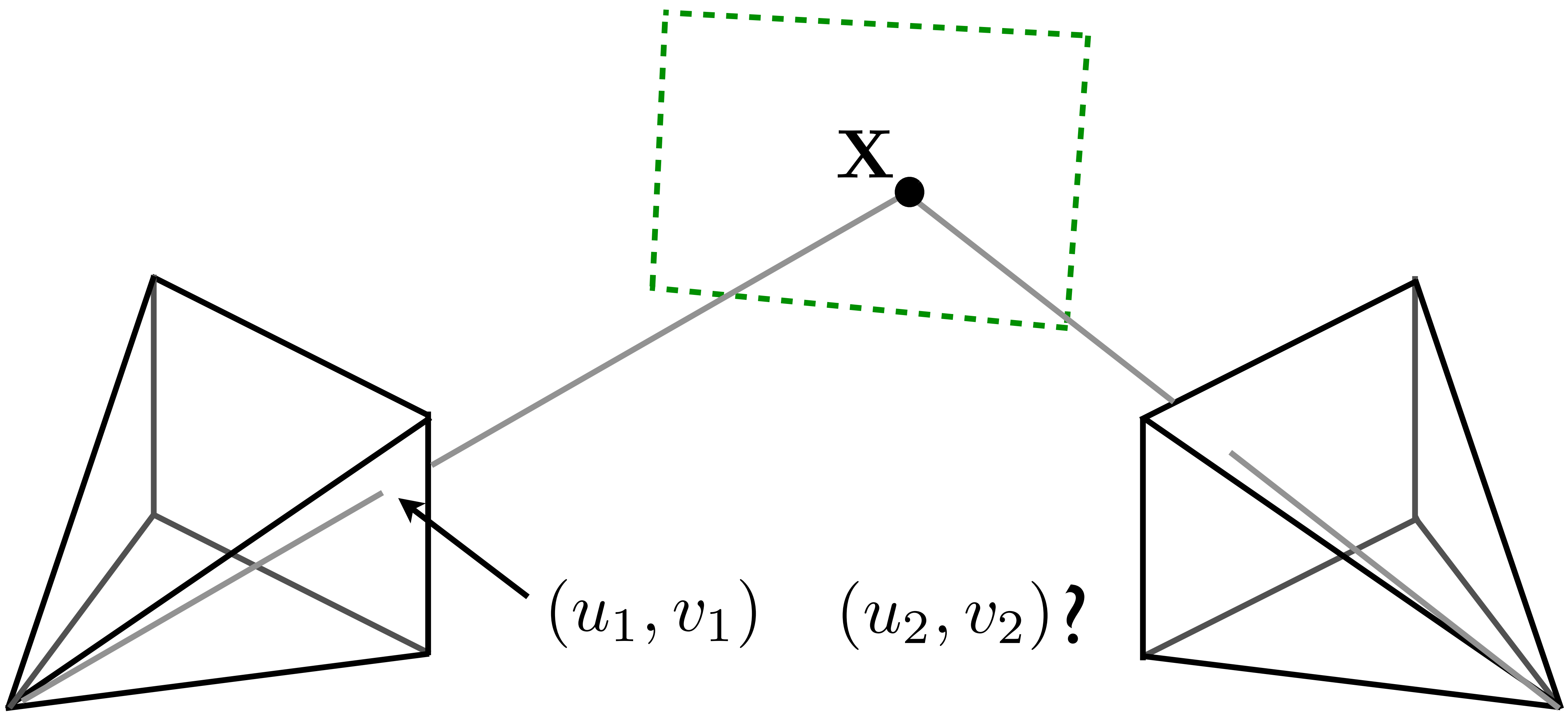
2-view Geometry

- How do we transfer points between 2 views?



