CSE-P590a

Deterministic Path Planning in Robotics

Courtesy of Maxim Likhachev
Carnegie Mellon University

Motion/Path Planning

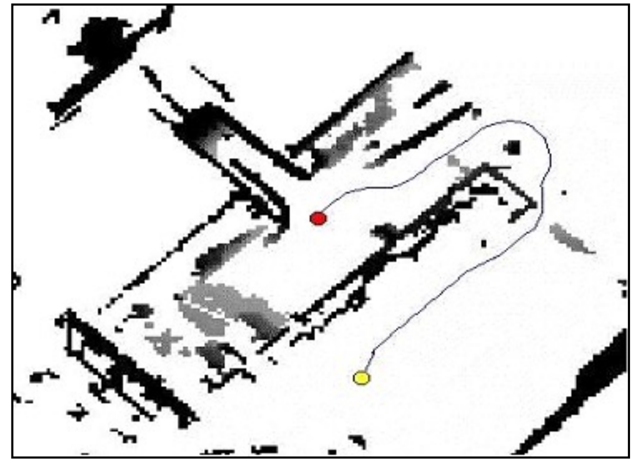
- Task:
  find a feasible (and cost-minimal) path/motion from the current configuration of the robot to its goal configuration (or one of its goal configurations)

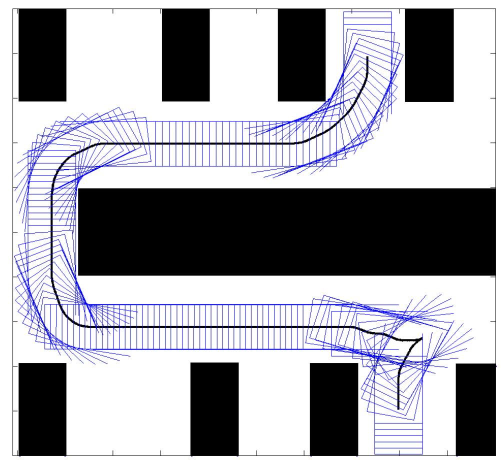
- Two types of constraints:
  environmental constraints (e.g., obstacles)
  dynamics/kinematics constraints of the robot

- Generated motion/path should (objective):
  be any feasible path
  minimize cost such as distance, time, energy, risk, …

Motion/Path Planning

Examples (of what is usually referred to as path planning):





Motion/Path Planning

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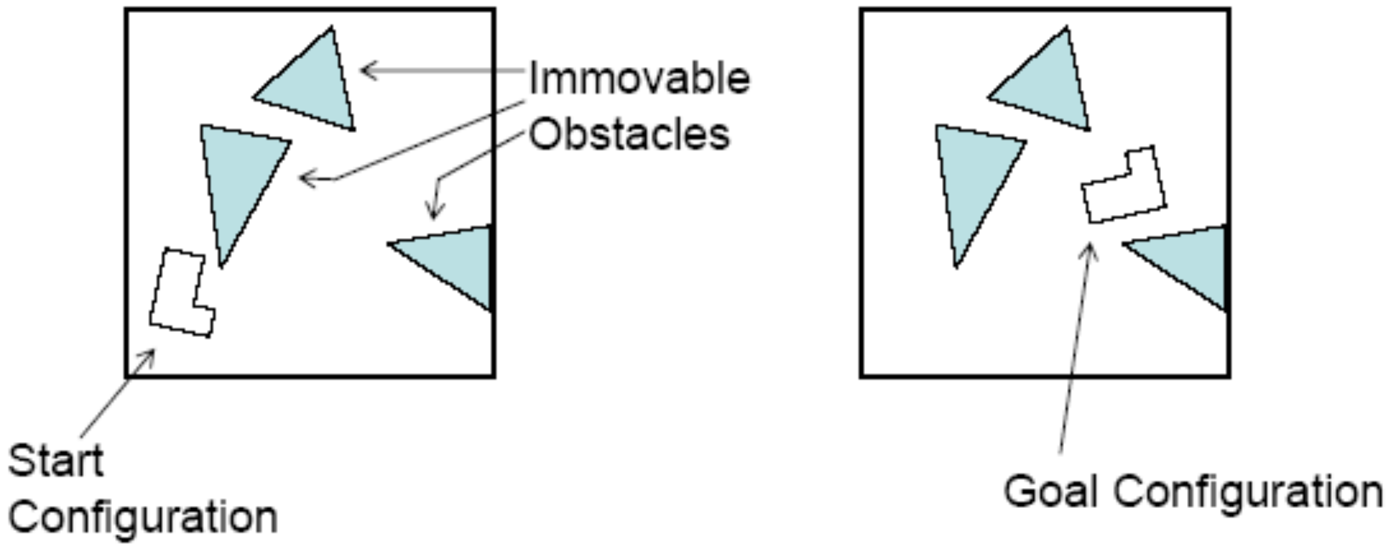

Piano Movers' problem

the example above is borrowed from www.cs.cmu.edu/~awm/tutorials

Motion/Path Planning

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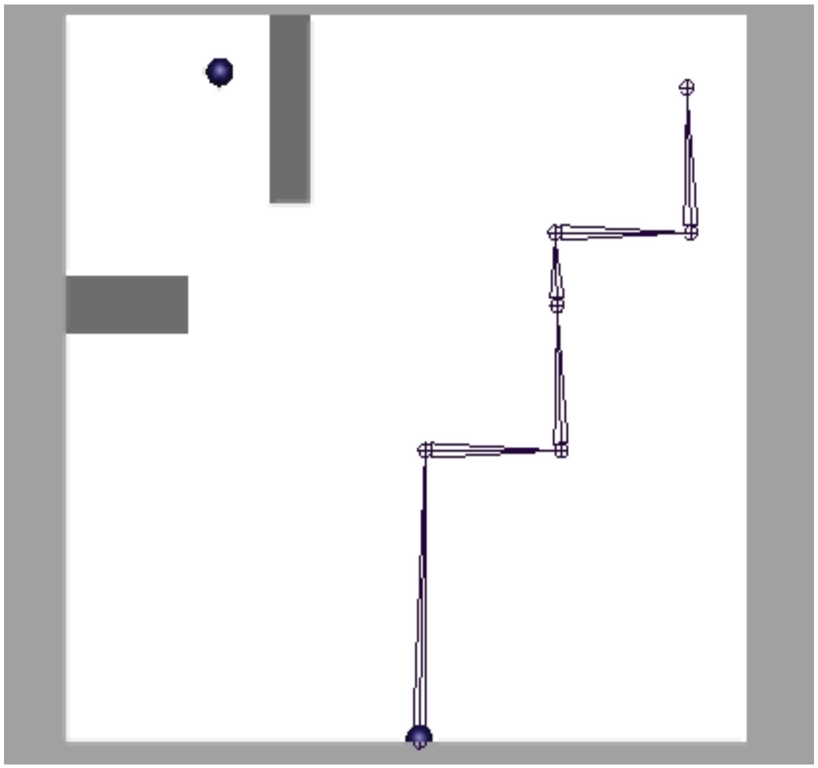
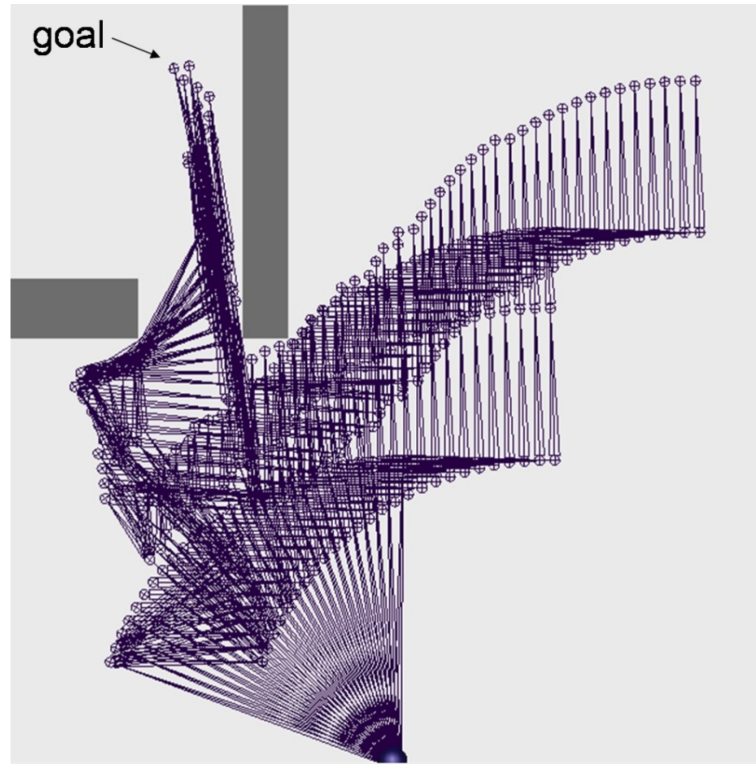
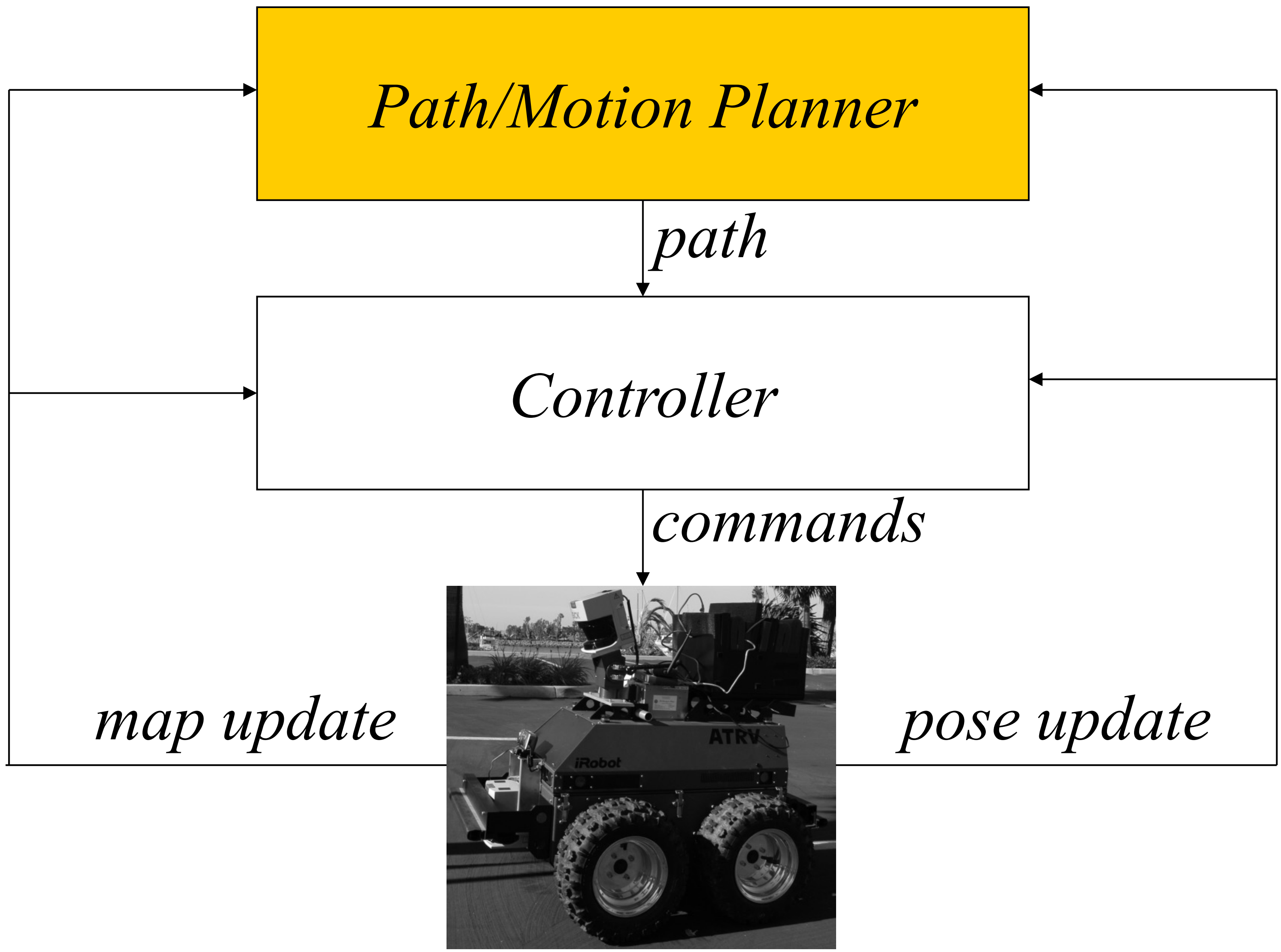


