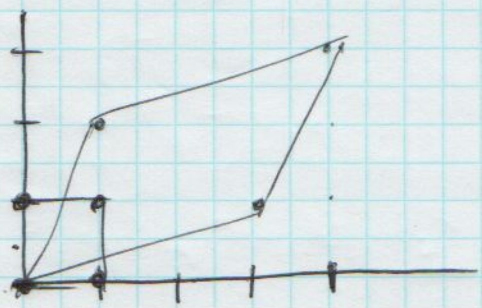
3.1

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3.2

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 & 4 \\ 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$A \begin{bmatrix} x_1 & x_2 & x_3 & \cdots \end{bmatrix} = \begin{bmatrix} Ax_1 & Ax_2 & Ax_3 & \cdots \end{bmatrix}$$



$\parallel$ lines $\quad l_1 = r + \underline{\lambda t}$

$\qquad\qquad\quad l_2 = s + \underline{\lambda t}$

Affine xform $\quad x' = Ax + b$

$$l_1' = Ar + b + \underline{\lambda A t}$$

$$l_2' = As + b + \underline{\lambda A t}$$

stay parallel.

3.3

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \end{bmatrix}$$

3.4

$$\left[\begin{array}{cc|c} \multicolumn{2}{c|}{\overset{2\times 2}{A}} & a_{13} \\ & & a_{23} \\ \hline 0 & 0 & 1 \end{array}\right]$$

"affine"

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

"similarity"

$$= s \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

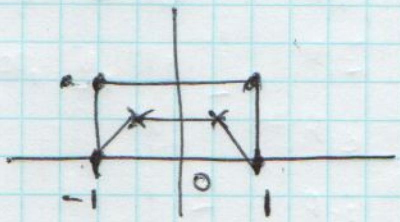
if $s = 1$, "Euclidean"

$A = I$, Translation

3.5

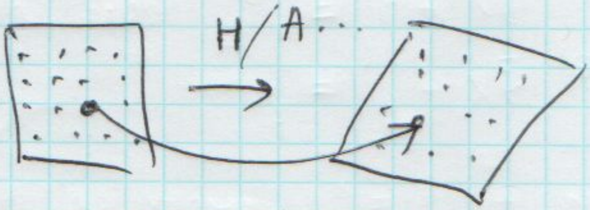
$$s\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} \rightarrow \begin{bmatrix} a/c \\ b/c \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \underset{1}{\bullet} & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} -1 & -0.5 & 1 & 0.5 \\ 0 & 0.5 & 0 & 0.5 \end{bmatrix}$$



3.6

$$\begin{array}{c} \text{camera} \\ \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} \end{array} = \overset{4}{\left(\underbrace{R}_{\substack{3\times 3 \\ \text{orthogonal} \\ \text{(rotation)}}} \;\middle|\; \underbrace{t}_{3\times 1} \right)_3} \begin{array}{c} \text{world} \\ \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \end{array}$$

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = K\left(R \mid t \right)\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

where $K = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix}$ intrinsic/calibration matrix

$$K = \begin{pmatrix} f_1 & \overset{\text{skew}}{s_1} & c_1 \\ \underset{0}{s_2} & f_2 & c_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$\begin{matrix} c_1 \\ c_2 \end{matrix}$ — principal point

$$\begin{pmatrix} \overline{} \\ 0 \; - \\ 0 \; 0 \; \cdot \end{pmatrix}\begin{pmatrix} R \end{pmatrix}\left(R \mid t \right)$$

$$K = T R_1 \left(R \mid t \right)$$

$$s\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underline{K}\boxed{R_1 \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix}}$$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot \\ 0 & 0 & \cdot \end{pmatrix} R$$

3.7 Linear camera
$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$
$$= {}_2\overset{3}{A} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + b \quad b = \begin{pmatrix} a_{14} \\ a_{24} \end{pmatrix}$$

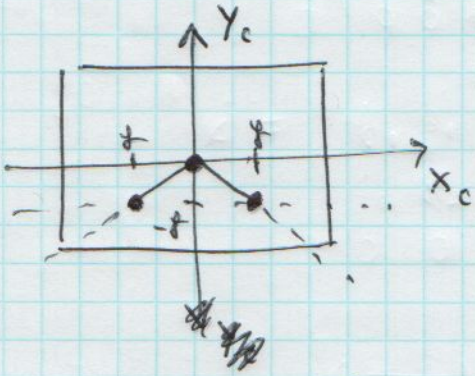
$\uparrow\uparrow$ lines in 3D $\quad l_i = r_i + \underline{\lambda} t \rightarrow A r_i + b + \underline{\lambda A t}$

stay parallel in the image

perspective camera

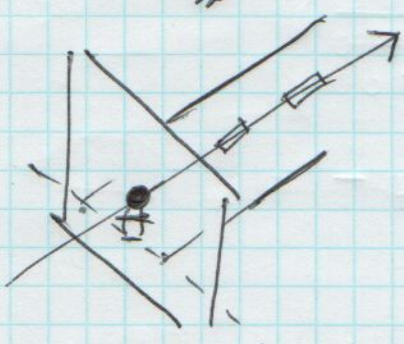
consider lines $\quad X_c = 1,\ Y_c = -1 \qquad u = f \frac{X_c}{Z_c}$

