CSEP 590
Data Compression
Autumn 2007

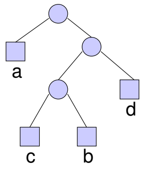
Adaptive Huffman Coding

Adaptive Huffman Coding

- One pass
- During the pass calculate the frequencies
- Update the Huffman tree accordingly
  - Coder – new Huffman tree computed after transmitting the symbol
  - Decoder – new Huffman tree computed after receiving the symbol
- Symbol set and their initial codes must be known ahead of time.
- Need NYT (not yet transmitted symbol) to indicate a new leaf is needed in the tree.

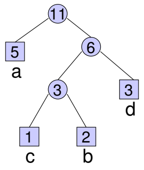
Optimal Tree Numbering

- a : 5, b: 2, c : 1, d : 3



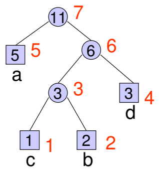
Weight the Nodes

- a : 5, b: 2, c : 1, d : 3



Number the Nodes

- a : 5, b: 2, c : 1, d : 3



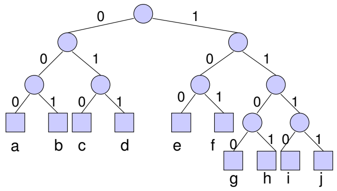
Number the nodes as they are removed from the priority queue.

Adaptive Huffman Principle

- In an optimal tree for n symbols there is a numbering of the nodes $y_1<y_2<\ldots <y_{2n-1}$ such that their corresponding weights $x_1,x_2, \ldots , x_{2n-1}$ satisfy:
  - $x_1 \leq x_2 \leq \ldots \leq x_{2n-1}$
  - siblings are numbered consecutively
- And vice versa
  - That is, if there is such a numbering then the tree is optimal. We call this the node number invariant.

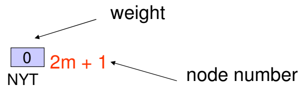
Initialization

- Symbols a_1, a_2, ... ,a_m have a basic prefix code, used when symbols are first encountered.
- Example: a, b ,c, d, e, f, g, h, i, j



Initialization

- The tree will encode up to m + 1 symbols including NYT.
- We reserve numbers 1 to 2m + 1 for node numbering.
- The initial Huffman tree consists of a single node



Coding Algorithm

1. If a new symbol is encountered then output the code for NYT followed by the fixed code for the symbol. Add the new symbol to the tree.
2. If an old symbol is encountered then output its code.
3. Update the tree to preserve the node number invariant.

Decoding Algorithm

1. Decode the symbol using the current tree.
2. If NYT is encountered then use the fixed code to decode the symbol. Add the new symbol to the tree.
3. Update the tree to preserve the node number invariant.

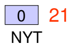
Updating the Tree

1. Let y be leaf (symbol) with current weight x.*
2. If y the root update x by 1, otherwise,
3. Exchange y with the largest numbered node with the same weight (unless it is the parent).**
4. Update x by 1
5. Let y be the parent with its weight x and go to 2.

*We never update the weight of NYT
** This exchange will preserve the node number invariant

