2-view Alignment and RANSAC

CSE P576

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These slides were developed by Dr. Matthew Brown for CSEP576 Spring 2020 and adapted (slightly) for Fall 2021
credit → Matt
blame → Vitaly

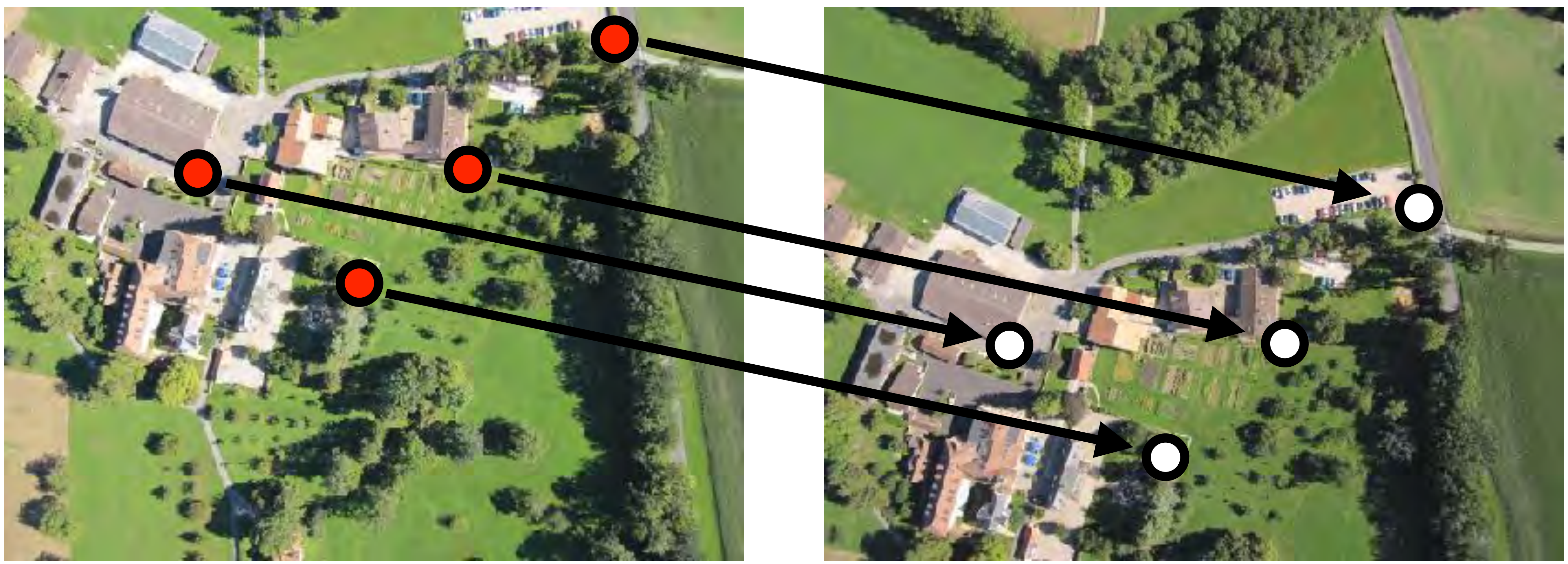
2-view Alignment + RANSAC

- 2-view alignment: linear equations
- Least squares and outliers
- Robust estimation via sampling

[Szeliski 8.1, 8.2]