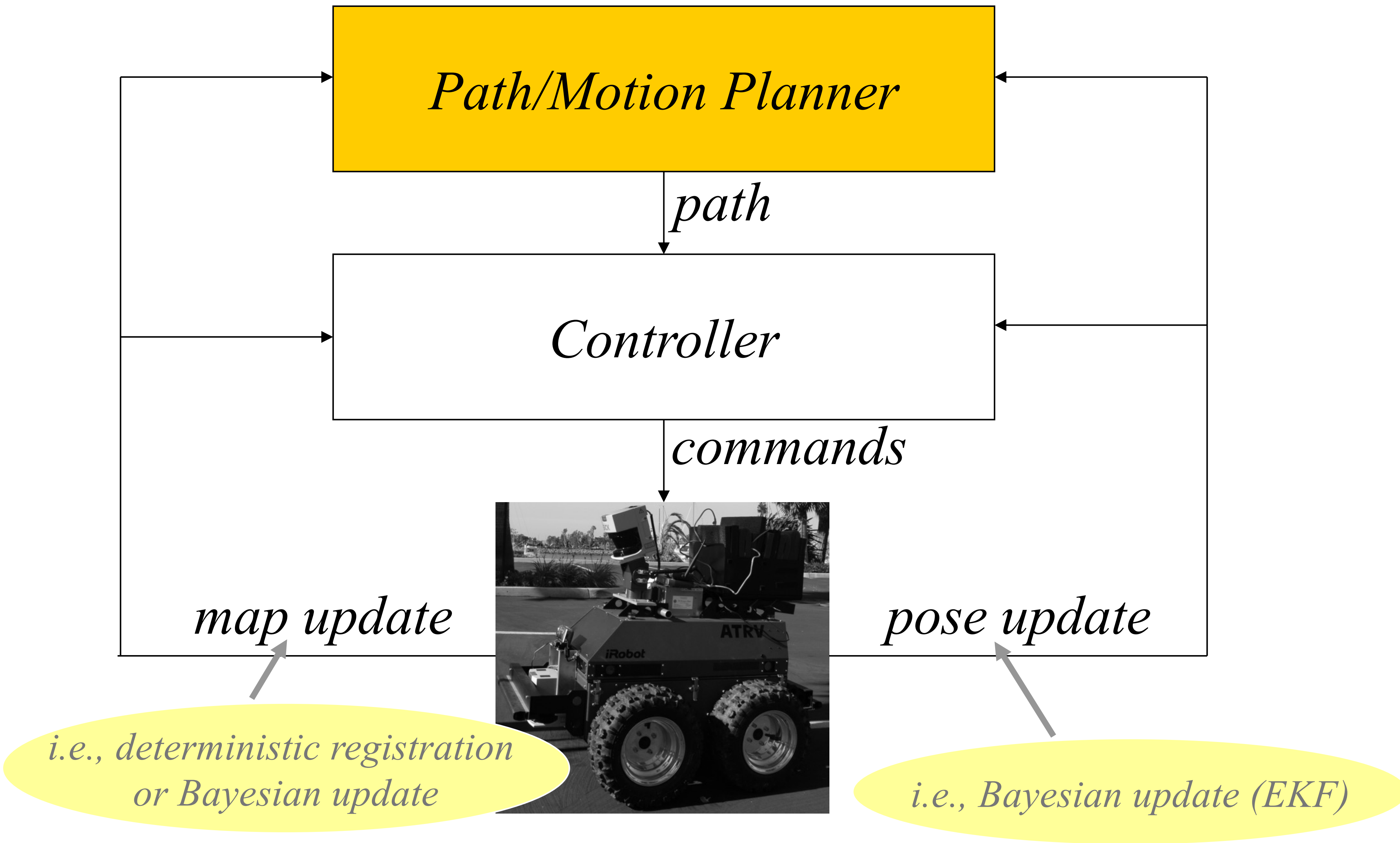


Planned motion for a 6DOF robot arm

Motion/Path Planning



Motion/Path Planning



Uncertainty and Planning

- Uncertainty can be in:
  - prior environment (i.e., door is open or closed)
  - execution (i.e., robot may slip)
  - sensing environment (i.e., seems like an obstacle but not sure)
  - pose

- Planning approaches:
  - deterministic planning:
    - assume some (i.e., most likely) environment, execution, pose
    - plan a single least-cost trajectory under this assumption
    - re-plan as new information arrives

  - planning under uncertainty:
    - associate probabilities with some elements or everything
    - plan a policy that dictates what to do for each outcome of sensing/action and minimizes expected cost-to-goal
    - re-plan if unaccounted events happen

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re-plan every time sensory data arrives or robot deviates off its path

re-planning needs to be FAST

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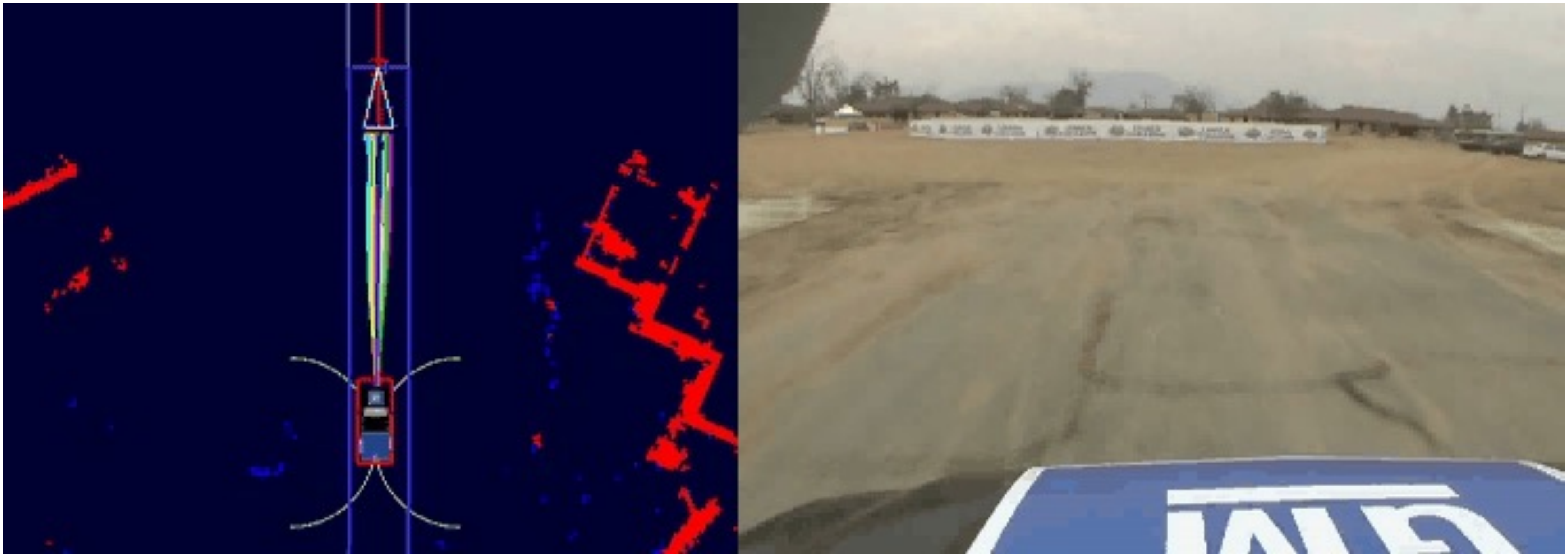
computationally MUCH harder

Example



*Urban Challenge Race, CMU team, planning with Anytime D**

Outline

- Deterministic planning
  - constructing a graph
  - search with A*
  - search with D*

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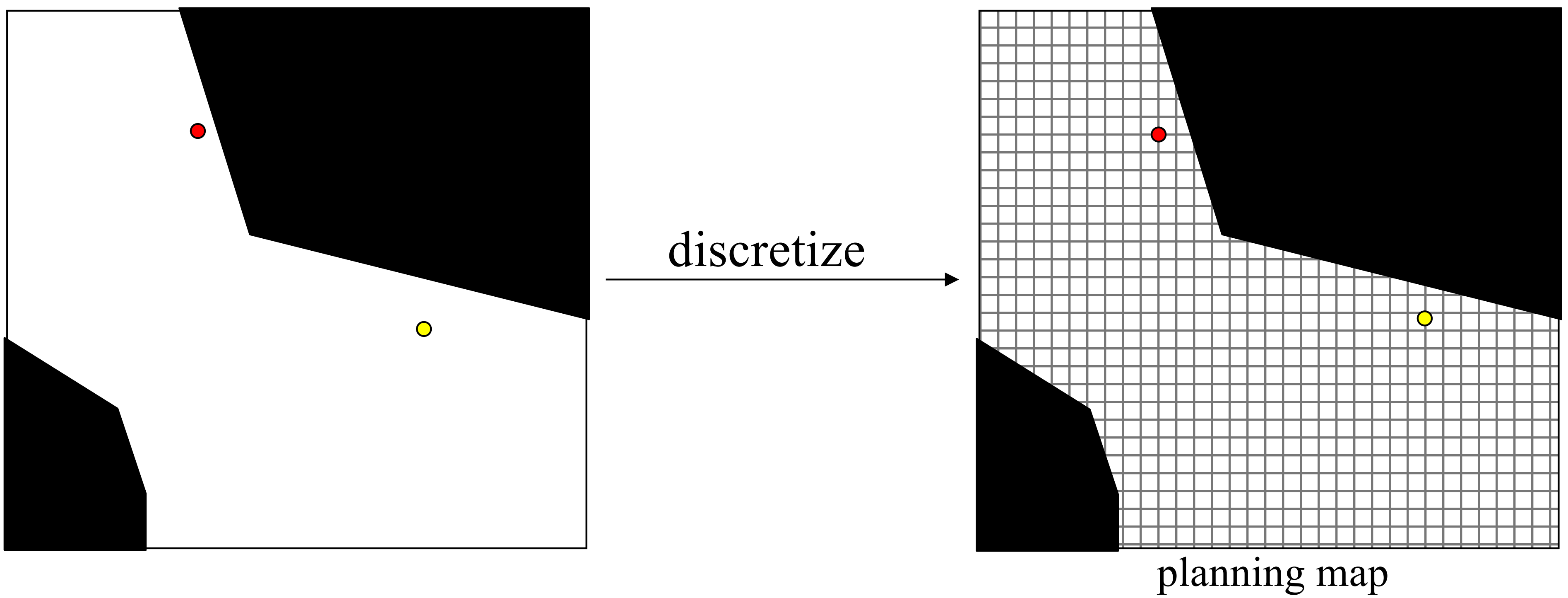
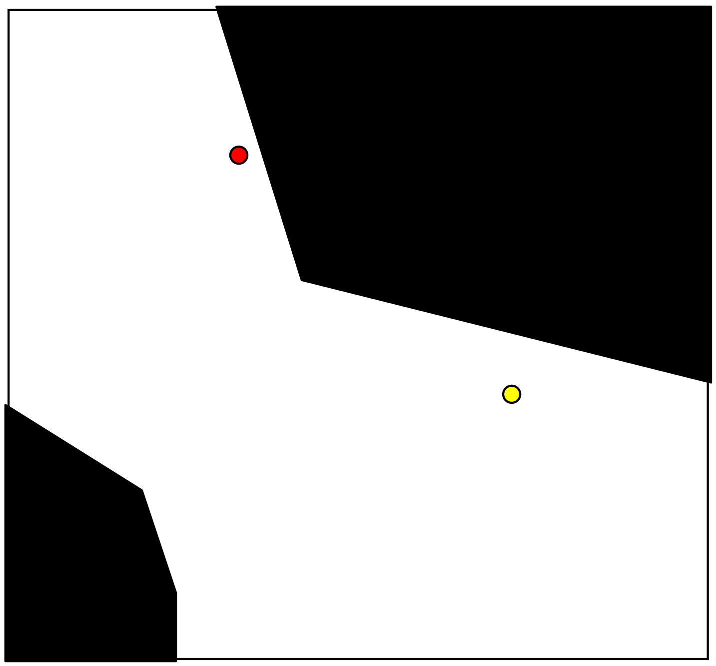
Planning via Cell Decomposition

- Approximate Cell Decomposition:
  - overlay uniform grid over the C-space (discretize)

