# 2-view Alignment and RANSAC 

CSE P576

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## 2-view Alignment + RANSAC

- 2-view alignment: linear equations
- Least squares and outliers
- Robust estimation via sampling


## Image Alignment

- Find corresponding (matching) points between the images

$\mathbf{u}=\mathbf{H x}$
2 points for Similarity
3 for Affine
4 for Homography


## Image Alignment

- In practice we have many noisy correspondences + outliers



## Linear Equations

- e.g., for an affine transform we have a linear system in the unknown parameters a:

$$
\begin{gathered}
{\left[\begin{array}{cccccc}
x_{1} & y_{1} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & x_{1} & y_{1} & 1 \\
x_{2} & y_{2} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & x_{2} & y_{2} & 1 \\
x_{3} & y_{3} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & x_{3} & y_{3} & 1
\end{array}\right]\left[\begin{array}{l}
a_{11} \\
a_{12} \\
a_{13} \\
a_{21} \\
a_{22} \\
a_{23}
\end{array}\right]=\left[\begin{array}{l}
x_{1}^{\prime} \\
y_{1}^{\prime} \\
x_{2}^{\prime} \\
y_{2}^{\prime} \\
x_{3}^{\prime} \\
y_{3}^{\prime}
\end{array}\right]}
\end{gathered}
$$

- It is overconstrained (more equations than unknowns)
- and subject to outliers (some rows are completely wrong)

Let's deal with these problems in a simpler context..

## Robust Line Fitting

- Consider fitting a line to noisy points
3.3


## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)



## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)



## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)


4 inliers (red, yellow, orange, brown),

## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)


4 outliers (blue, light blue, purple, pink)

## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)


4 inliers (red, yellow, orange, brown),
4 outliers (blue, light blue, purple, pink)

## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

cbberlentingraindigequcese \#inliers = 2


## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)



## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

cheblowappplrmadigelaneces
\#inliers = 2


## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)



## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

clatrdobrapcimadigeargies
\#inliers $=4$


## RANSAC Example

- RANSAC solution for Similarity Transform (2 points)



## RANSAC algorithm

I. Match feature points between 2 views
2. Select minimal subset of matches*
3. Compute transformation T using minimal subset
4. Check consistency of all points with T - compute projected position and count \#inliers with distance < threshold
5. Repeat steps 2-4 to maximise \#inliers

* Similarity transform = 2 points, Affine $=3$, Homography $=4$


## Project 2

- Try out the RANSAC Implementation section in Project 2.


## 2-view Rotation Estimation

- Find features + raw matches, use RANSAC to find Similarity



## 2-view Rotation Estimation

- Remove outliers, can now solve for $R$ using least squares



## 2-view Rotation Estimation

- Final rotation estimation



## Rotation Estimation

- We can solve for 3D rotation by forming a correlation matrix of corresponding rays (unit vectors in camera coordinates)

$$
\begin{aligned}
\boldsymbol{C}=\sum_{i} \hat{\boldsymbol{x}}^{\prime} \hat{\boldsymbol{x}}^{T} & =\boldsymbol{U} \boldsymbol{\Sigma} \boldsymbol{V}^{T} \\
\boldsymbol{R} & =\boldsymbol{U} \boldsymbol{V}^{T}
\end{aligned}
$$

- The solution for R minimizes the squared distance between corresponding rays, this is known as an "Orthogonal Procrustes Problem", see Szeliski p32I,Arun et al 1987.
- You can use this to complete the Rotation Estimation section in Project 2


## Next Lecture

- Epipolar Geometry, Multiview Reconstruction