Image Alignment

- Find corresponding (matching) points between the images

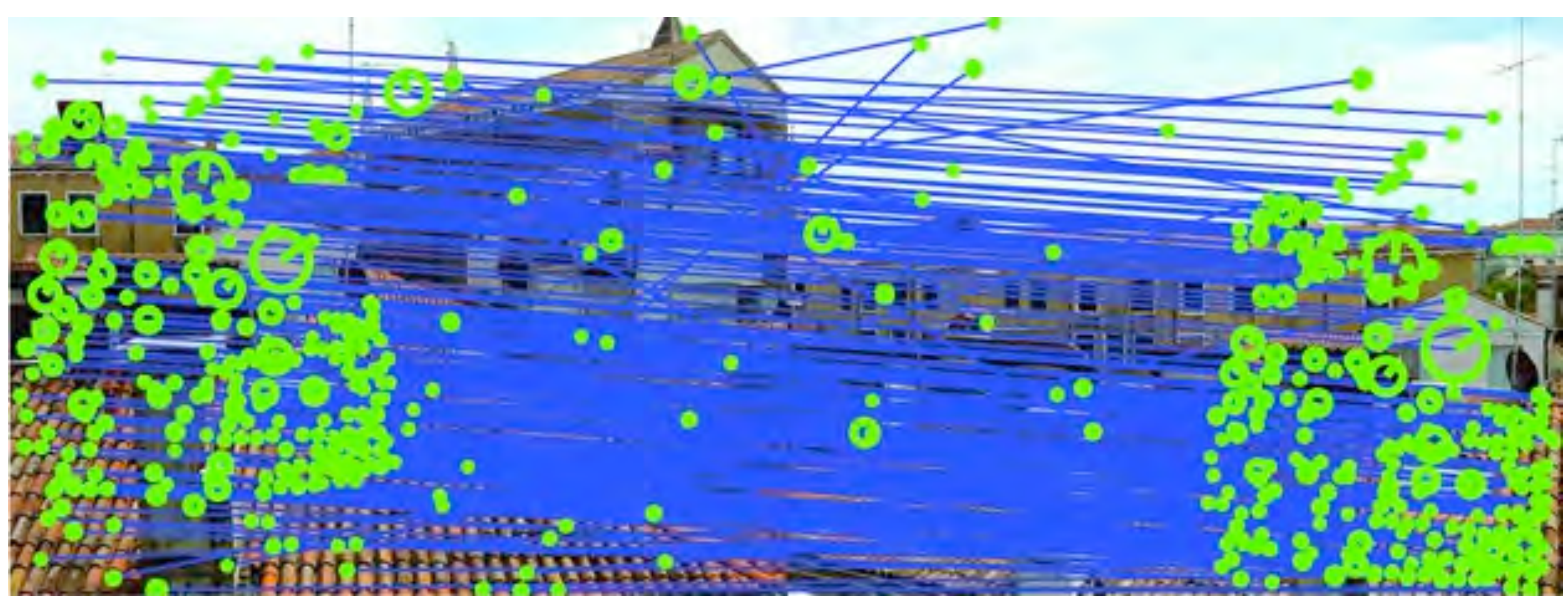


$$\mathbf{u} = \mathbf{H}\mathbf{x}$$

2 points for Similarity
3 for Affine
4 for Homography

Image Alignment

- In practice we have many noisy correspondences + **outliers**



Linear Equations

- e.g., for an affine transform we have a linear system in the unknown parameters **a**:

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix}$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

- It is **overconstrained** (more equations than unknowns)
- and subject to **outliers** (some rows are completely wrong)

Let's deal with these problems in a simpler context..

Robust Line Fitting

- Consider fitting a line to noisy points

RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

4 inliers (red, yellow, orange, brown),

RANSAC Example

- RANSAC solution for Similarity Transform (2 points)


4 outliers (blue, light blue, purple, pink)