Example

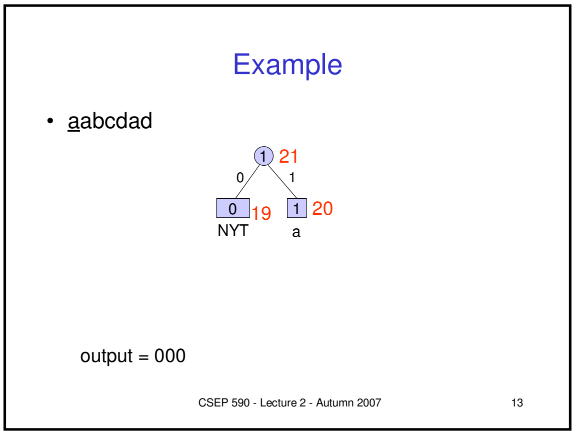
- aabcdad in alphabet {a,b,..., j}



fixed code for a

output = 000

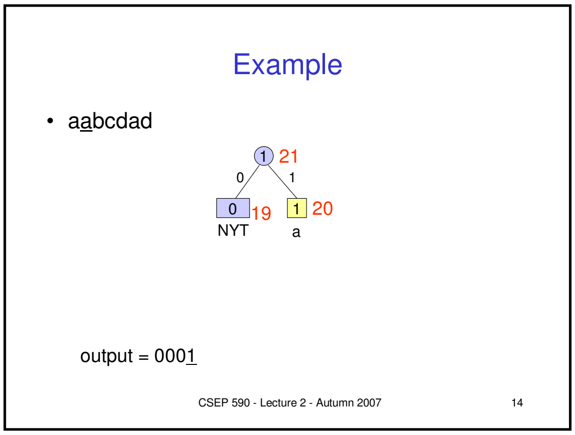
fixed code



Example

- aabcdad

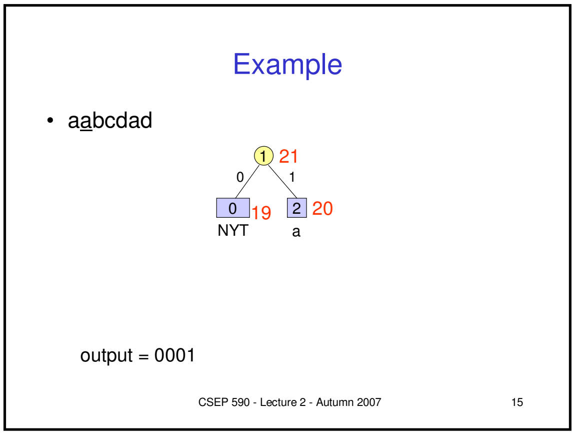
output = 000



Example

- aabcdad

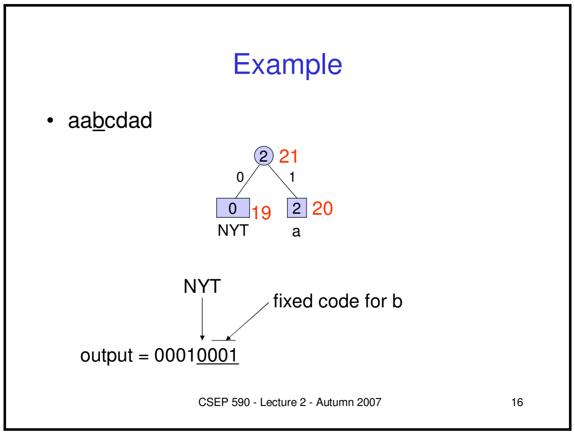
output = 0001



Example

- aabcdad

output = 0001



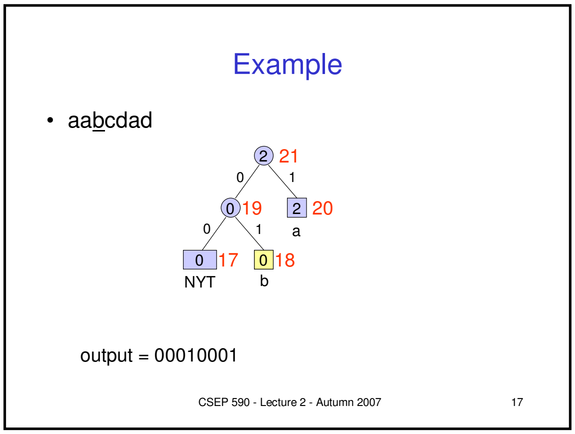
Example

- aabcdad

NYT

fixed code for b

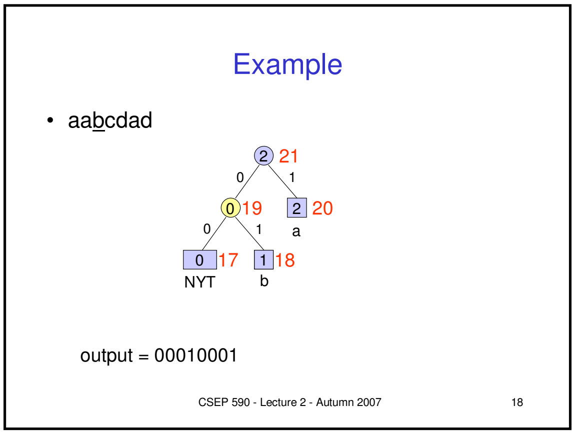
output = 00010001



Example

- aabcdad

output = 00010001



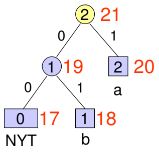