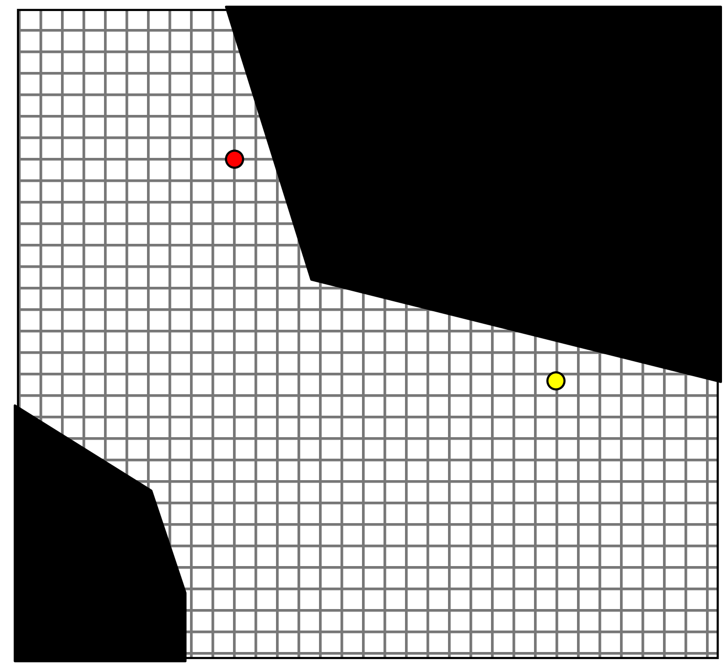


Planning via Cell Decomposition

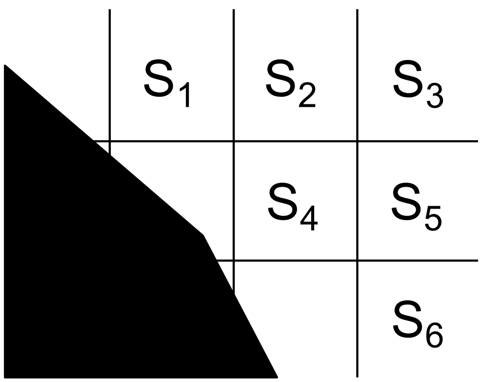
- Approximate Cell Decomposition:
  - construct a graph and search it for a least-cost path



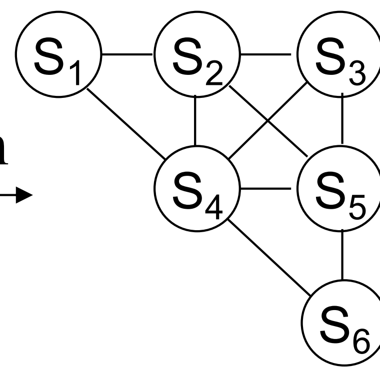
discretize



planning map



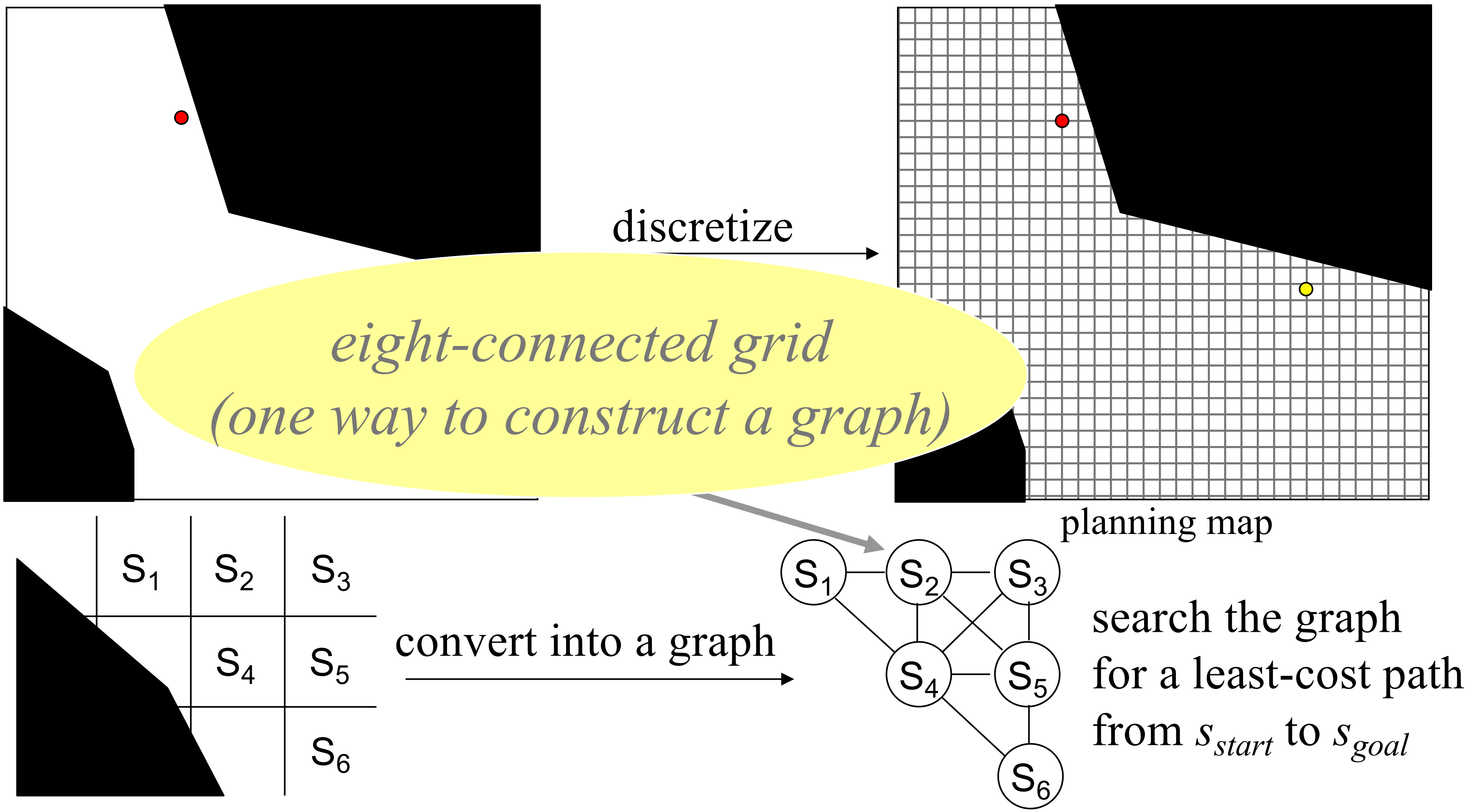
convert into a graph



search the graph for a least-cost path from s_{start} to s_{goal}

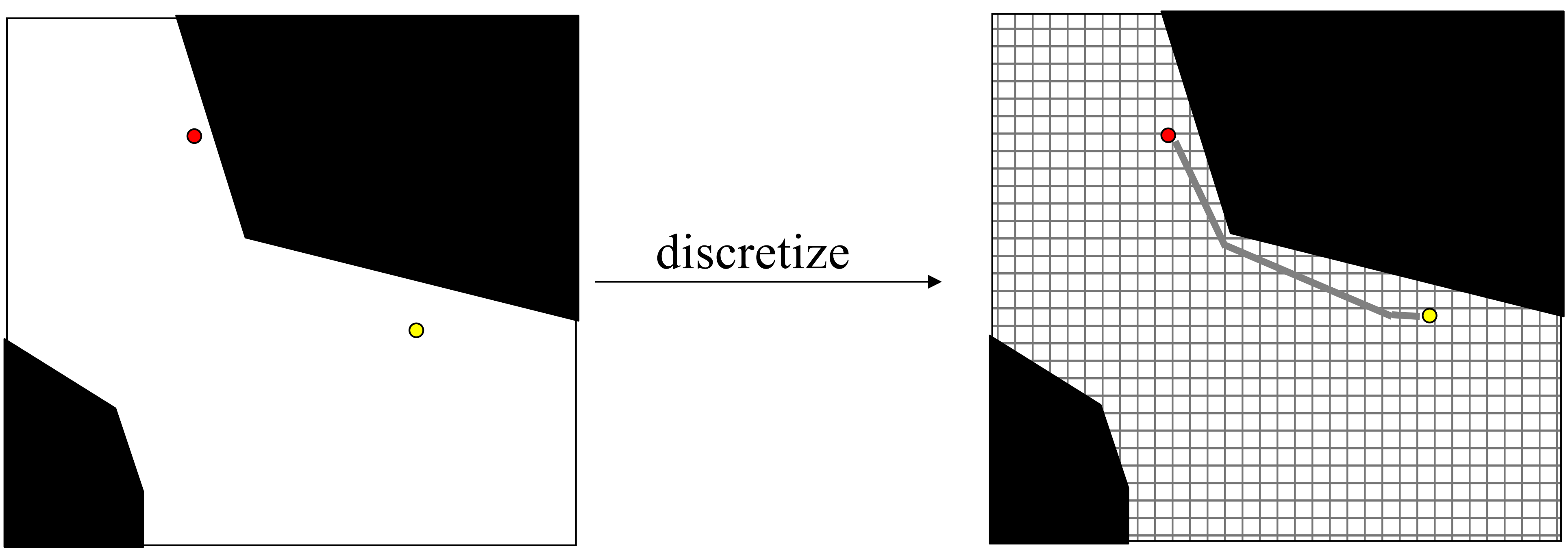
Planning via Cell Decomposition

- Approximate Cell Decomposition:
  - construct a graph and search it for a least-cost path



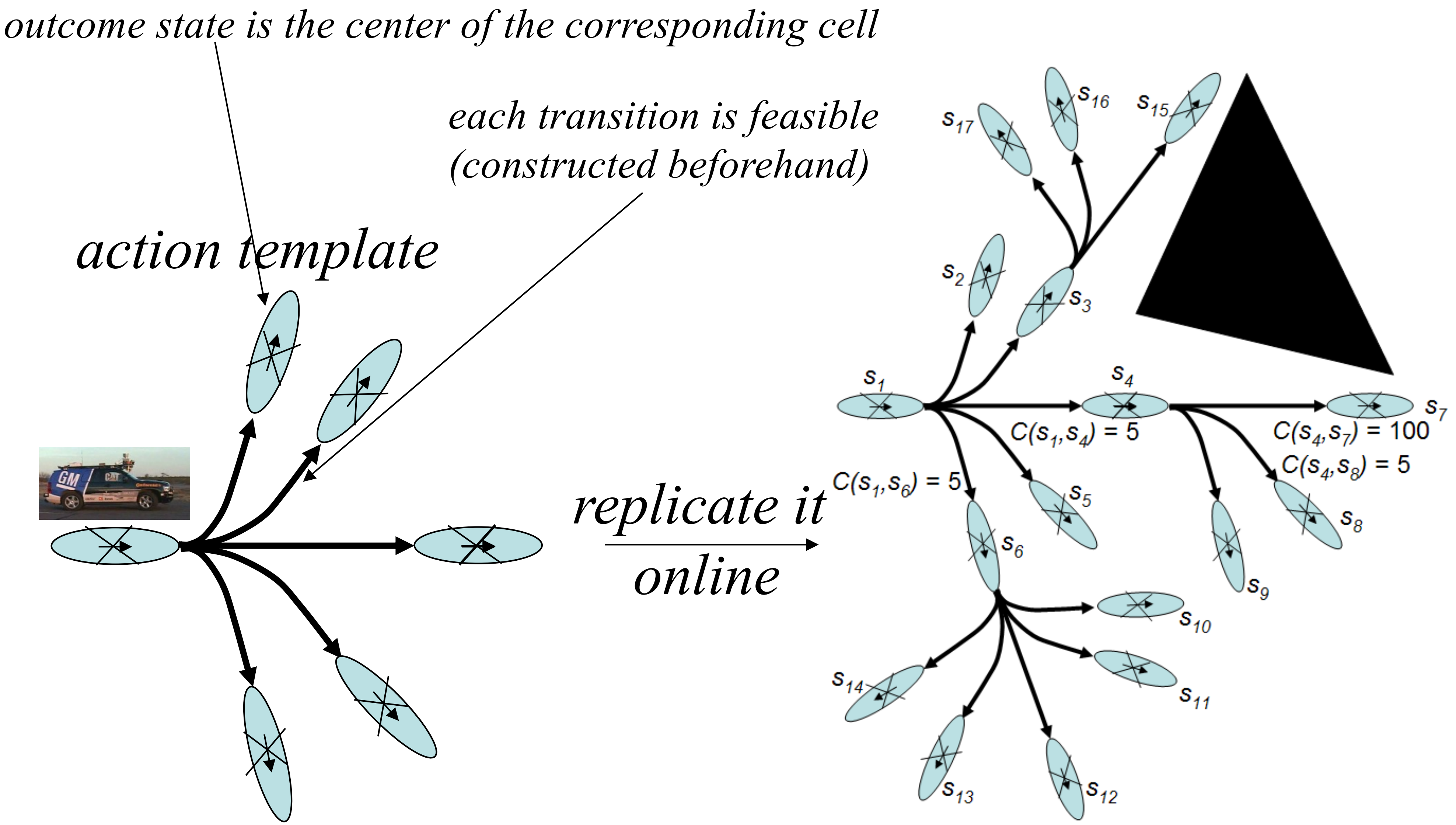
Planning via Cell Decomposition

- Approximate Cell Decomposition:
  - construct a graph and search it for a least-cost path
    - VERY popular due to its simplicity and representation of arbitrary obstacles
    - Problem: transitions difficult to execute on non-holonomic robots