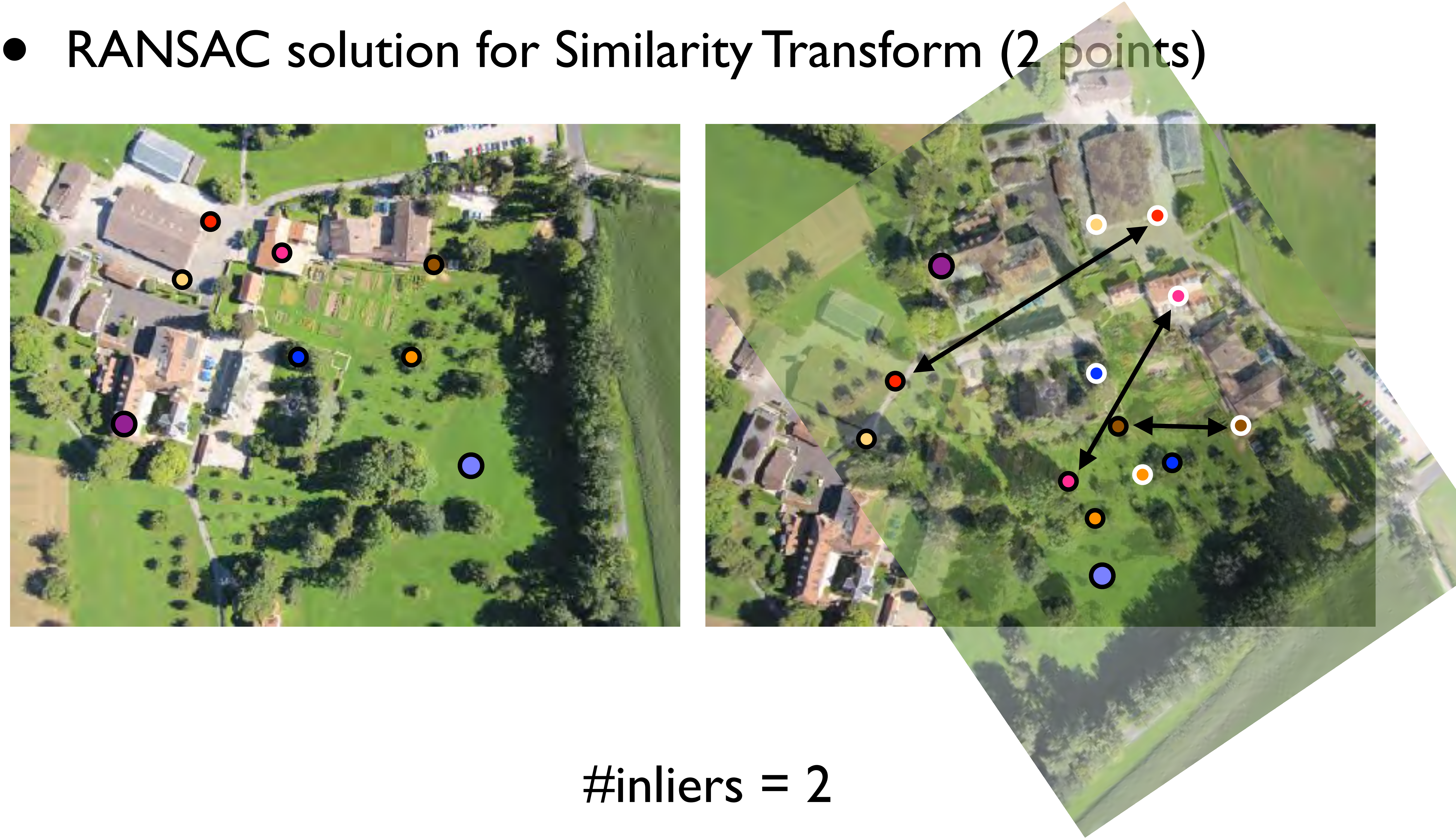
RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

4 inliers (red, yellow, orange, brown),
4 outliers (blue, light blue, purple, pink)

RANSAC Example

- RANSAC solution for Similarity Transform (2 points)