2-view Geometry

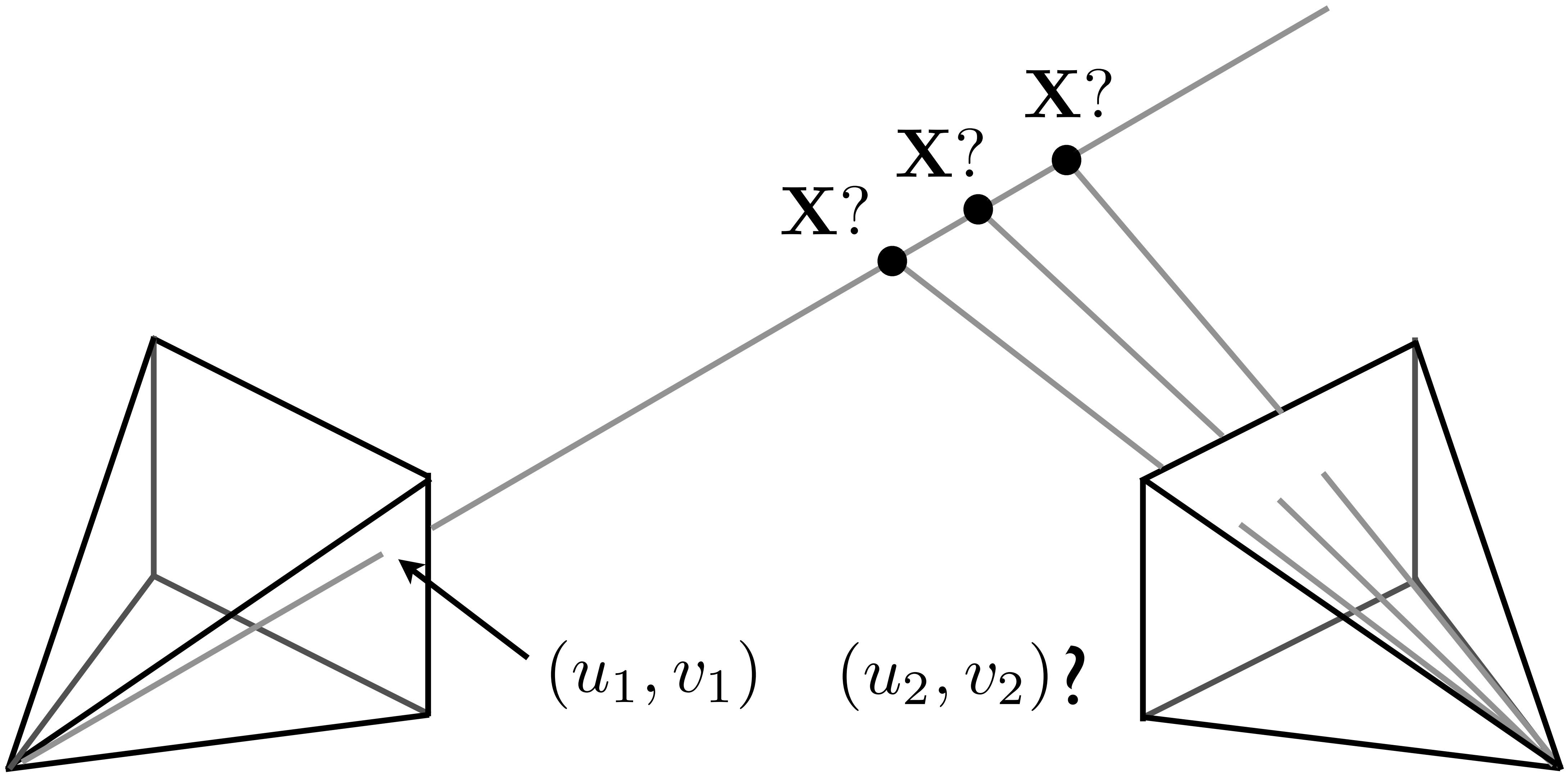
- How do we transfer points between 2 views? (planar case)



Planar case: one-to-one mapping via plane

2-view Geometry

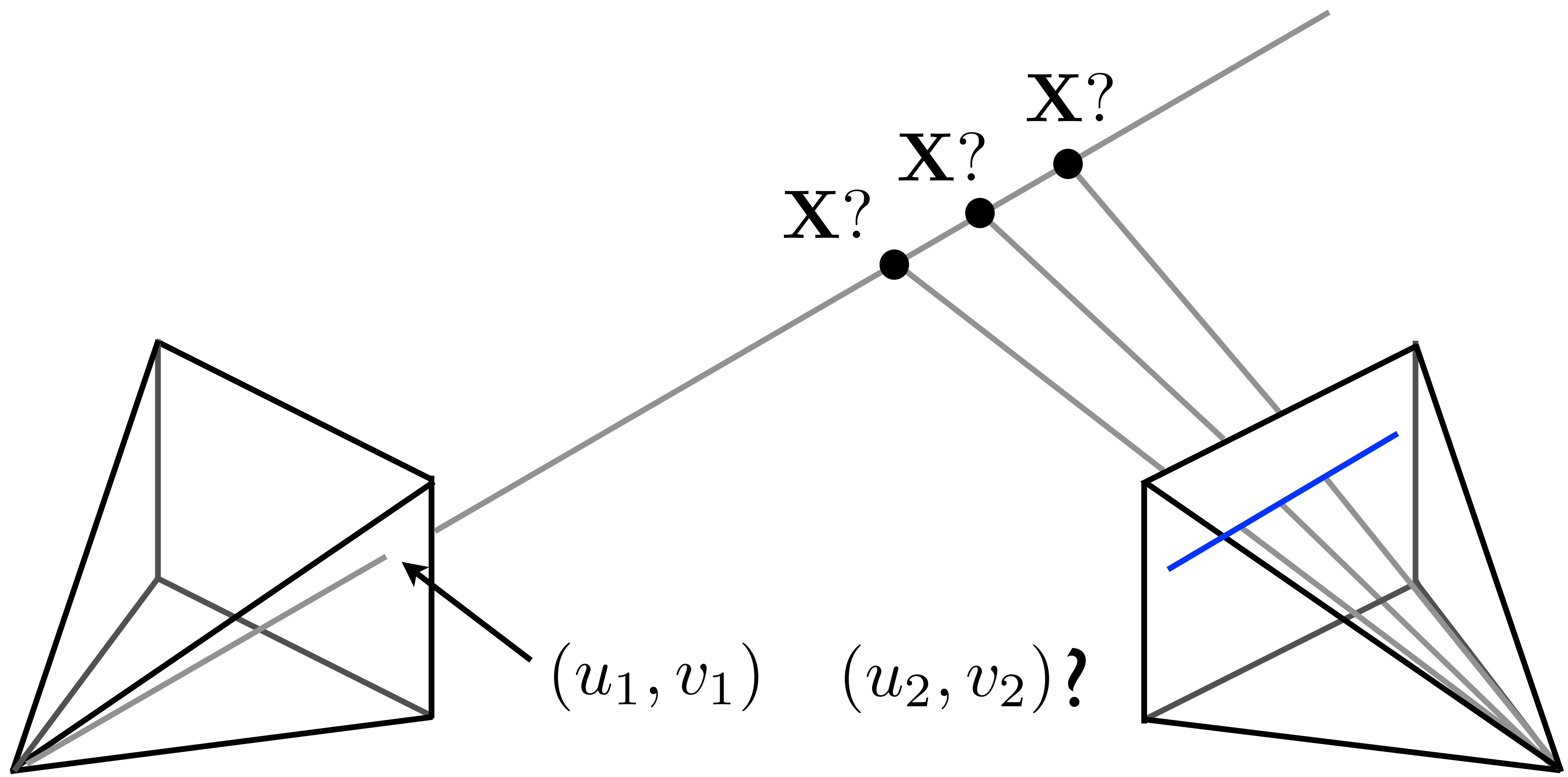
- How do we transfer points between 2 views? (non-planar)