Example

- aabcdad

output = 00010001

Example

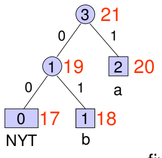
- aabcdad



output = 00010001

Example

- aabcdad



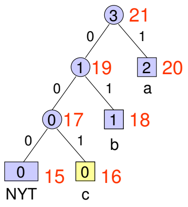
fixed code for c

NYT

output = 000100010001000

Example

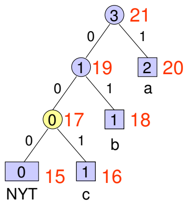
- aabcdad



output = 0001000100010

Example

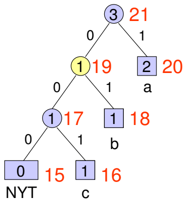
- aabcdad



output = 0001000100010

Example

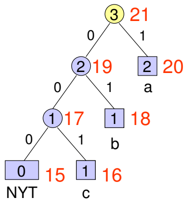
- aabcdad



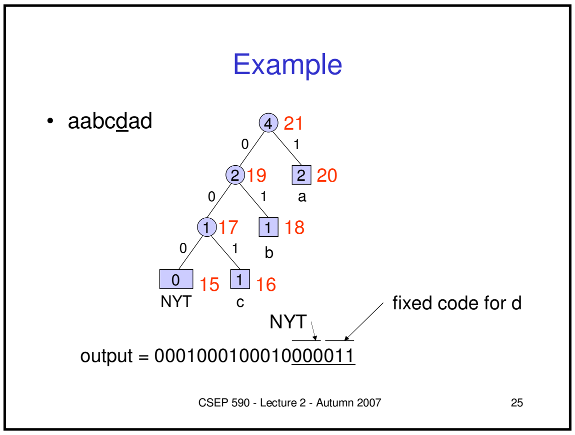
output = 0001000100010

Example

- aabcdad



output = 0001000100010