#inliers = 2

RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

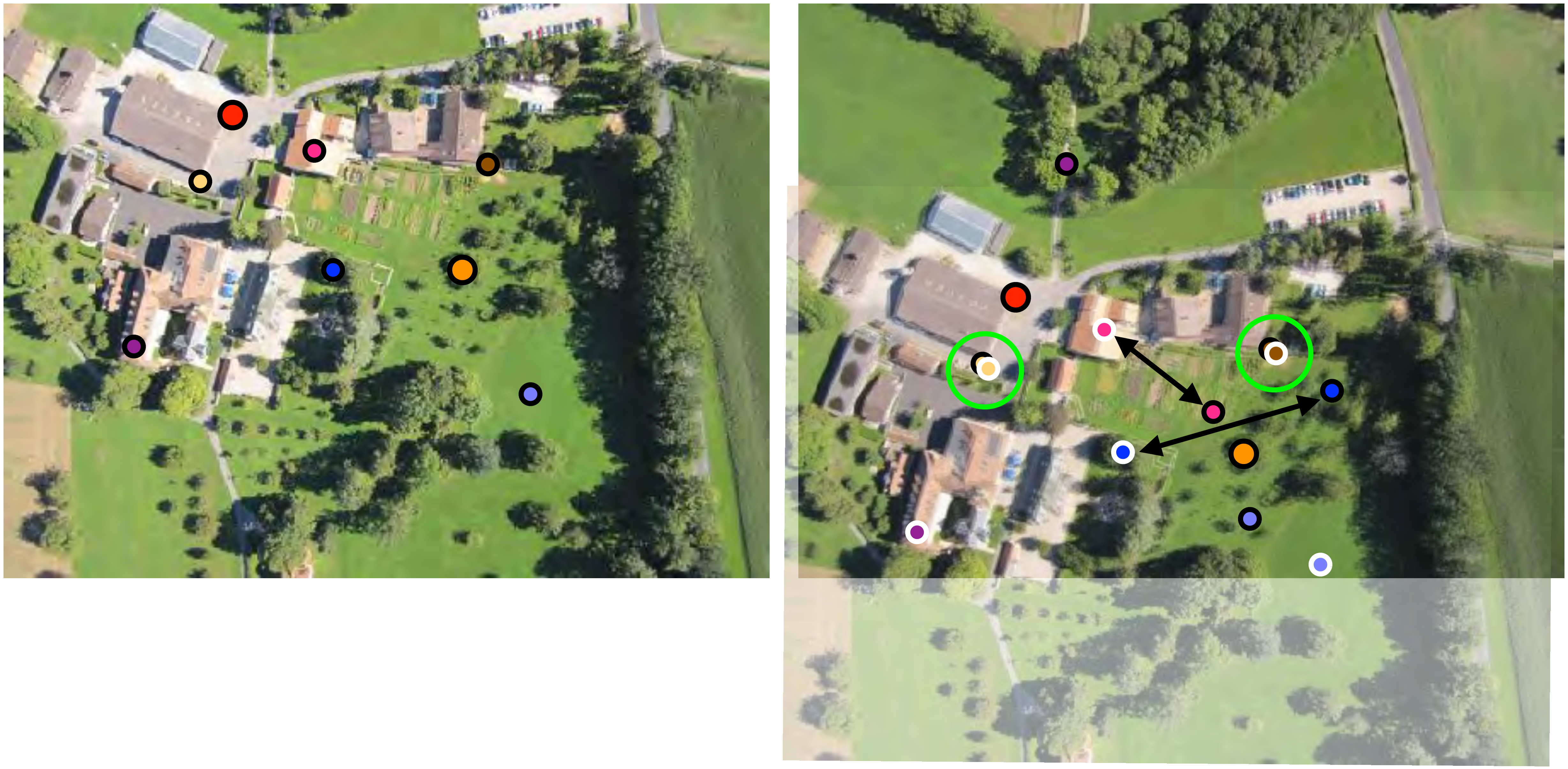
#inliers = 2

RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

RANSAC Example

- RANSAC solution for Similarity Transform (2 points)



#inliers = 4

RANSAC Example

- RANSAC solution for Similarity Transform (2 points)

RANSAC algorithm

1. Match feature points between 2 views
2. Select minimal subset of matches*
3. Compute transformation T using minimal subset
4. Check consistency of all points with T — compute projected position and count #inliers with distance < threshold
5. Repeat steps 2-4 to maximise #inliers