Non-planar case: depends on the depth of the 3D point

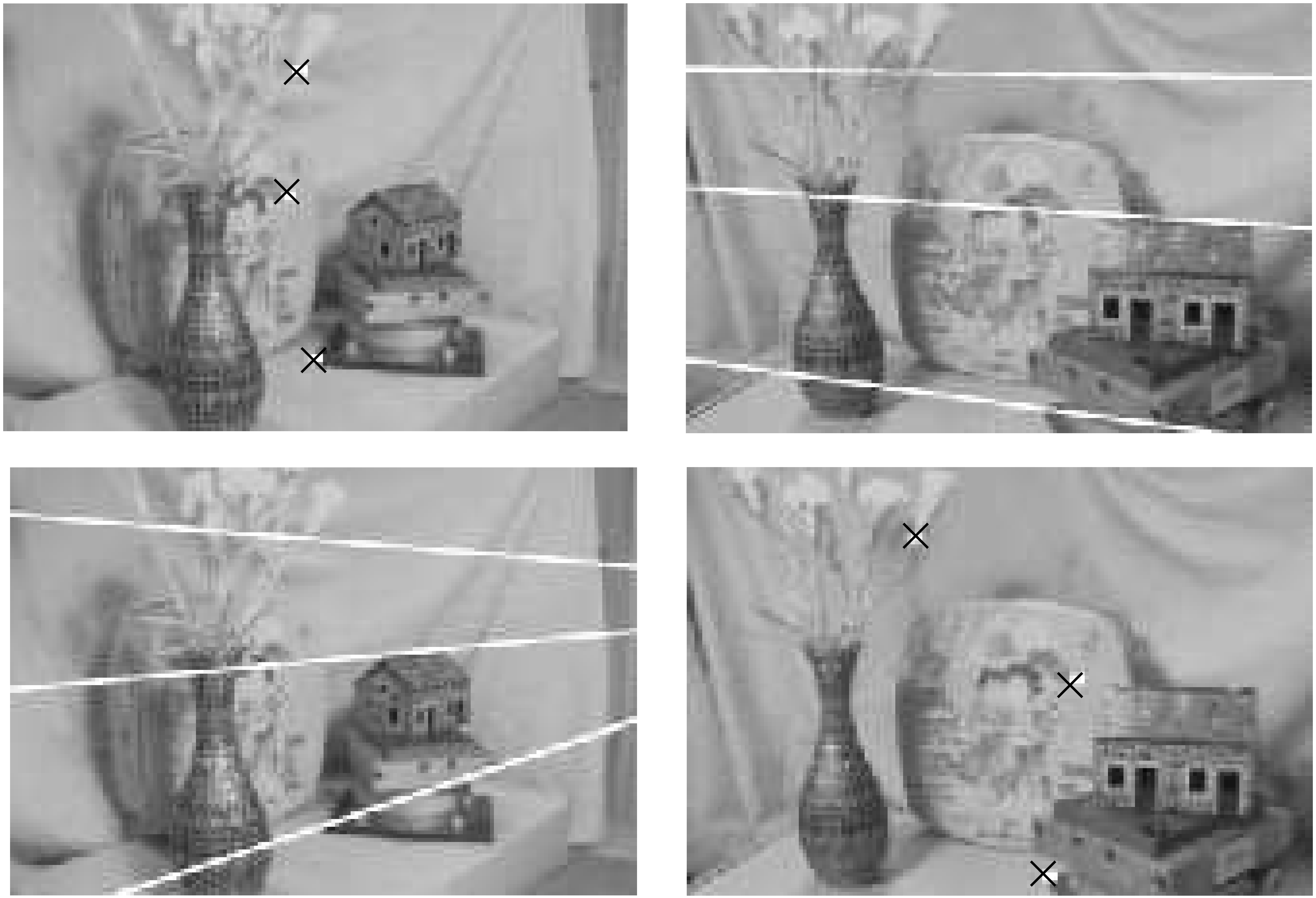
Epipolar Line

- How do we transfer points between 2 views? (non-planar)