Planning via Cell Decomposition

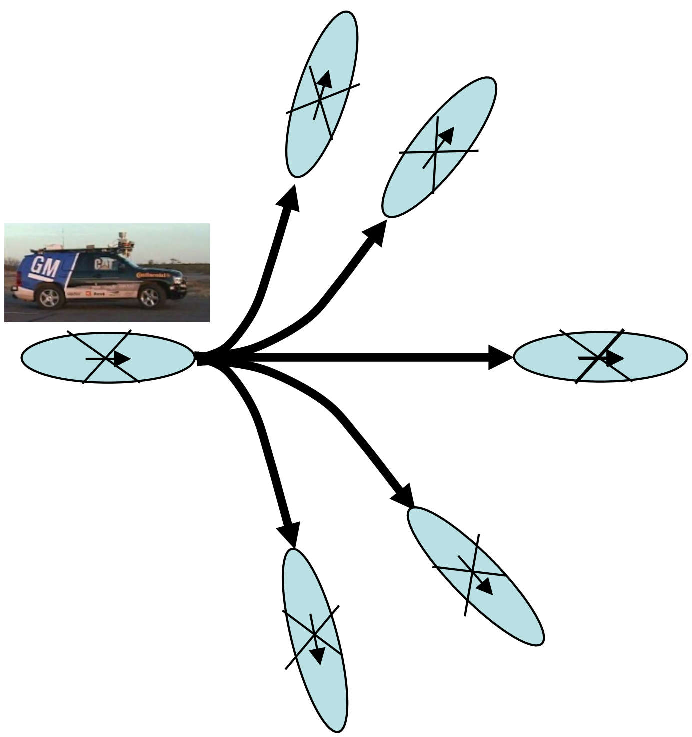
- Graph construction:
  - lattice graph



Planning via Cell Decomposition

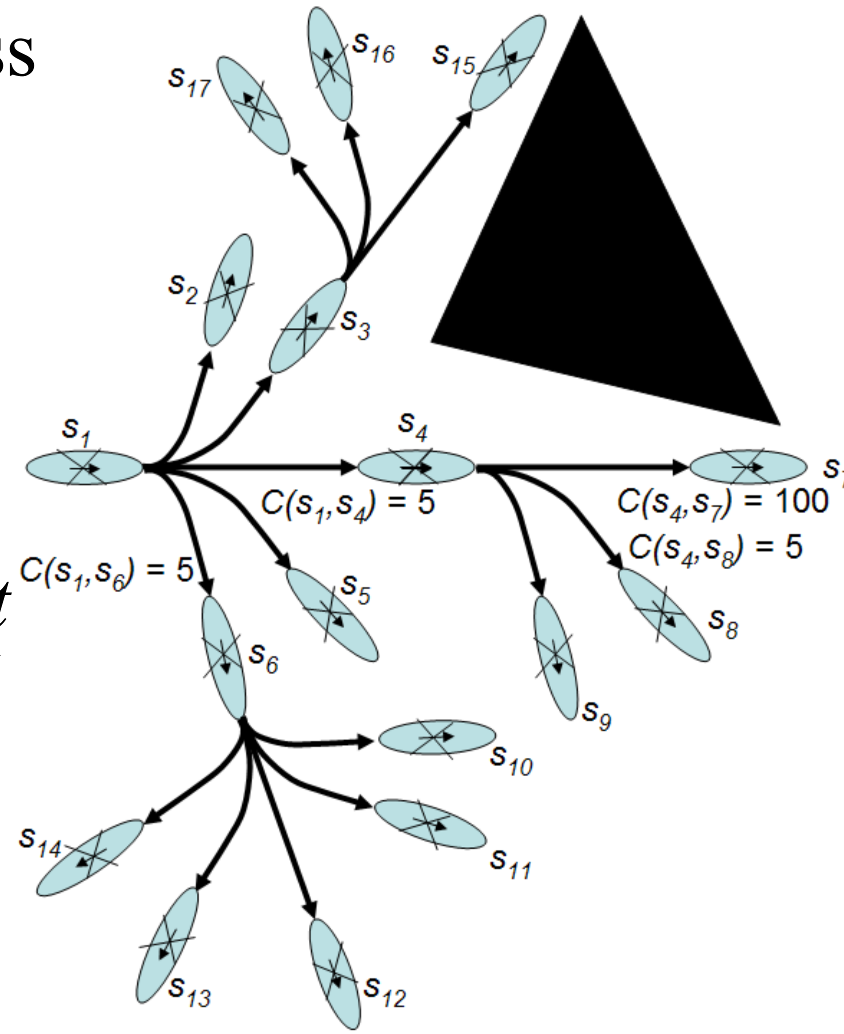
- Graph construction:
  - lattice graph
  - pros: sparse graph, feasible paths
  - cons: possible incompleteness

action template



replicate it

online



Outline

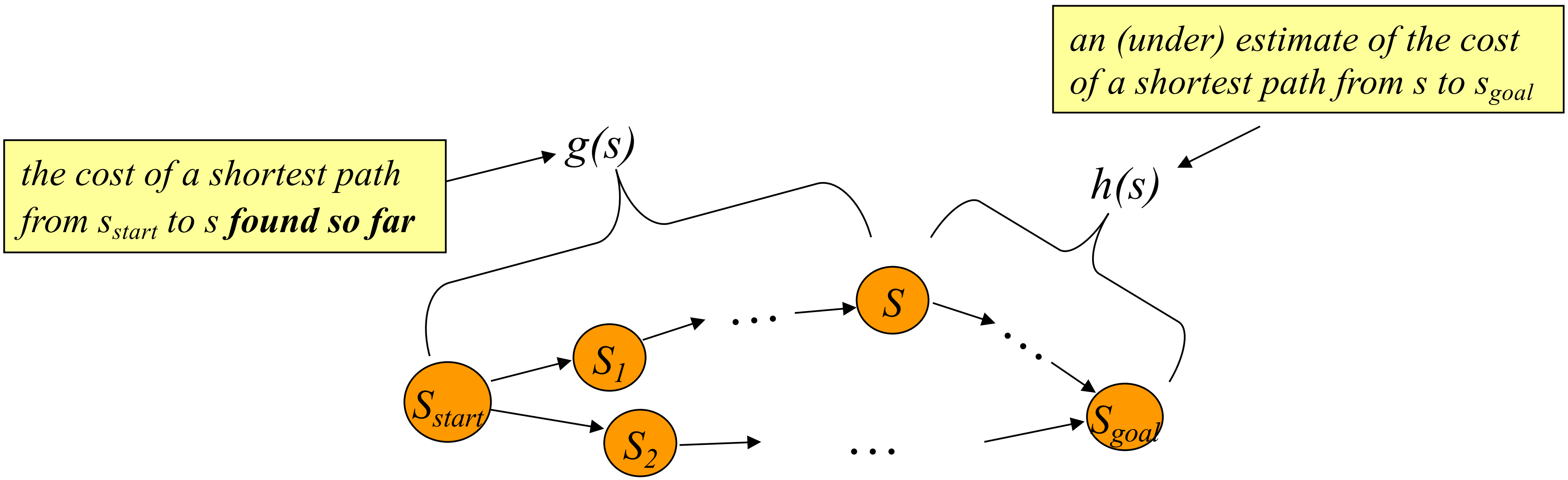
- Deterministic planning
  - constructing a graph
  - search with A*
  - search with D*

- Planning under uncertainty
  -Markov Decision Processes (MDP)
  -Partially Observable Decision Processes (POMDP)

A* Search

- Computes optimal g-values for relevant states

at any point of time:



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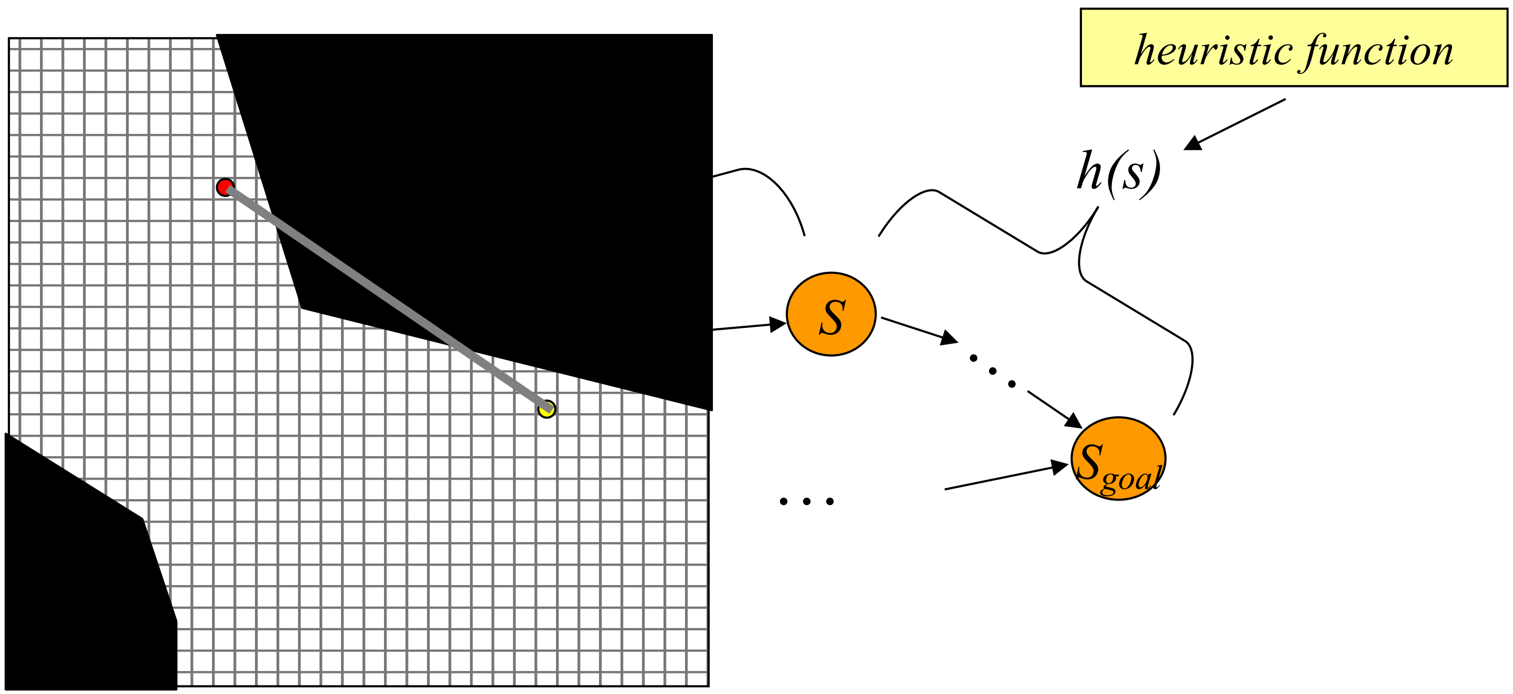


one popular heuristic function – Euclidean distance

A* Search

- Computes optimal g-values for relevant states

ComputePath function

while(s_{goal} is not expanded)

remove s with the smallest $[f(s) = g(s)+h(s)]$ from $OPEN$;

insert s into $CLOSED$;

for every successor s' of s such that s' not in $CLOSED$

if $g(s') > g(s) + c(s,s')$

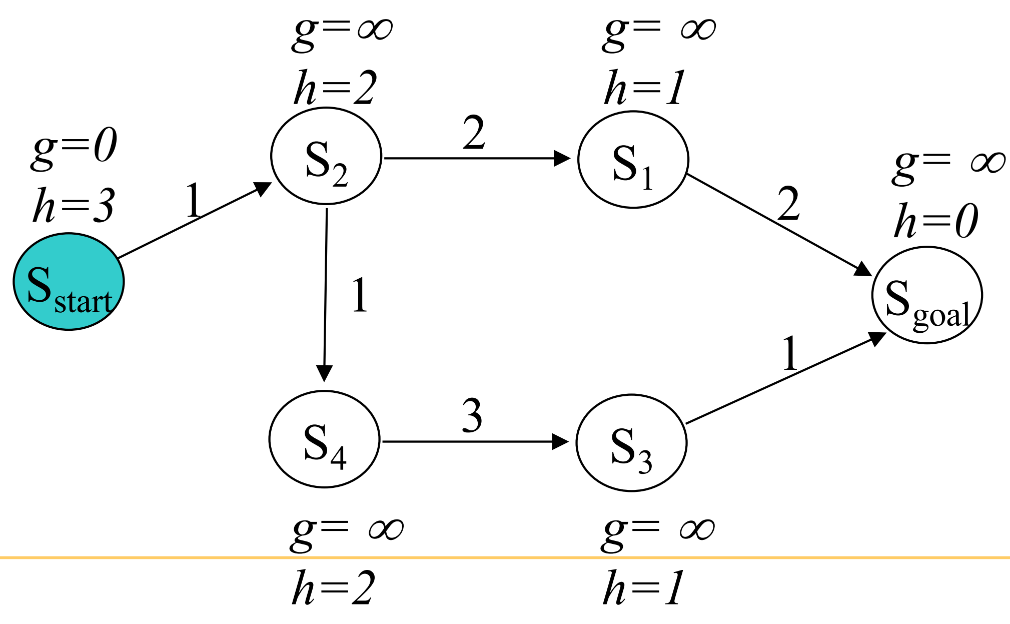
$g(s') = g(s) + c(s,s')$;

insert s' into $OPEN$;