* Similarity transform = 2 points, Affine = 3, Homography = 4

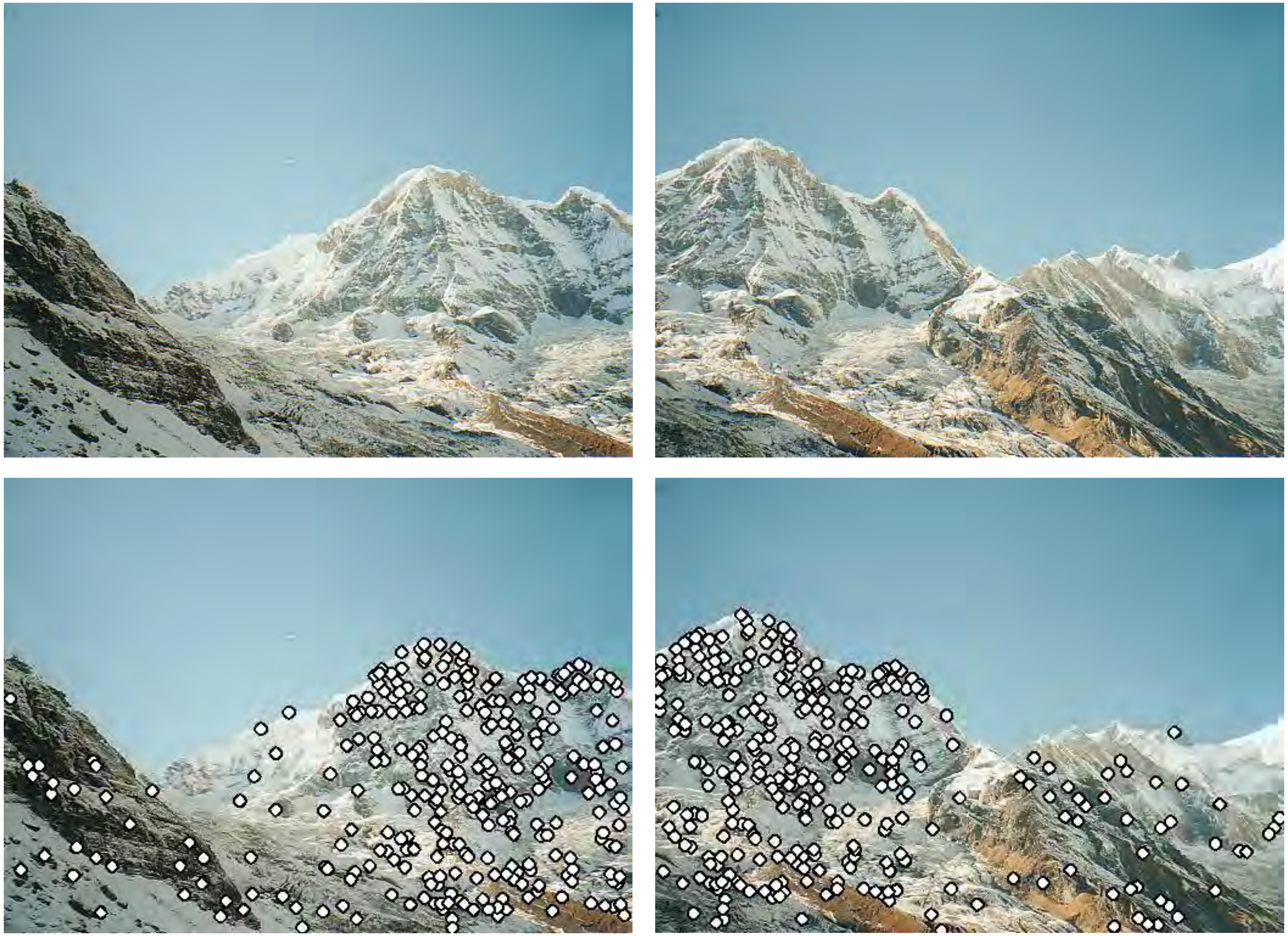
Project 2

P2

- Try out the **RANSAC Implementation** section in Project 2.