A point in image 1 gives a **line** in image 2

Epipolar Lines



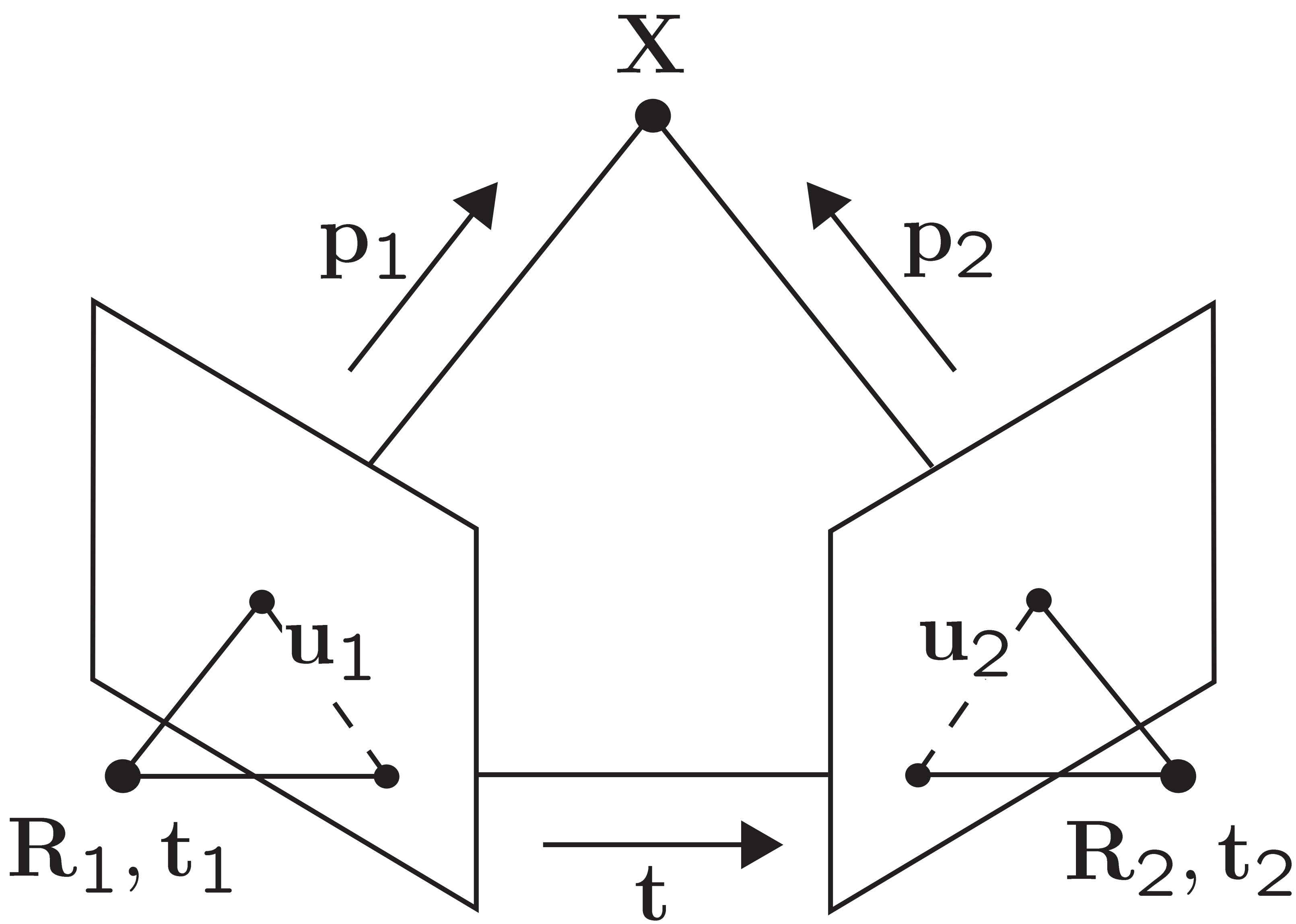
[R. Cipolla]

Epipolar Lines

[R. Cipolla]

The Epipolar Constraint

- For rays to intersect at a point (X), the two rays and the camera translation must lie in the same plane