$CLOSED = \{\}$

$OPEN = \{s_{start}\}$

next state to expand: s_{start}



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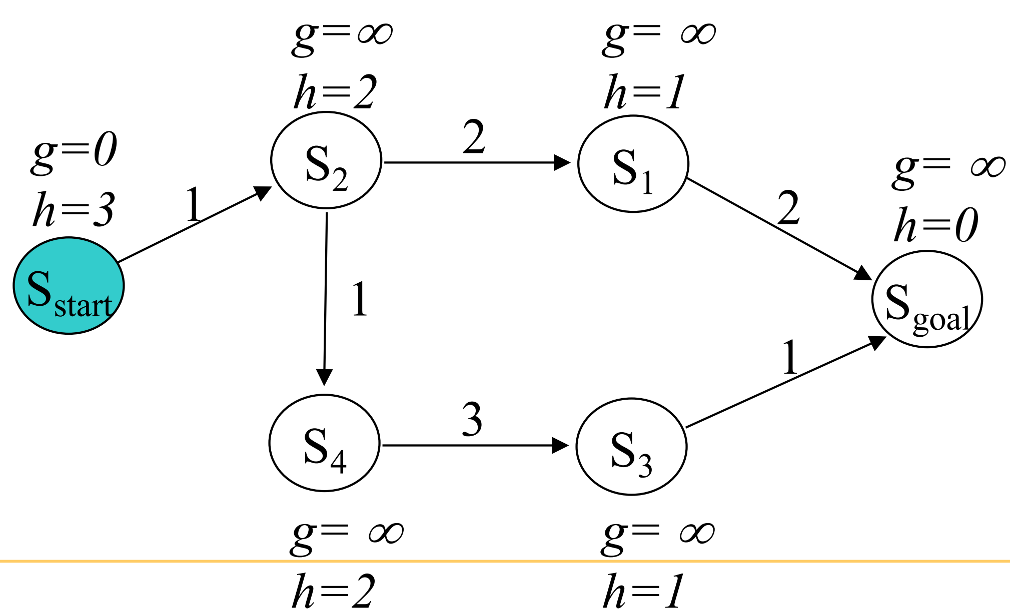
if $g(s') > g(s) + c(s,s')$

$g(s') = g(s) + c(s,s')$;

insert s' into *OPEN*;

$g(s_2) > g(s_{start}) + c(s_{start},s_2)$



CLOSED = {}

OPEN = $\{s_{start}\}$

next state to expand: s_{start}

A* Search

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remove s with the smallest $[f(s) = g(s)+h(s)]$ from *OPEN*;

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if $g(s') > g(s) + c(s,s')$

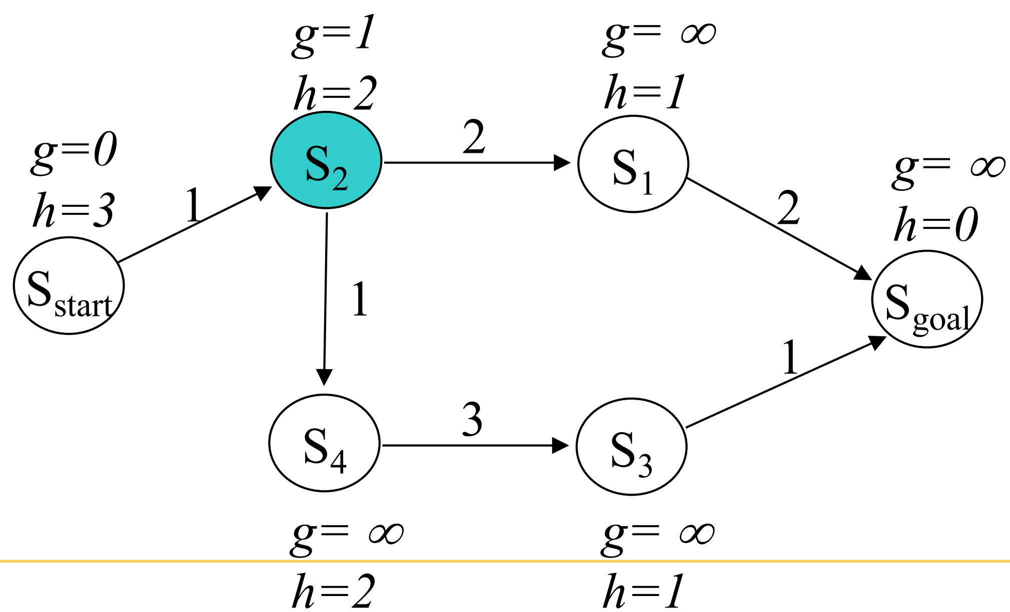
$g(s') = g(s) + c(s,s')$;

insert s' into *OPEN*;

$CLOSED = \{s_{start}\}$

$OPEN = \{s_2\}$

next state to expand: s_2



A* Search

- Computes optimal g-values for relevant states

ComputePath function

while(s_{goal} is not expanded)

remove s with the smallest $[f(s) = g(s)+h(s)]$ from *OPEN*;

insert s into *CLOSED*;

for every successor s' of s such that s' not in *CLOSED*

if $g(s') > g(s) + c(s,s')$

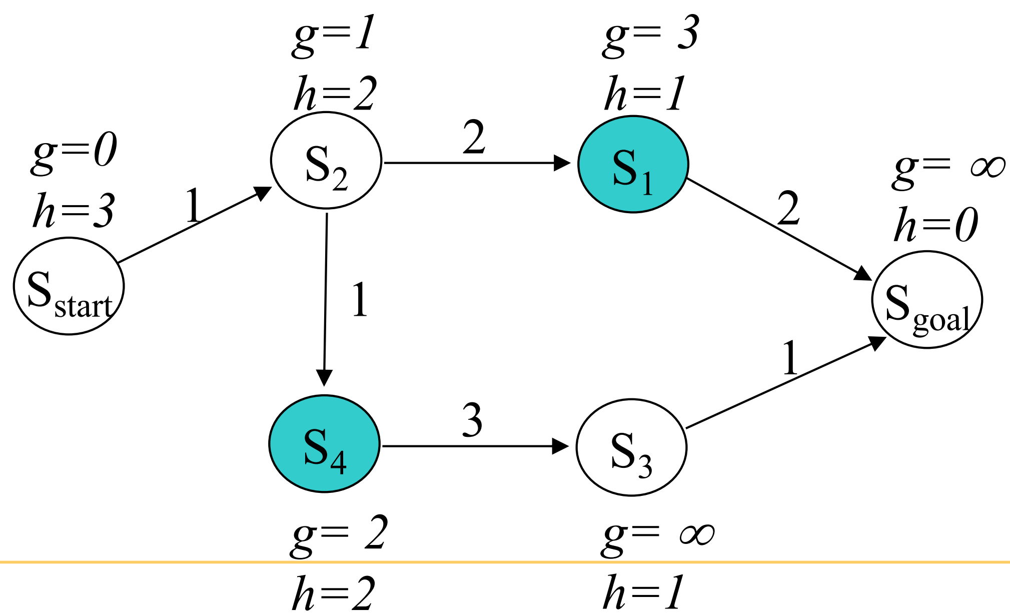
$g(s') = g(s) + c(s,s')$;

insert s' into *OPEN*;

$CLOSED = \{s_{start}, s_2\}$

$OPEN = \{s_1, s_4\}$

next state to expand: s_1



A* Search

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ComputePath function

while(s_{goal} is not expanded)

remove s with the smallest $[f(s) = g(s)+h(s)]$ from *OPEN*;

insert s into *CLOSED*;

for every successor s' of s such that s' not in *CLOSED*

if $g(s') > g(s) + c(s,s')$

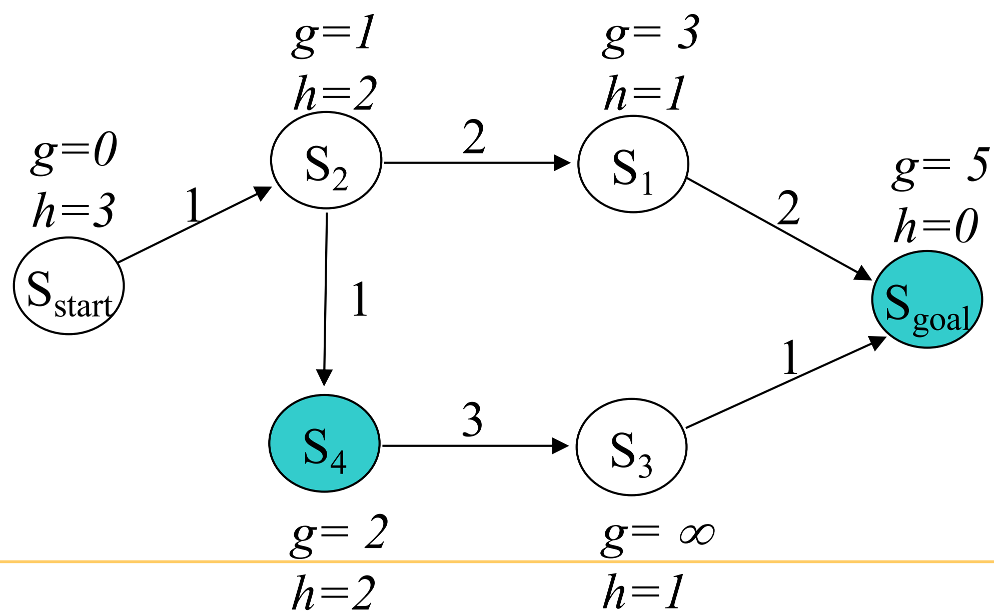
$g(s') = g(s) + c(s,s')$;

insert s' into *OPEN*;

$CLOSED = \{s_{start}, s_2, s_1\}$

$OPEN = \{s_4, s_{goal}\}$

next state to expand: s_4



A* Search

- Computes optimal g-values for relevant states

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while(s_{goal} is not expanded)

remove s with the smallest $[f(s) = g(s)+h(s)]$ from $OPEN$;

insert s into $CLOSED$;

for every successor s' of s such that s' not in $CLOSED$

if $g(s') > g(s) + c(s,s')$

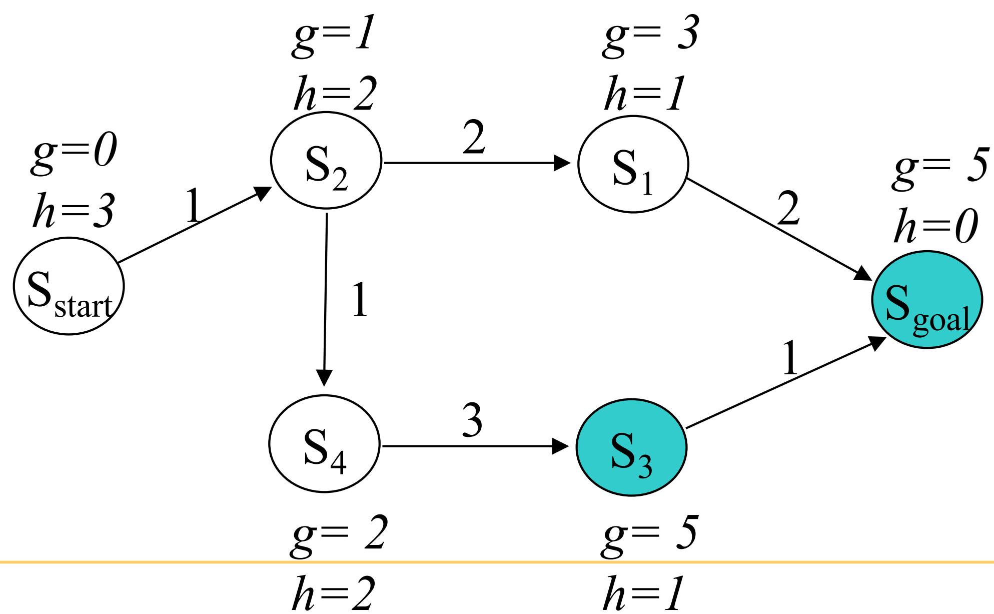
$g(s') = g(s) + c(s,s')$;

insert s' into $OPEN$;