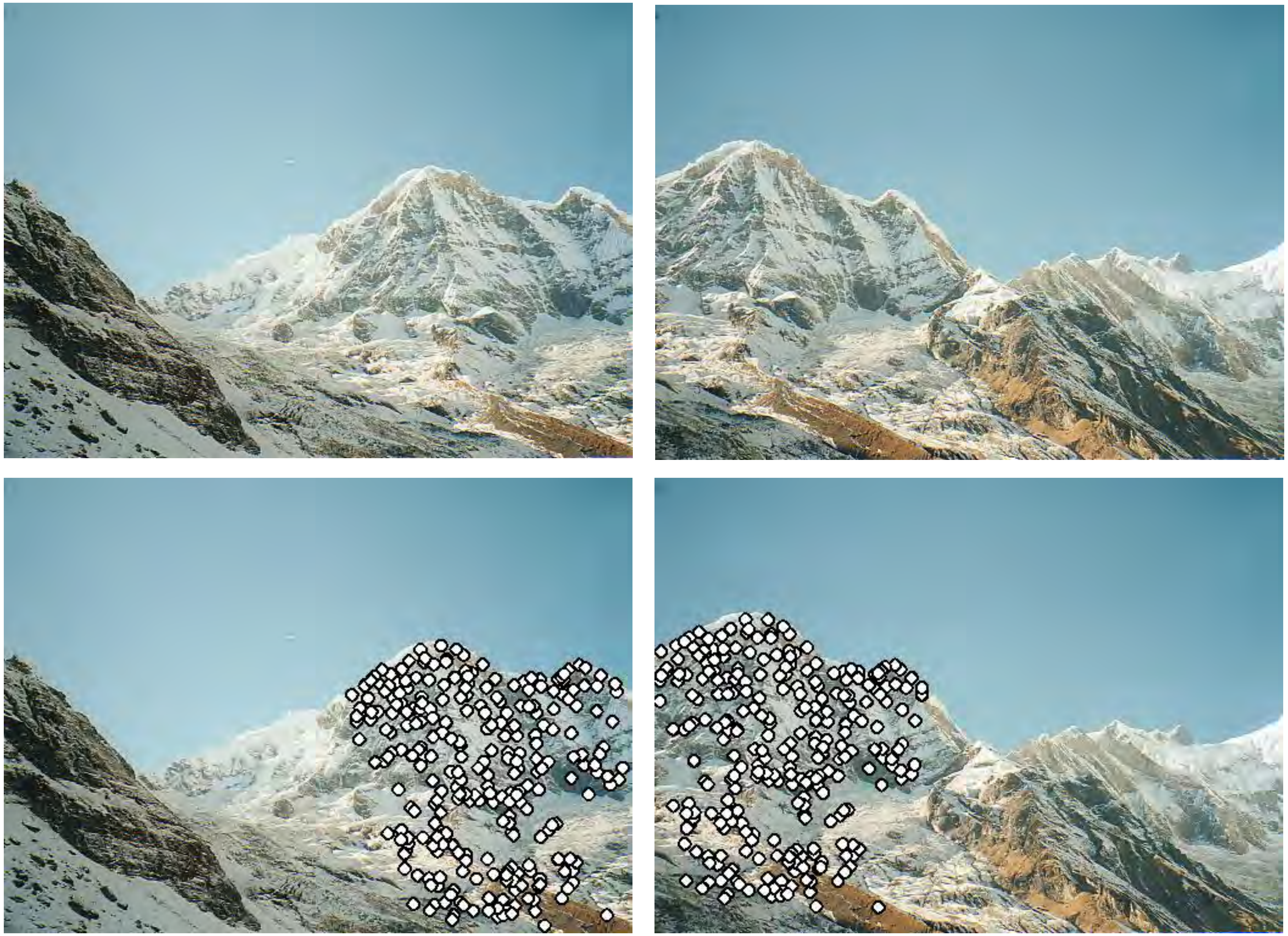
2-view 3D Rotation Estimation

- Find features + raw matches, use RANSAC to find Similarity



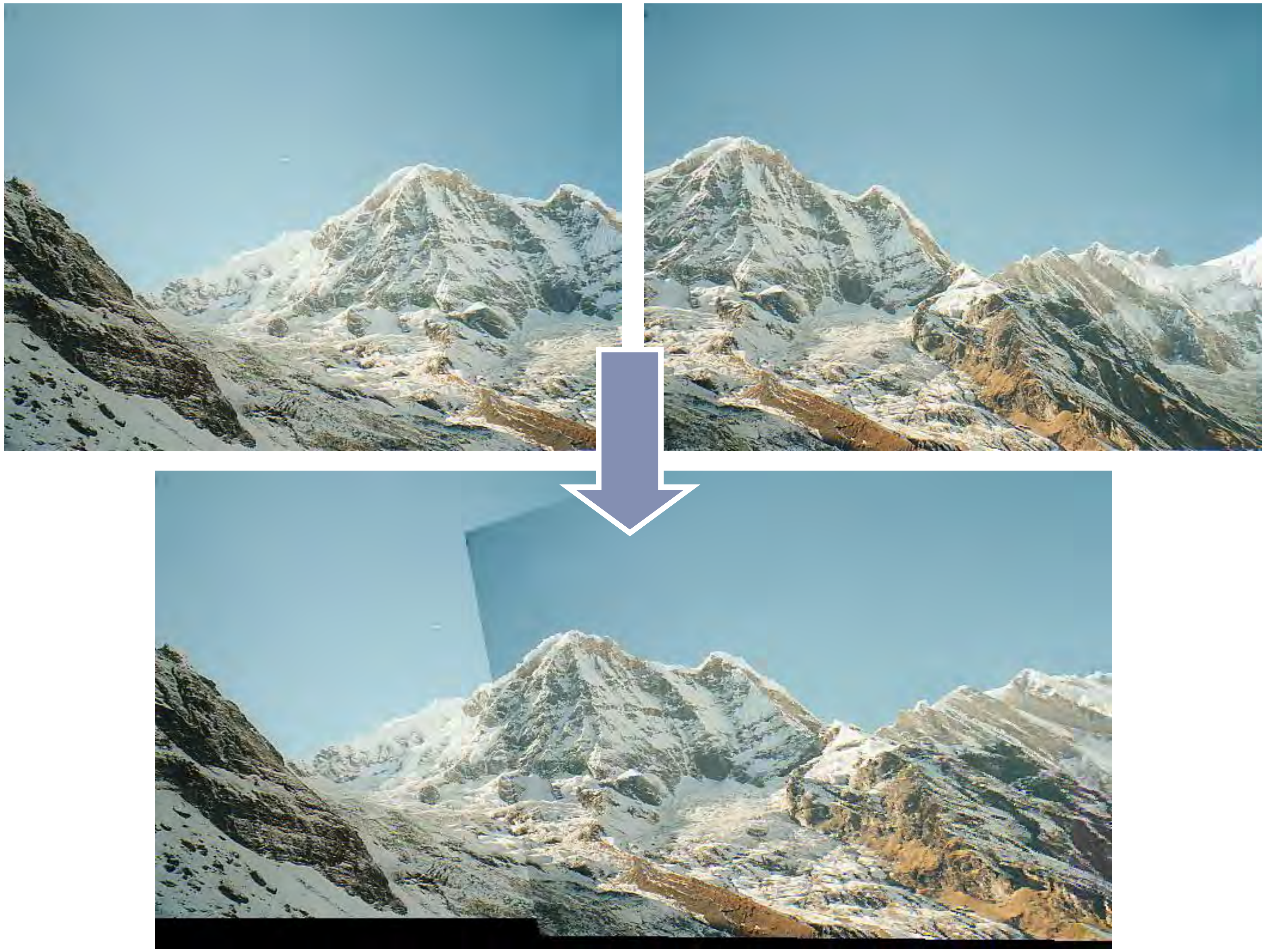
2-view 3D Rotation Estimation

- Remove outliers, can now solve for 3D rotation



2-view 3D Rotation Estimation

- Final homography given the 3D rotation



Rotation Estimation in 3D

- We can solve for 3D rotation by forming a correlation matrix of corresponding rays (unit vectors in camera coordinates)

- The solution for R minimizes the squared distance between corresponding rays, this is known as an "Orthogonal Procrustes Problem"; see Szeliski Ch. 8.1, Arun et al 1987.

Matrix Norm

matrix norm $\|A\| \geq 0$, $\|A\| = 0 \iff A = 0$

$\|\alpha A\| = |\alpha| \|A\|$ $\|A+B\| \leq \|A\| + \|B\|$

induced norms: $\sup_{x \neq 0} \frac{\|Ax\|_p}{\|x\|_q}$

entry-wise norms $\|A\| = f[(\text{vec}(A))]$

Frobenius norm $\|A\|_F = \sum_i \sum_j |a_{ij}|^2 = [\text{trace}\, A^T A]^{\frac{1}{2}}$

$= [\sum_i^{\min(m,n)} \sigma_i^2(A)]$ where

$A = U \cdot \text{diag}[\sigma_1, \ldots \sigma_{\min(m,n)}] V^T$

