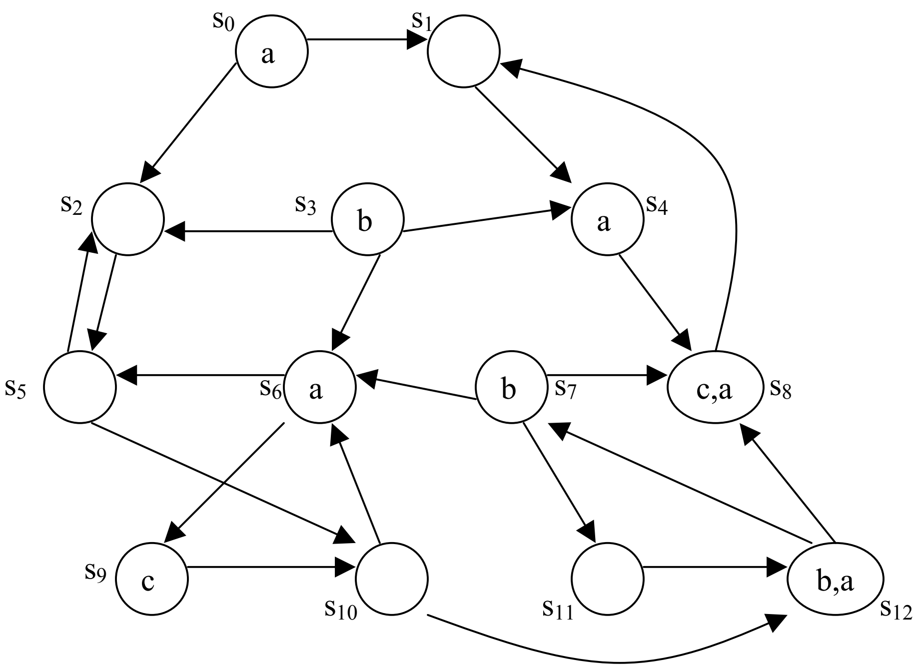
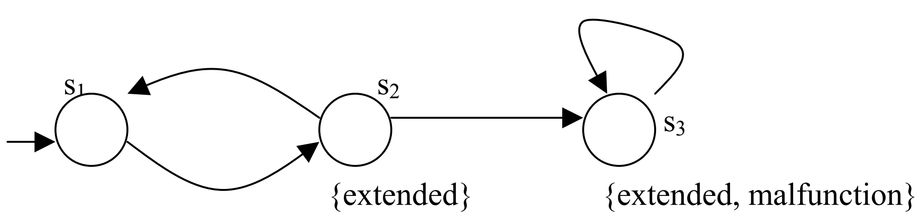
CSEP590 – Problem Set 5
Summer 2003
Due: Thursday, August 7 by midnight.
Directions: Answer each of the following questions. Email your solutions in doc, pdf, or html format to evan@cs.washington.edu

1. Consider the following transition system M. All states are labeled from the set of labels $\{s_0 \ldots s_{12}\}$. The atomic proposition p is true in all states. In addition, the system has a set of fairness constraints C = {a,b,c}. States are labeled with the fairness constraints true in that state.



a. How many maximal strongly connected components are there in this graph? Name the set of states comprising each of the maximal components.
b. Does M, s_0 |= E_CGp hold? Why or why not? Justify your answer.
c. Now, let's suppose that p does not hold in state s_6. With this assumption, does M, s_0 |= E_CGp hold? Why or why not? Justify your answer.

2. Consider the following Kripke Structure which we will call *Spring*.



This structure represents a very simple model of a spring. One can pull the spring, and later release it. After being pulled, the spring may lose its elasticity and stay permanently extended, or it may just return to its original form. There are 3 states: s_1,s_2,s_3 and 2 atomic propositions, namely *extended* and *malfunction*. The atomic propositions true in a given state are shown in brackets underneath the state. The spring starts in its unextended form in state s_1. Consider the following system run: $\pi = s_1\, s_2\, s_1\, s_2\, s_3\, s_3\, s_3 \ldots$ For each of the following LTL formulae, decide whether or not the formula is satisfied by the run π. Be sure to justify your answer.

a. *extended*
b. X *extended*
c. XX *extended*
d. F *extended*
e. G *extended*
f. FG *extended*
g. !*extended* U *malfunction*

For each for the following LTL formulae, decide whether they are satisfied by the system *Spring*. Justify your answers.

a. F *extended*
b. G(!*extended* => X *extended*)
c. FG *extended*
d. !FG *extended*
e. G(*extended* => X !*extended*)

3. For parts a-d, check whether each of the following pairs of LTL-formula are equivalent. If they are not equivalent, give an example of a structure that demonstrates why they are not equivalent. Answer part e as it appears. Assume that p, q, r are atomic propositions.

a. XFp, FXp
b. (FGp) $\wedge$ (FGq), F(Gp $\wedge$ Gq)
c. (p U q) U q, p U q
d. (p U q) $\wedge$ (q U r), (p U r)
e. Find a CTL* path formula that says that p is true for every second state along a path.

4. This problem will deal with monotone functions and fixed points. Consider the functions H_1, H_2, H_3 : P({1,2,3,4,5,6,7,8,9,10}) $\rightarrow$ P({1,2,3,4,5,6,7,8,9,10}), where P denotes "power set." The functions are defined as follows for all $Y \subseteq \{1,2,3,4,5,6,7,8,9,10\}$:

$$H_1(Y) = Y - \{1,4,7\}$$
$$H_2(Y) = \{2,5,9\} - Y$$
$$H_3(Y) = \{1,2,3,4,5\} \cap (\{2,4,8\} \cup Y)$$

(Note: X – Y means the set that results from removing from X all elements appearing in Y. For example, {a,b,c} – {b,c} = {a})

a. Which of these functions are monotone? Which ones aren't? In each case, provide solid justification for you answer.

b. Compute the least and greatest fixed points of H_3.
c. Does H_2 have any fixed points? Why or why not?

5. The final question deals a bit with the relational mu-calculus. Recall the definition of what it means for a valuation p to satisfy the formula $\nu Z.f$, namely that $p \models \nu Z.f$ iff ($p \models \nu_m Z.f$ for all $m \geq 0$). Using this definition of satisfaction and the definition of the satisfaction relation for formulas of the mu-calculus, use mathematical induction to prove that $p \models \nu Z.Z$ for *all* valuations p.