$CLOSED = \{s_{start}, s_2, s_1, s_4\}$

$OPEN = \{s_3, s_{goal}\}$

next state to expand: s_{goal}



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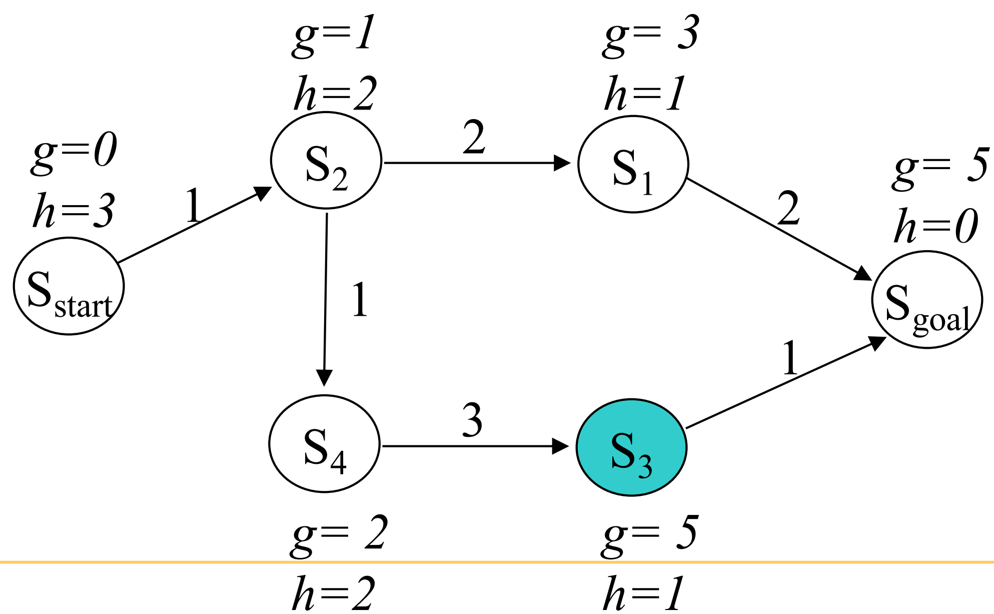
insert s' into *OPEN*;

$CLOSED = \{s_{start}, s_2, s_1, s_4, s_{goal}\}$

$OPEN = \{s_3\}$

done



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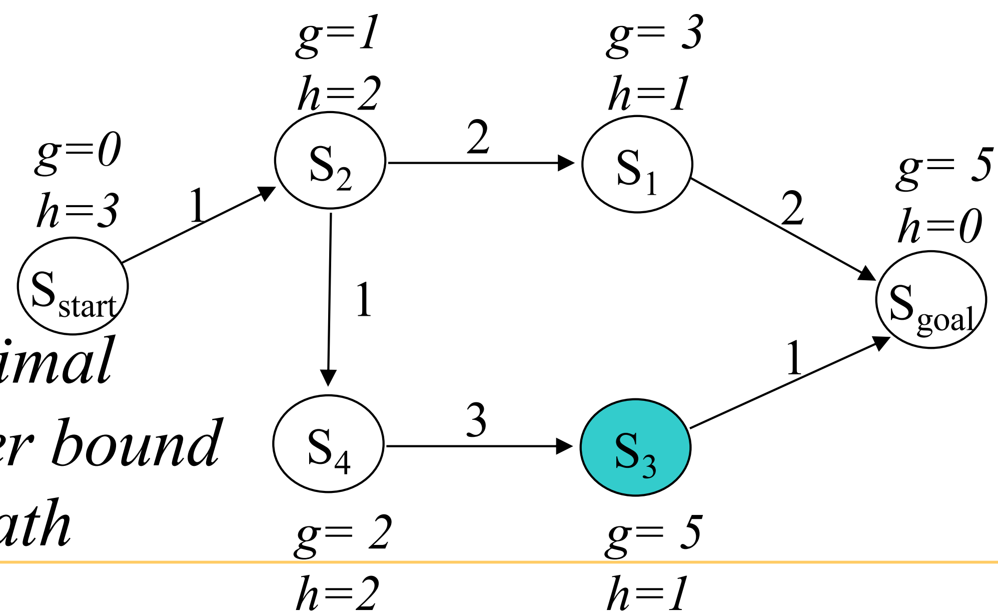
      insert *s'* into *OPEN*;



for every expanded state g(s) is optimal
for every other state g(s) is an upper bound
we can now compute a least-cost path

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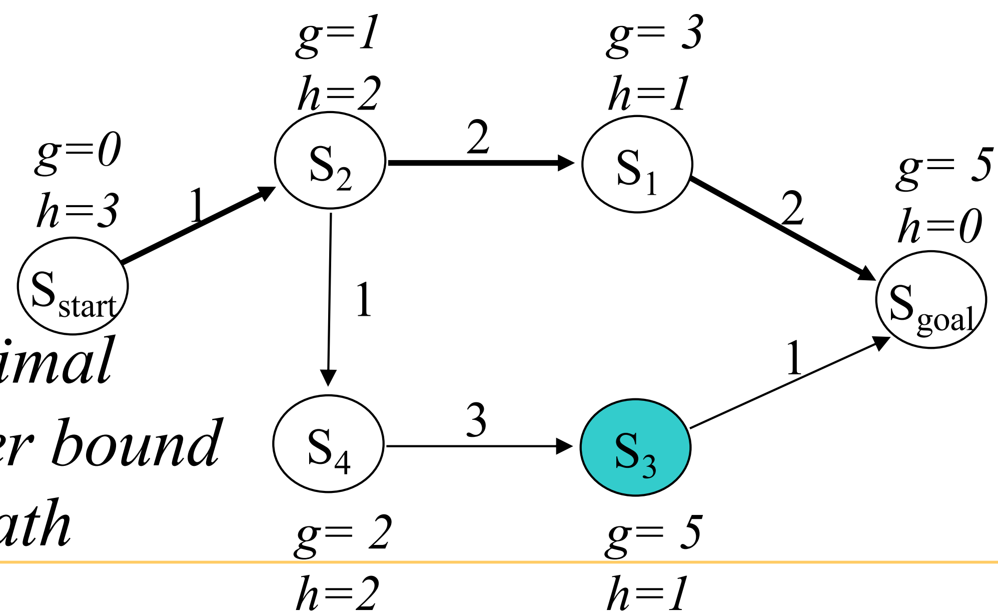
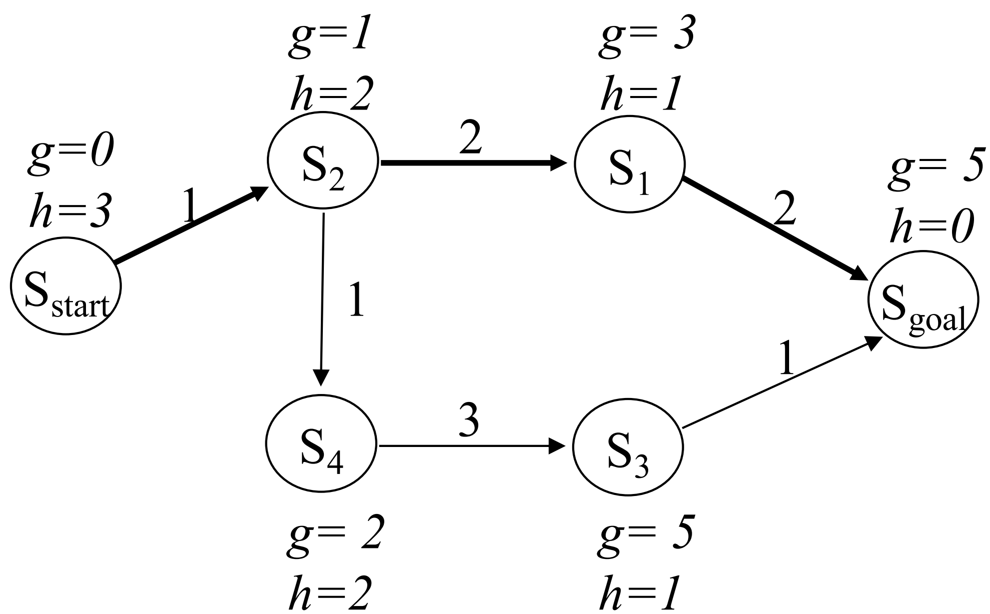


for every expanded state g(s) is optimal
for every other state g(s) is an upper bound
we can now compute a least-cost path

A* Search

- Is guaranteed to return an optimal path (in fact, for every expanded state) – optimal in terms of the solution

- Performs provably minimal number of state expansions required to guarantee optimality – optimal in terms of the computations

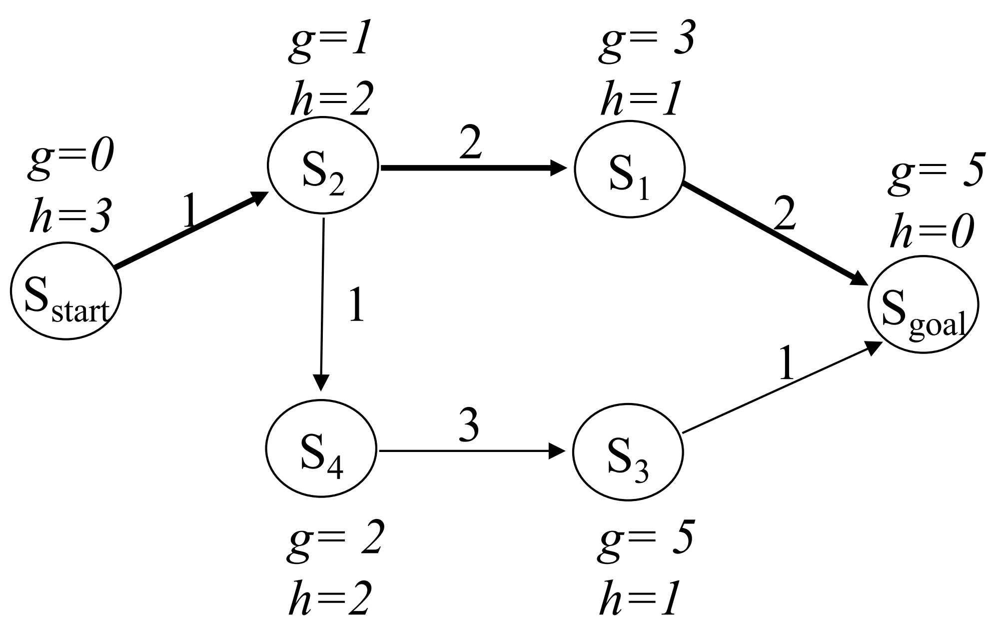


A* Search

- Is guaranteed to return an optimal path (in fact, for every expanded state) – o ion

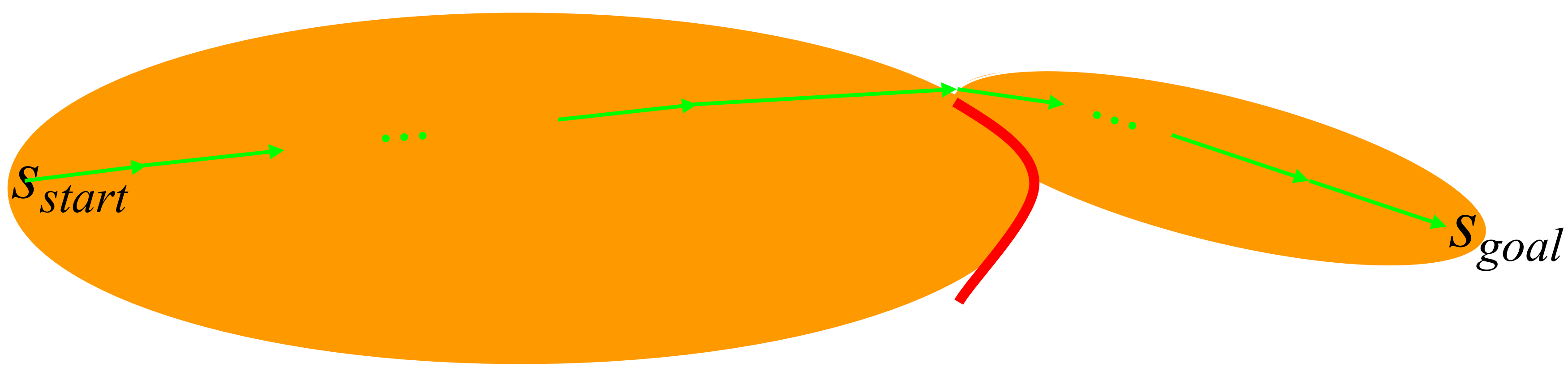
helps with robot deviating off its path if we search with A backwards (from goal to start)*