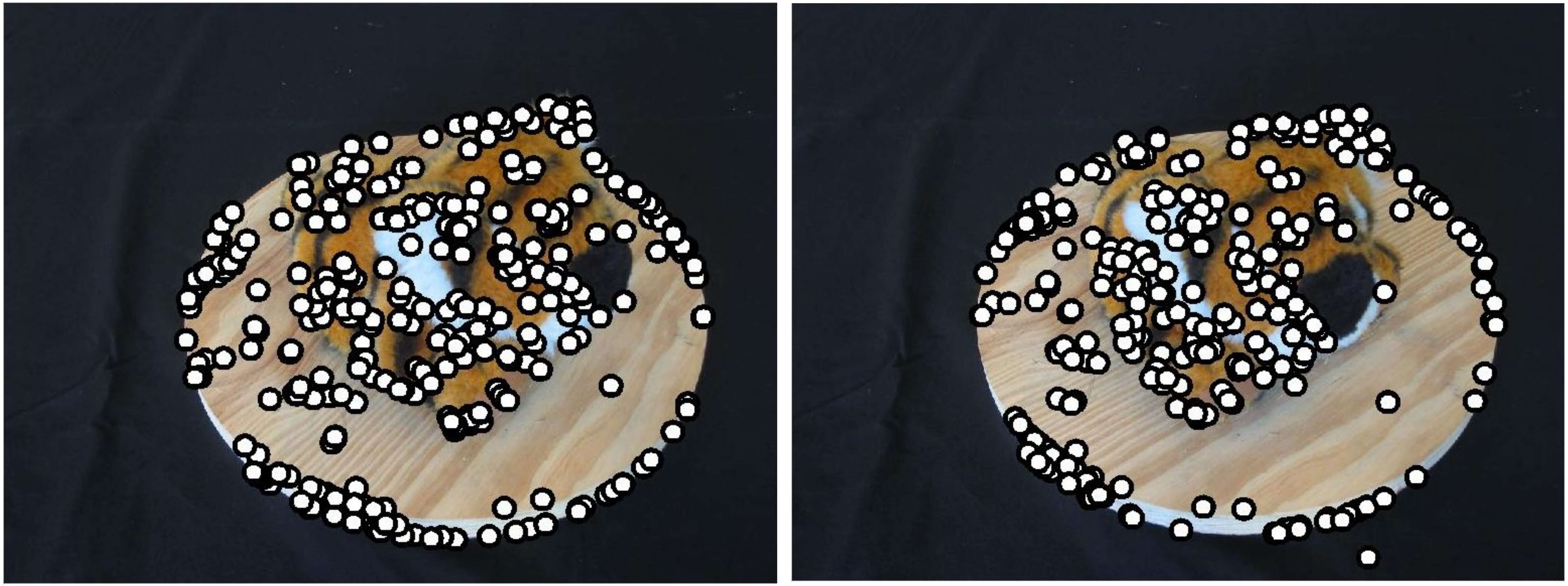
3.10

Epipolar Geometry

- Example: 2-view matching in 3D

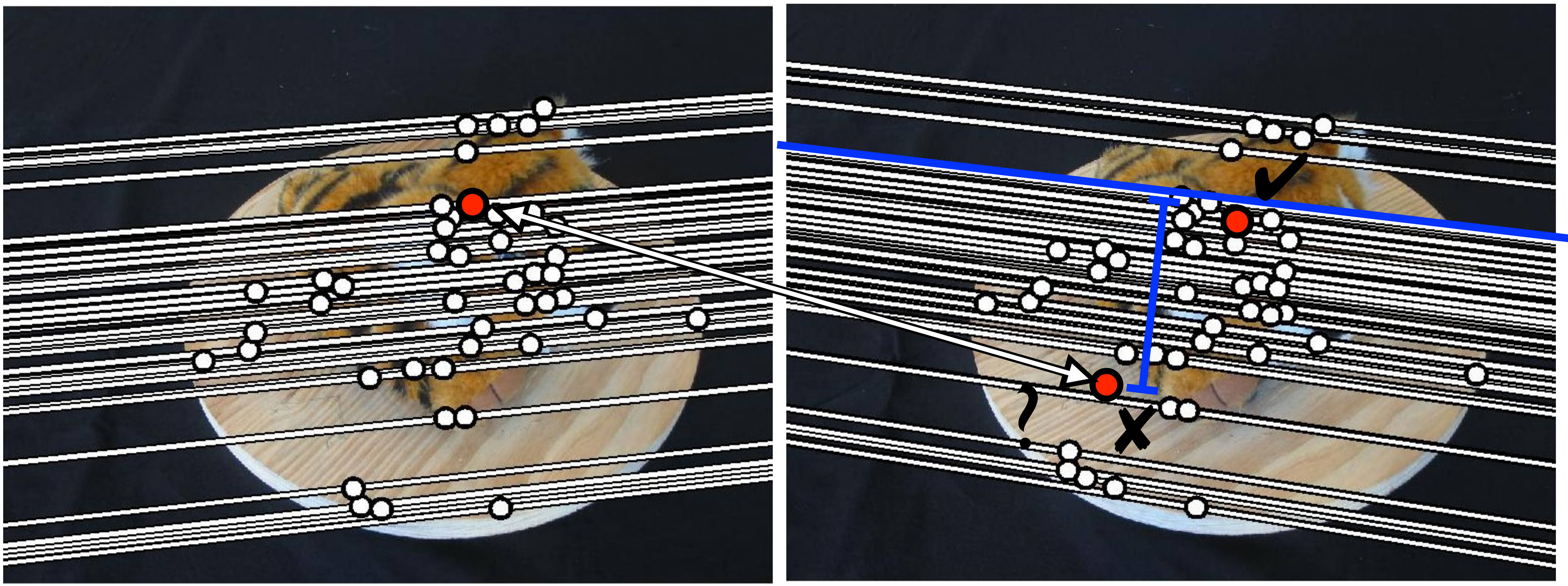
Epipolar Geometry

- Raw SIFT matches



Epipolar Geometry

- Epipolar lines



Can use RANSAC to find inliers with small distance from epipolar line

Epipolar Geometry

- Consistent matches

Computing F

- Single correspondence gives us one equation

$$\begin{bmatrix} u_1 & v_1 & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = 0$$