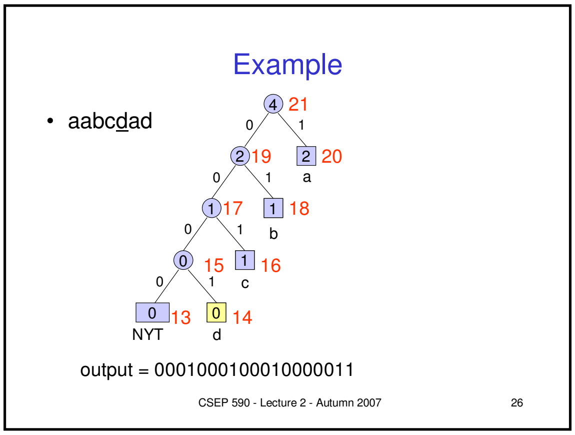
Example

- aabcdad

NYT

fixed code for d

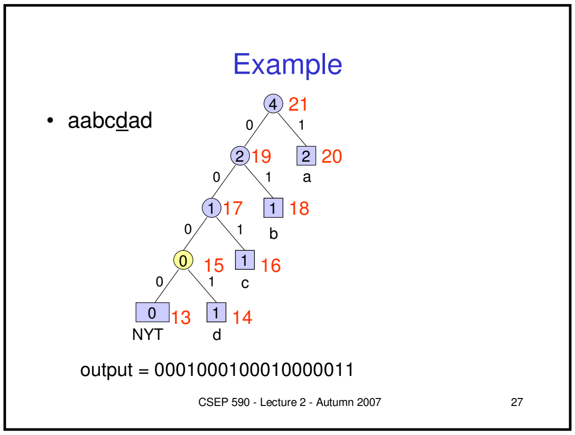
output = 0001000100010000011



Example

- aabcdad

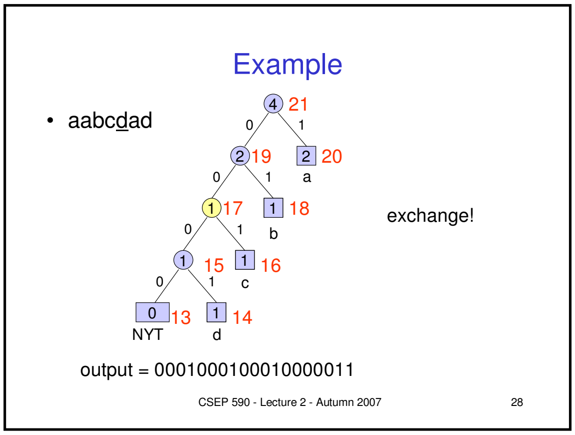
output = 0001000100010000011



Example

- aabcdad

output = 0001000100010000011

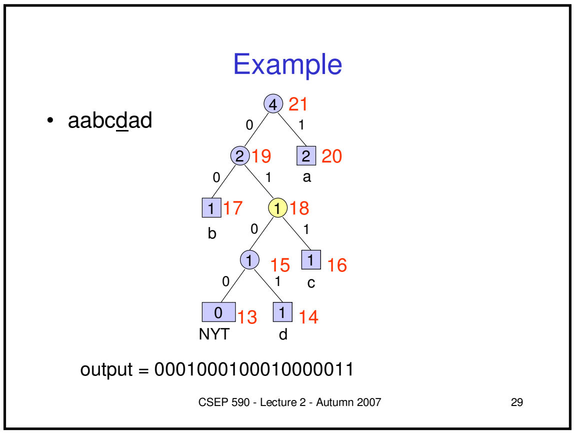


Example

- aabcdad

exchange!

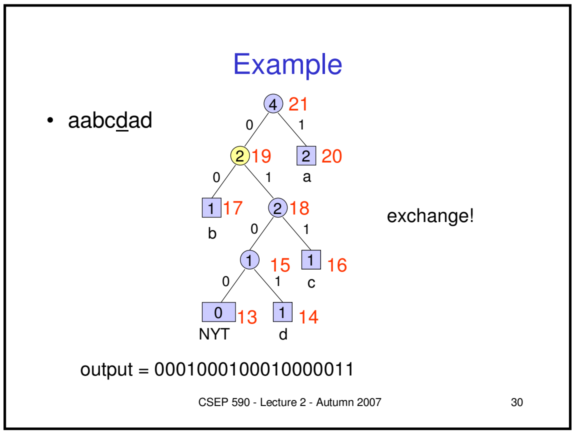
output = 0001000100010000011



Example

- aabcdad

output = 0001000100010000011

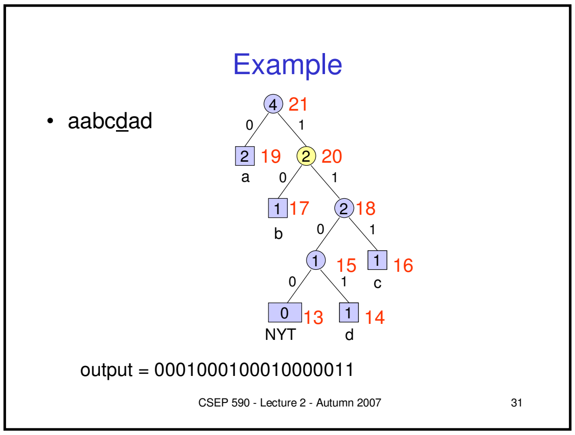


Example

- aabcdad

exchange!