- Performs provably minimal number of state expansions required to guarantee optimality – optimal in terms of the computations



Effect of the Heuristic Function

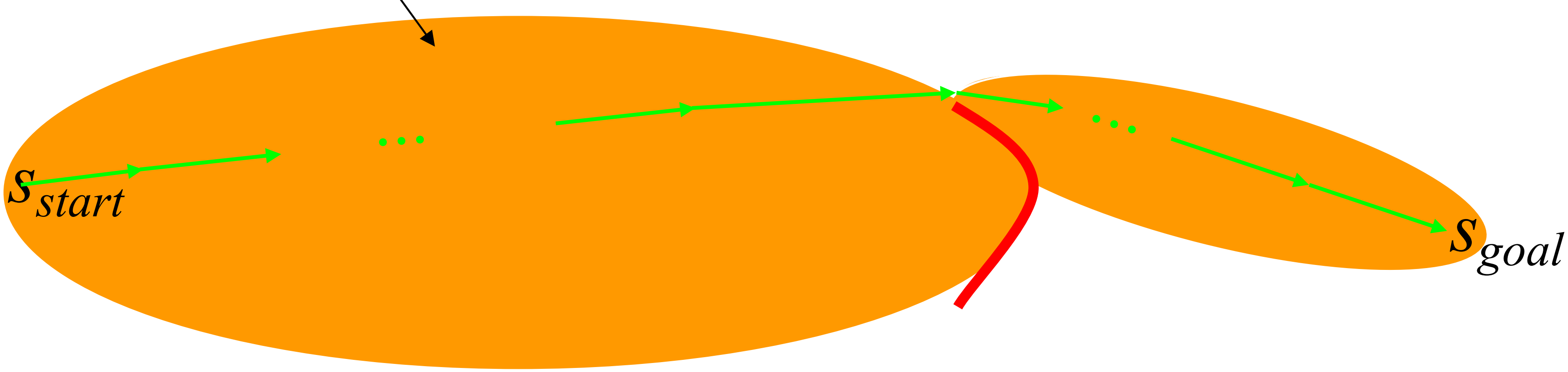
- A* Search: expands states in the order of $f = g+h$ values



Effect of the Heuristic Function

- A* Search: expands states in the order of $f = g+h$ values

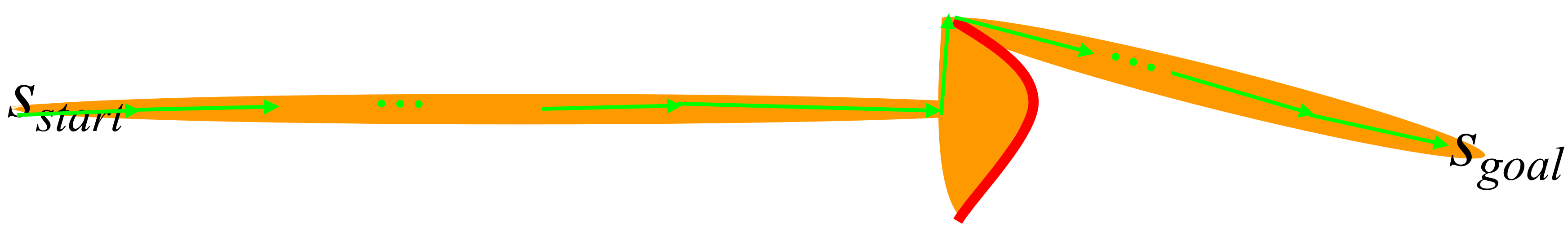
for large problems this results in A quickly running out of memory (memory: O(n))*



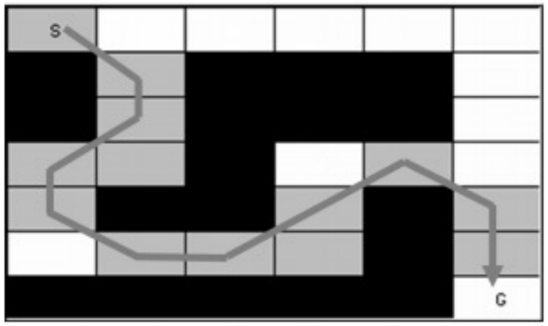
Effect of the Heuristic Function

- Weighted A* Search: expands states in the order of $f = g+\varepsilon h$ values, $\varepsilon > 1$ = bias towards states that are closer to goal

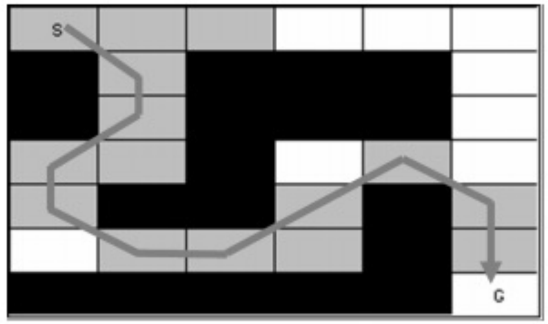
solution is always ε-suboptimal:
cost(solution) ≤ ε·cost(optimal solution)



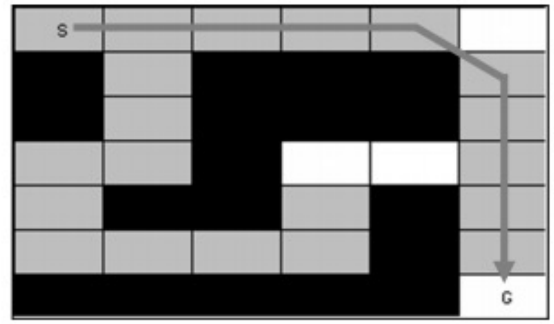
Adaptive Real-Time A*



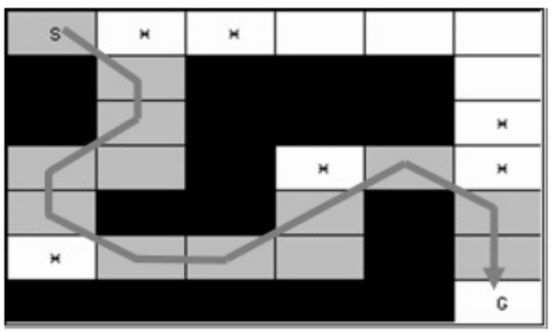
$\epsilon = 2.5$



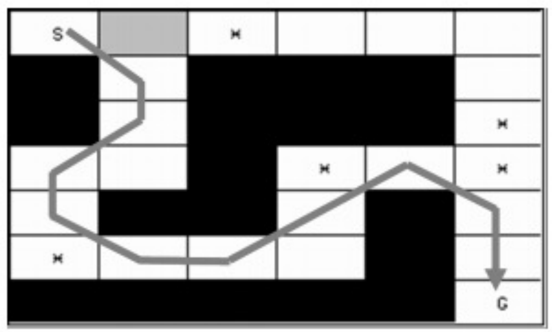
$\epsilon = 1.5$



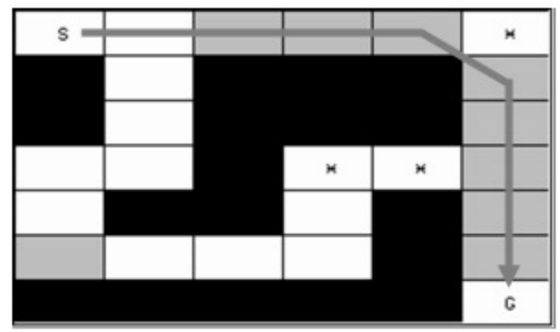
$\epsilon = 1.0$ (optimal search)



initial search ($\epsilon = 2.5$)



second search ($\epsilon = 1.5$)



third search ($\epsilon = 1.0$)

Effect of the Heuristic Function

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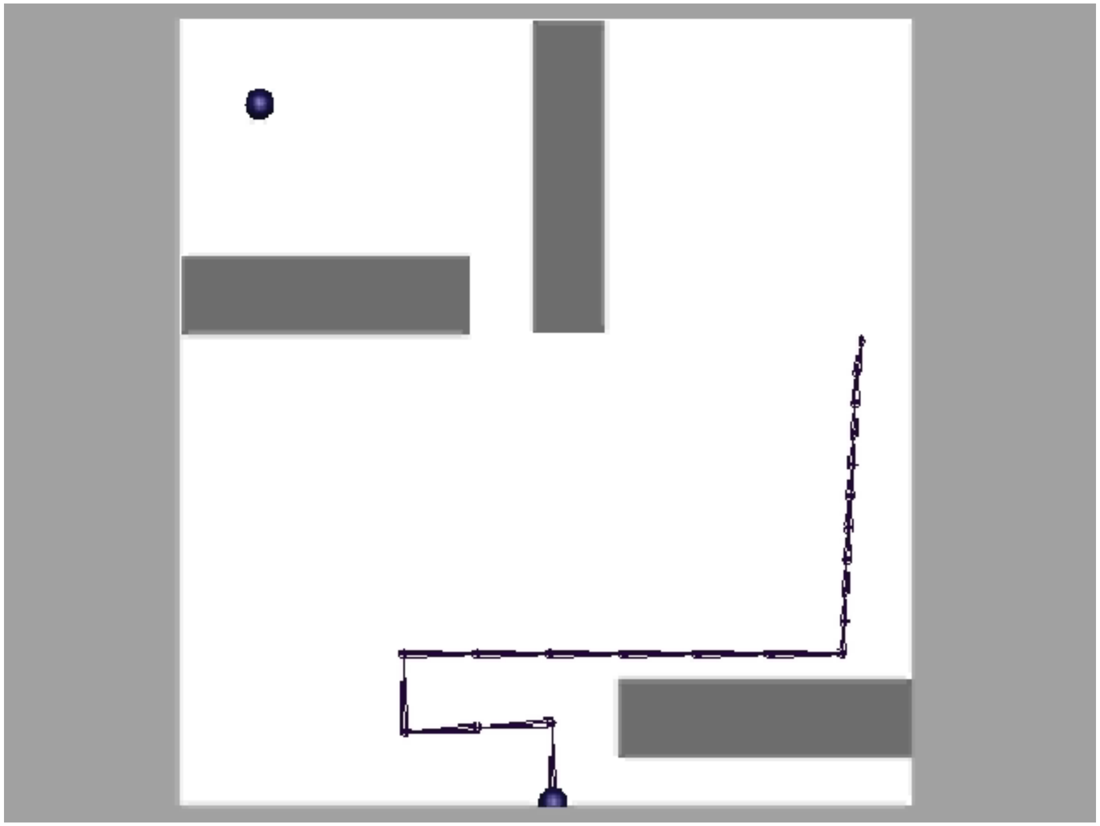
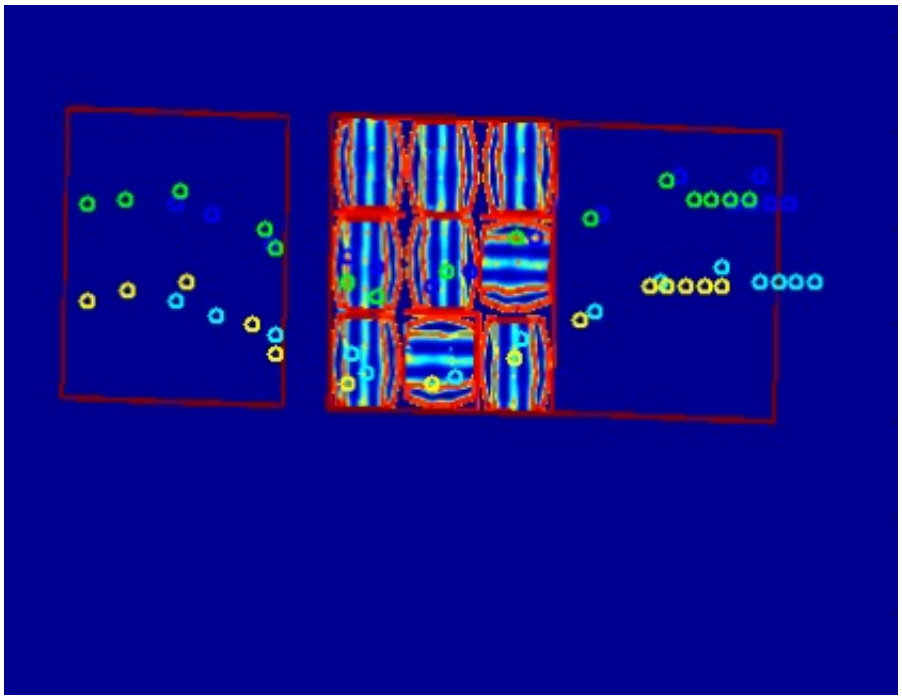
20DOF simulated robotic arm
state-space size: over 10^{26} states



planning with ARA* (anytime version of weighted A*)