- Multiply out

$$u_1 x_1 f_{11} + u_1 y_1 f_{12} + u_1 f_{13} + v_1 x_1 f_{21} + v_1 y_1 f_{22} \\ + v_1 f_{23} + x_1 f_{31} + y_1 f_{32} + f_{33} = 0$$

Computing F

- Rearrange for unknowns, add points by stacking rows

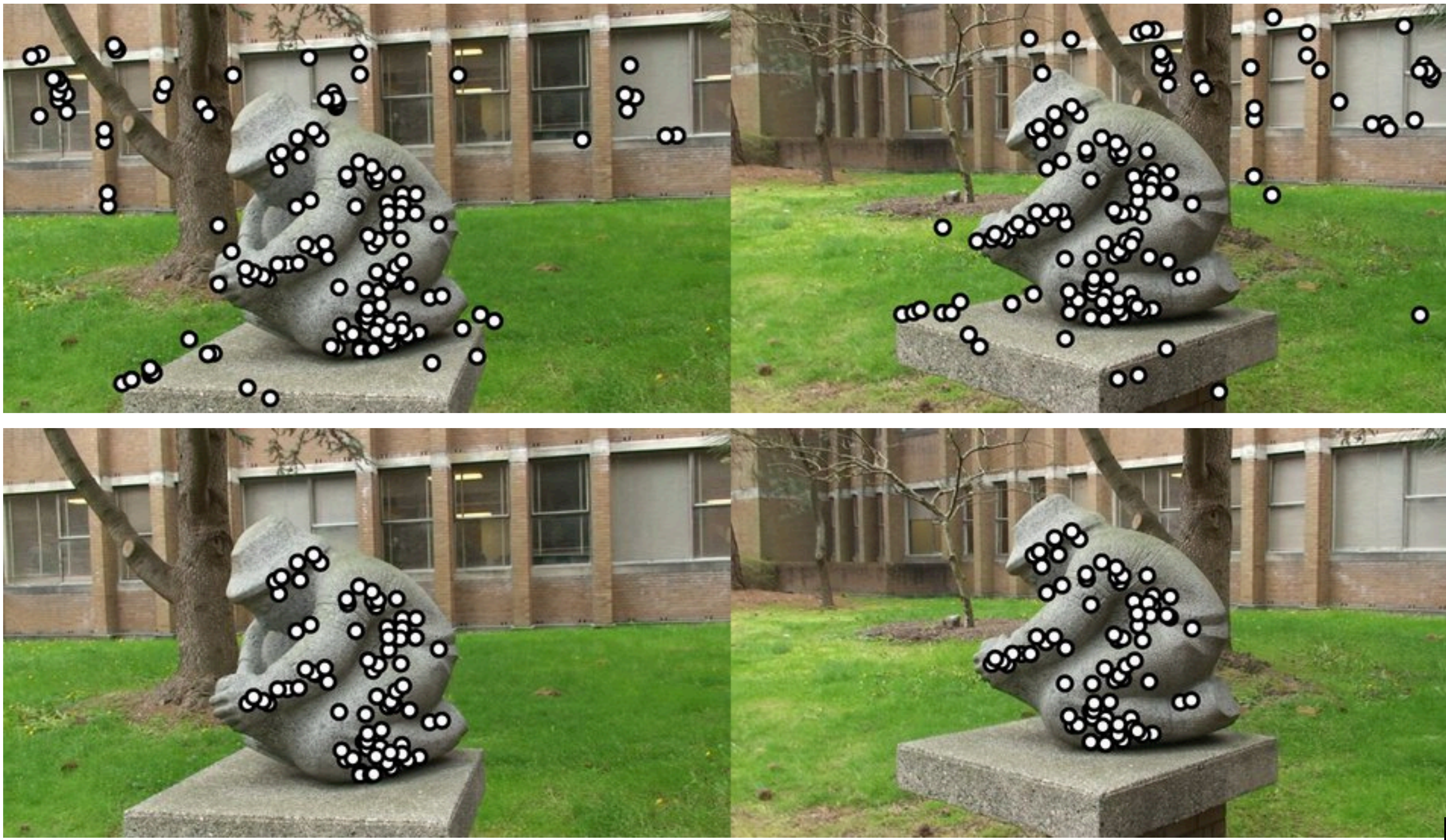
$$\begin{bmatrix} u_1x_1 & u_1y_1 & u_1 & v_1x_1 & v_1y_1 & v_1 & x_1 & y_1 & 1 \\ u_2x_2 & u_2y_2 & u_2 & v_2x_2 & v_2y_2 & v_2 & x_2 & y_2 & 1 \\ u_3x_3 & u_3y_3 & u_3 & v_3x_3 & v_3y_3 & v_3 & x_3 & y_3 & 1 \\ u_4x_4 & u_4y_4 & u_4 & v_4x_4 & v_4y_4 & v_4 & x_4 & y_4 & 1 \\ u_5x_5 & u_5y_5 & u_5 & v_5x_5 & v_5y_5 & v_5 & x_5 & y_5 & 1 \\ u_6x_6 & u_6y_6 & u_6 & v_6x_6 & v_6y_6 & v_6 & x_6 & y_6 & 1 \\ u_7x_7 & u_7y_7 & u_7 & v_7x_7 & v_7y_7 & v_7 & x_7 & y_7 & 1 \\ u_8x_8 & u_8y_8 & u_8 & v_8x_8 & v_8y_8 & v_8 & x_8 & y_8 & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