$X_c = -1,\ Y_c = -1 \qquad v = f \frac{Y_c}{Z_c}$

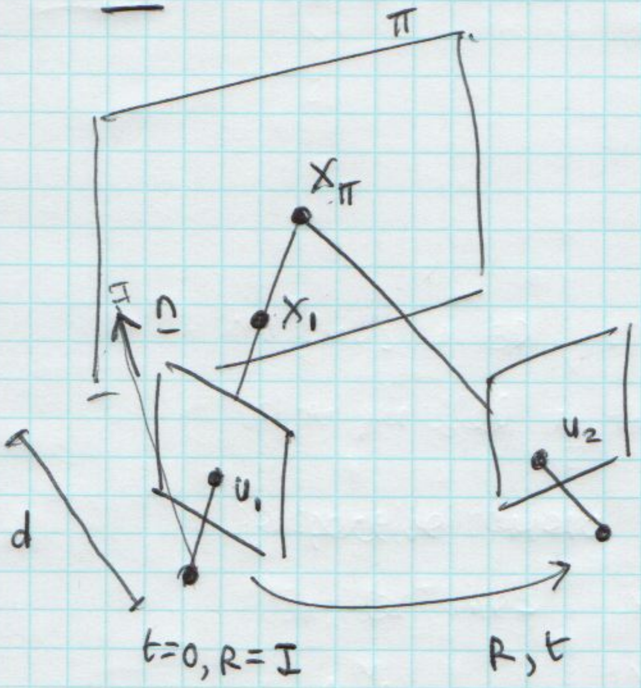


for $Z_c = 1$, $(u, v) = (f, -f), (-f, -f)$

as $Z_c \rightarrow \infty \quad (u, v) \rightarrow (0, 0)$



3.8



plane $n^T X = d$

scale X_1 to plane $s_1 X_1 = X_\pi$

$$n^T s_1 X_1 = d \,, \quad s_1 = \frac{d}{n^T X_1}$$

transform to camera 2.

$$X_2 = R \underbrace{s_1 X_1}_{X_\pi} + t = \frac{d}{n^T X_1} R X_1 + t$$

$$\frac{n^T X_1}{d} X_2 = R X_1 + \frac{t n^T X_1}{d}$$

$$s_2 X_2 = \left(R + \frac{t n^T}{d}\right) X_1$$

pixel coords $\check{u}_1 = s \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix} = K_1 X_1$

$$\check{u}_2 = K_2 X_2$$

$$\check{u}_2 = K_2 \left(R + \frac{t n^T}{d}\right) K_1^{-1} \check{u}_1$$

Q: What is the form of H when the camera rotates about it's centre?

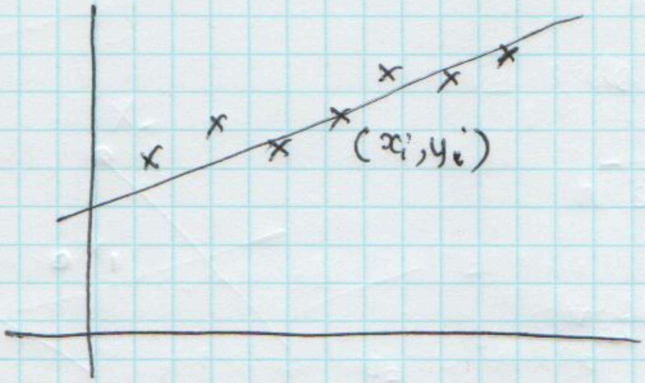
rotation $\check{u}_2 = \underline{K_2 R K_1^{-1}} \check{u}_1$

$$s \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K \left(R \mid \underset{=0}{\cancel{t}} \right) \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\check{u}_1 = s \begin{pmatrix} u_1 \\ v_1 \\ 1 \end{pmatrix}$$

3.9

x outlier



$y = ax + b$

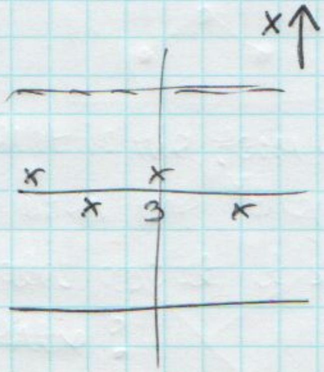
solve for a, b

$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ \vdots & \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \end{bmatrix}$$

$M\theta = y$

$$\hat{\theta} = \arg\min_{\theta} |M\theta - y|^2$$

let $a = 0$



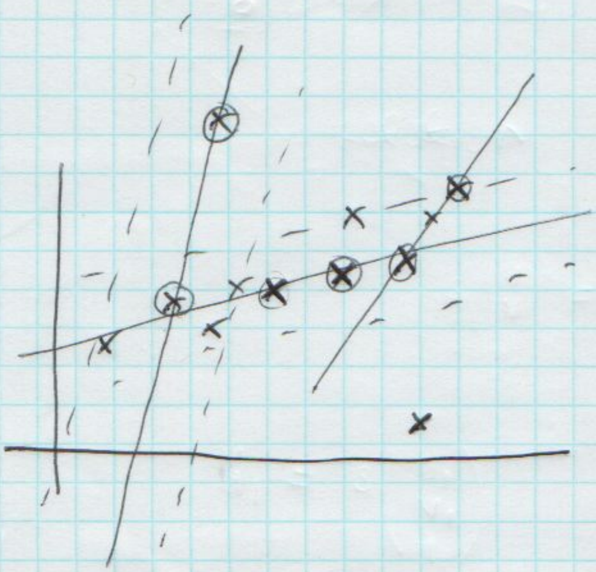
$y_i = \{3.1, 2.9, 3.2, 3.0, 10000\}$

LSquares $\quad b^* = \arg\min_{b} \sum_i (y_i - b)^2$

$$b^* = \frac{1}{N} \sum_i y_i$$

$$b^* = (3.1 + 2.9 + 3.2 + 3.0 + 10000)/5$$

$\simeq 2000$



select minimal subset (2)

solve for θ

check "consensus"

repeat to maximise # inliers

<u>Ran</u>dom <u>Sa</u>mple <u>C</u>onsensus

"RANSAC"