Computing Optimal 3D Rotation

Consider two sets of column vectors

$$P = [\cdots p_i \cdots] \quad P' = [\cdots p_i' \cdots]$$

column vectors

For example, let $P, P' \in \mathbb{R}^{3\times n}$, $\{p_i\}, \{p_i'\}$ zero-mean

Suppose we want to rotate vectors in P to "align" with vectors in P'

i.e. find R^* rotation matrix

$$R^* = \arg\min_{R \in SO(3)} \| P - R \cdot P' \|_F^2$$

Computing Optimal 3D Rotation

$$R^* = \arg\min_{R \in SO(3)} \|P - R \cdot P'\|_F^2$$

$$= \arg\min_{R \in SO(3)} \operatorname{trace}(PP^T + P'P'^T - 2RP'P^T)$$

$$= \arg\max_{R \in SO(3)} \operatorname{trace}[RP'P^T]$$

$$\operatorname{tr}[R \cdot U\Sigma V^T]$$ ← SVD of $P'P^T$

$$\operatorname{tr}[V^T R U \Sigma]$$ ← $\operatorname{tr}(AB) = \operatorname{tr}(BA)$

$$= \arg\max_{\tilde{R} \in SO(3)} \operatorname{tr}[\tilde{R}\Sigma]$$ ← $\Sigma = \operatorname{diag}(\sigma_1, .. \sigma_3)$, $\sigma_i \geq 0$

since $\tilde{R}\tilde{R}^T = I$ (by definition), $\Rightarrow r_{ii} \leq 1$

thus $\tilde{R}^* = I \Rightarrow R^* = V^T \tilde{R}^* U = V^T U$

Next Lecture

- Epipolar Geometry, Multiview Reconstruction