- This is a linear system of the form $\mathbf{Af} = \mathbf{0}$

  can be solved using Singular Value Decomposition (SVD)

Pairwise Matching

- Find feature matches and use RANSAC to find inliers to F



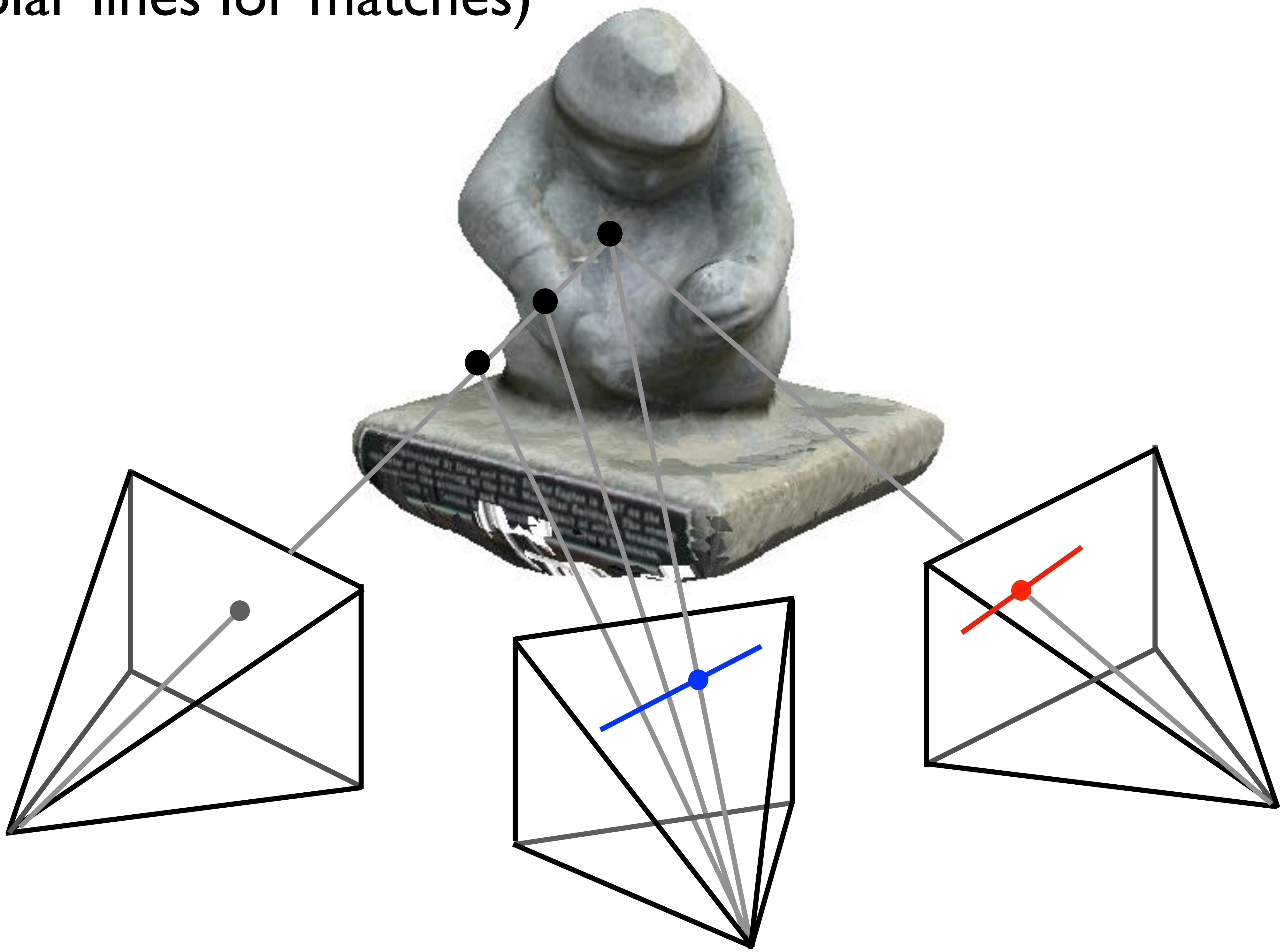
Structure from Motion

Given an (unordered) set of input images, compute camera positions and 3D structure of the scene