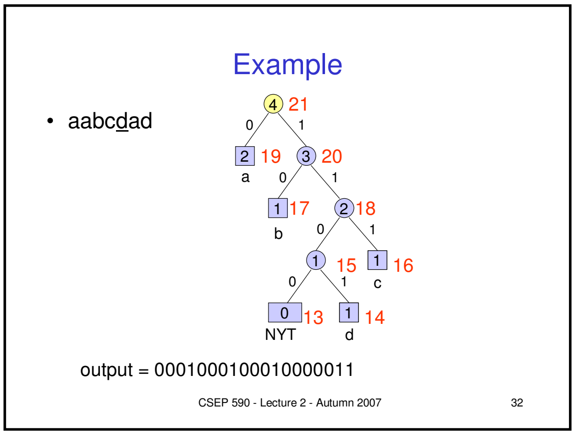
output = 0001000100010000011



Example

- aabcdad

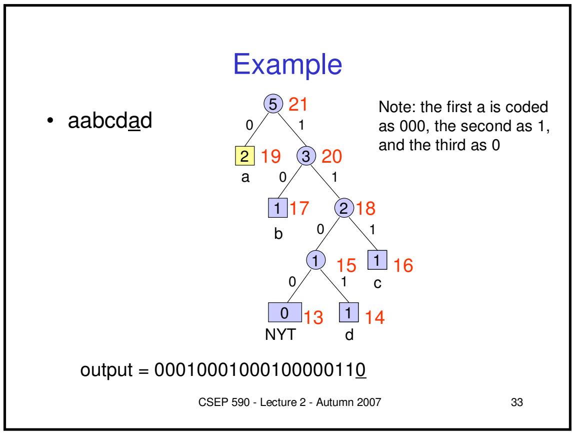
output = 0001000100010000011



Example

- aabcdad

output = 0001000100010000011

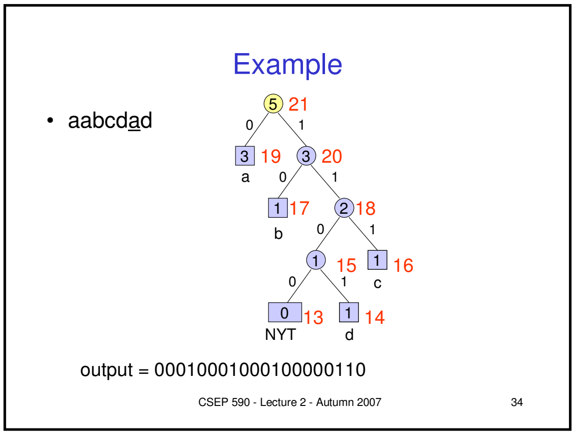


Example

- aabcdad

Note: the first a is coded as 000, the second as 1, and the third as 0

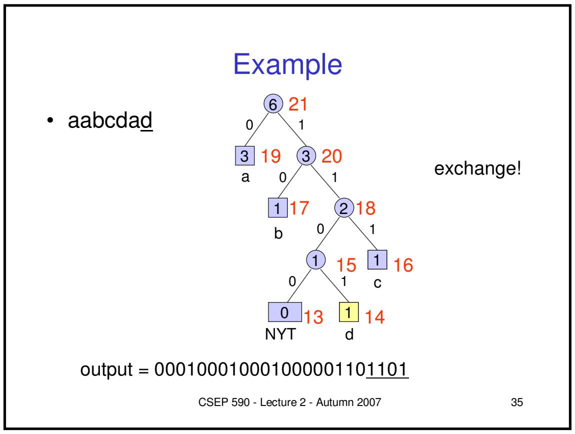
output = 00010001000100000110



Example

- aabcdad

output = 00010001000100000110

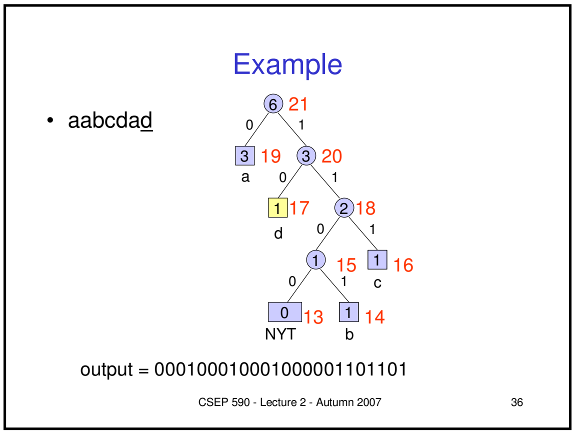


Example

- aabcdad

exchange!

output = 000100010001000001101101



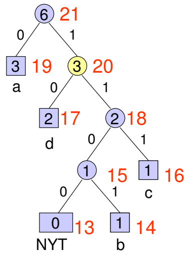
Example

- aabcdad

output = 000100010001000001101101

Example

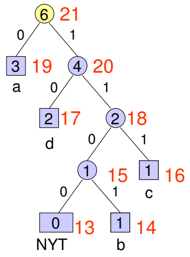
- aabcdad



output = 00010001000100000110110

Example

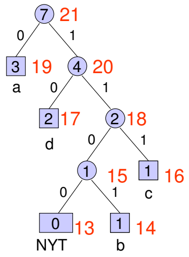
- aabcdad



output = 00010001000100000110110

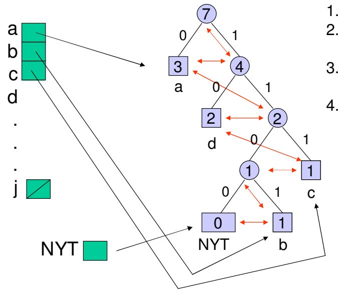
Example

- aabcdad



output = 00010001000100000110110

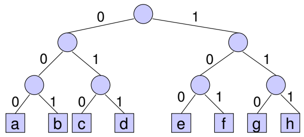
Data Structure for Adaptive Huffman



1. Fixed code table
2. Binary tree with parent pointers
3. Table of pointers nodes into tree
4. Doubly linked list to rank the nodes

In Class Exercise

- Decode using adaptive Huffman coding assuming the following fixed code



- 00110000

Huffman Summary

- Statistical compression algorithm
- Prefix code
- Fixed-to-variable length code
- Optimization to create a best code
- Symbol merging
- Context
- Adaptive coding
- Decoder and encoder behave almost the same
- Need for data structures and algorithms