Sparse Reconstruction

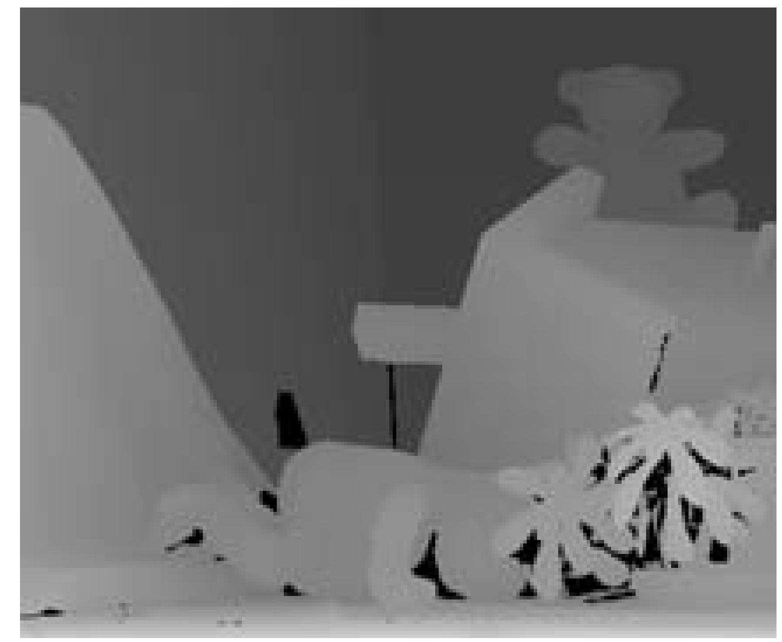
Dense Reconstruction

- Use multiview stereo to compute dense depth (search along epipolar lines for matches)



Stereo Matching

- Stereo cameras are typically arranged by horizontal translation so epipolar lines (thus point matches) are horizontal

Stereo algorithms attempt to match along scanlines to find depth

[D. Scharstein]

Disparity and Depth

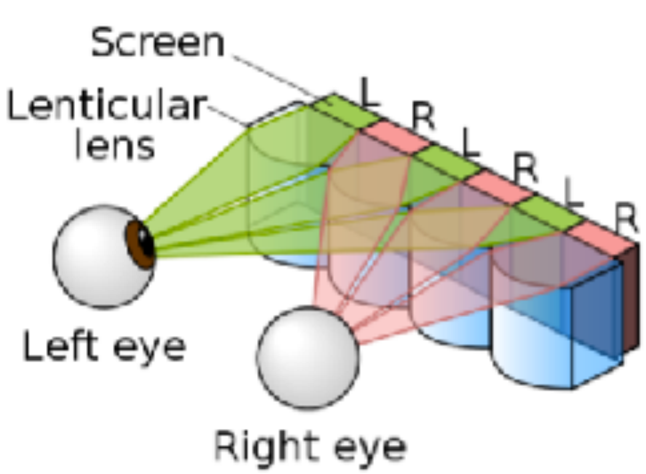
[D. Scharstein]

Stereo Images

- Image pair with depth dependent disparity

3D TV

- Most common use field sequential (shutter) glasses
- Transmit alternate left/right image at 120Hz



Glasses free 3D systems send different images directly to each eye

Recap: 2-view Geometry

- Planar geometry: one to one mapping of points

$$\mathbf{u} = \mathbf{H}\mathbf{x}$$

viewing a plane, rotation

Recap: 2-view Geometry

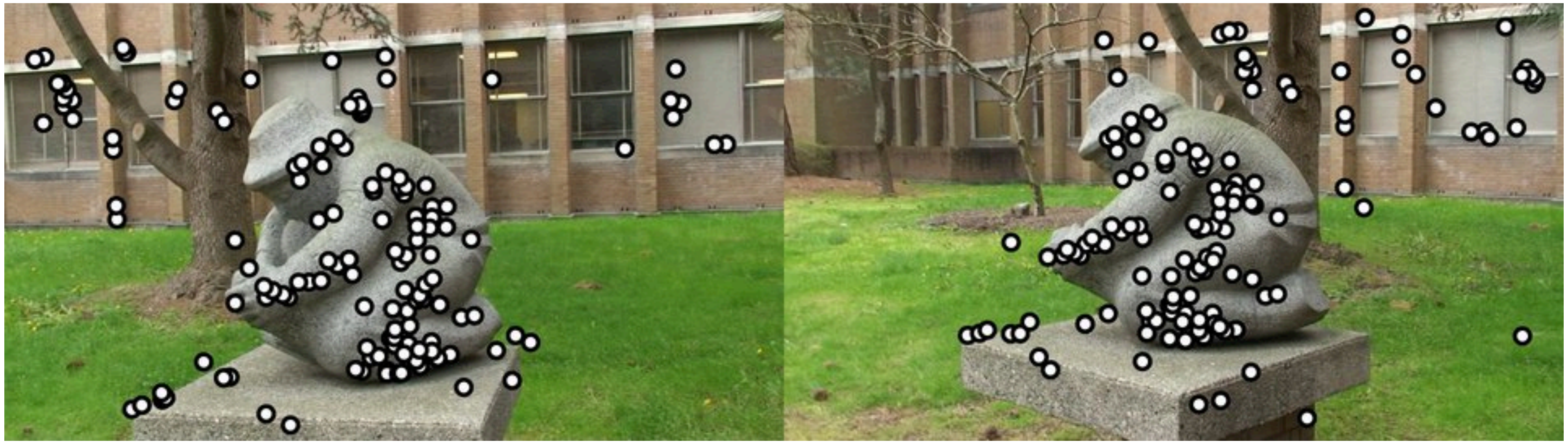
- Epipolar geometry: point to line mapping

$$\mathbf{u}^T \mathbf{F} \mathbf{x} = 0$$

moving camera, 3D scene

Recap: RANSAC

- Robust estimation of H/F and consistent matches



Raw feature matches

RANSAC for F

Next Lecture

- Multiview geometry + optimization