Effect of the Heuristic Function

- planning in 8D (<*x*,*y*> for each foothold)
- heuristic is Euclidean distance from the center of the body to the goal location
- cost of edges based on kinematic stability of the robot and quality of footholds



planning with R* (randomized version of weighted A*)

joint work with Subhrajit Bhattacharya, Jon Bohren, Sachin Chitta, Daniel D. Lee, Aleksandr Kushleyev, Paul Vernaza

Outline

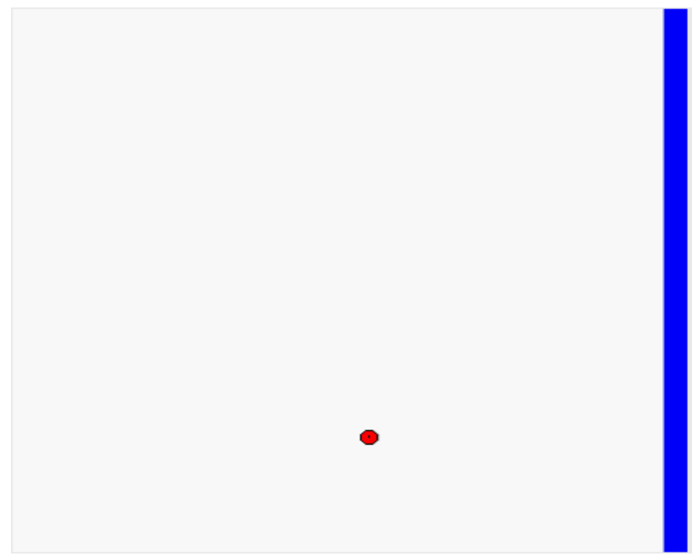
- Deterministic planning
  - constructing a graph
  - search with A*
  - search with D*

Incremental version of A* (D*/D* Lite)

- Robot needs to re-plan whenever
  - new information arrives (partially-known environments or/and dynamic environments)
  - robot deviates off its path

ATRV navigating initially-unknown environment

planning map and path

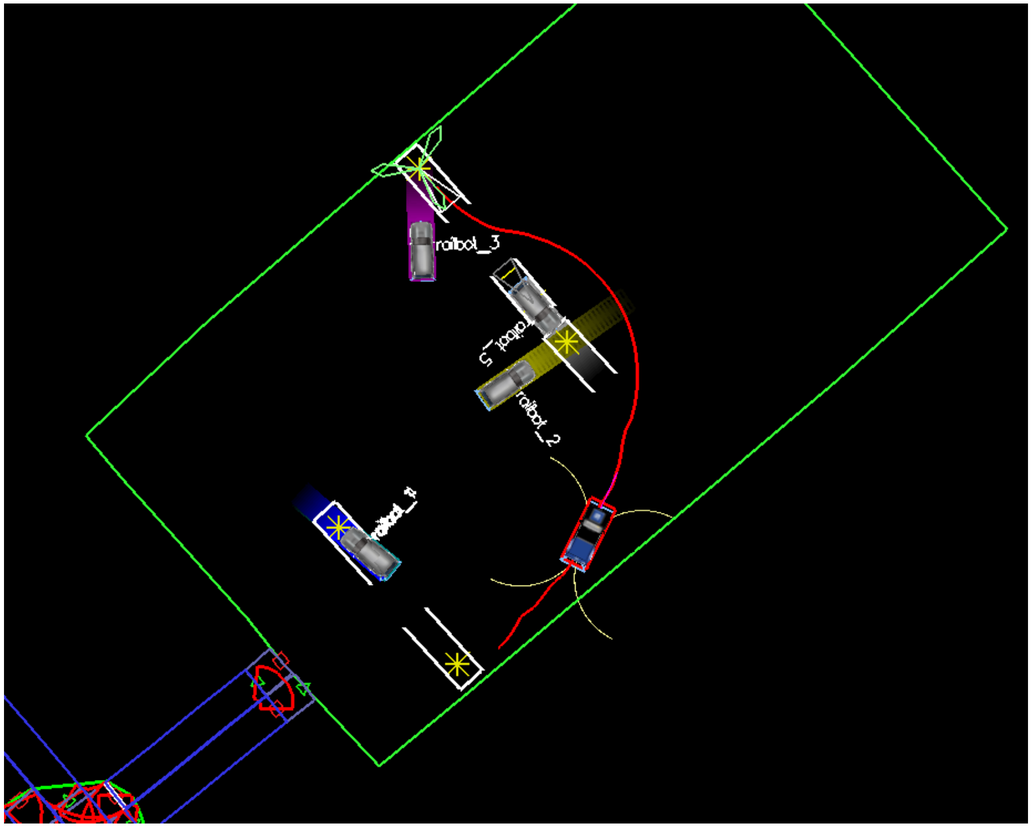


Incremental version of A* (D*/D* Lite)

- Robot needs to re-plan whenever
  - new information arrives (partially-known environments or/and dynamic environments)
  - robot deviates off its path

***incremental planning (re-planning)**: reuse of previous planning efforts*

planning in dynamic environments

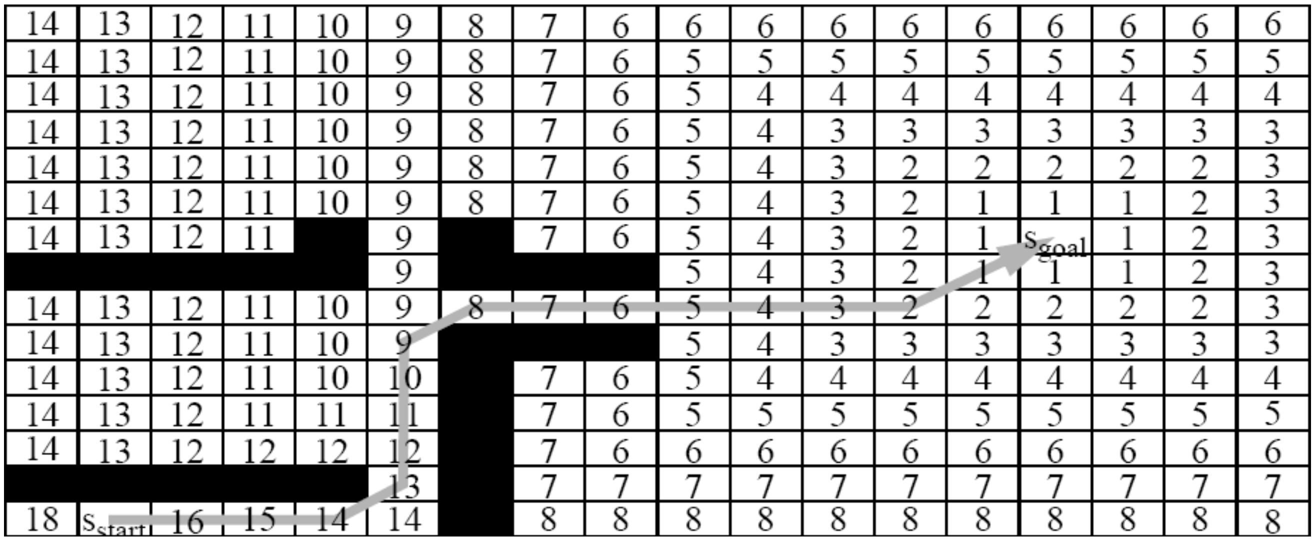


Tartanracing, CMU

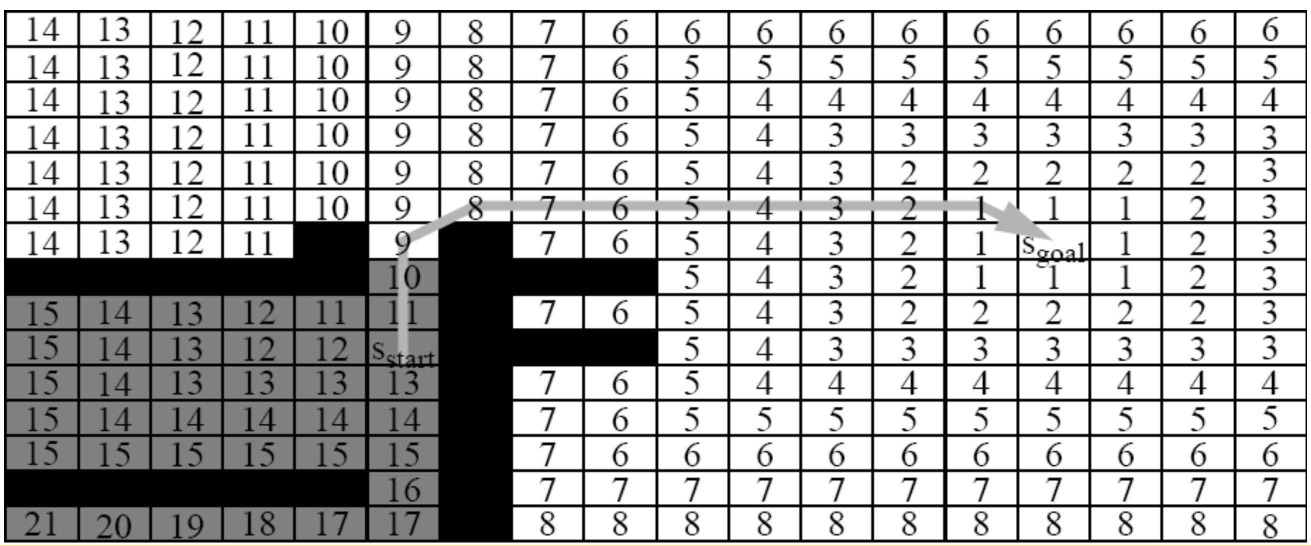
Motivation for Incremental Version of A*

- Reuse state values from previous searches

cost of least-cost paths to s_{goal} initially



cost of least-cost paths to s_{goal} after the door turns out to be closed

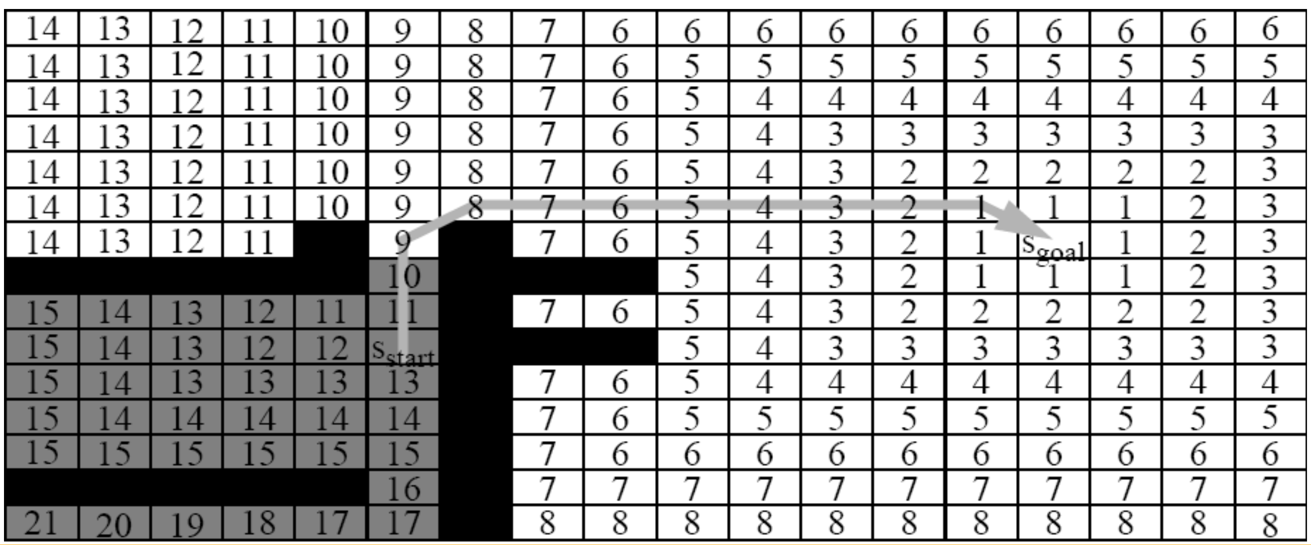


Motivation for Incremental Version of A*

- Reuse state values from previous searches

cost of least-cost paths to s_{goal} initially

cost of least-cost paths to s_{goal} after the door turns out to be closed



Motivation for Incremental Version of A*

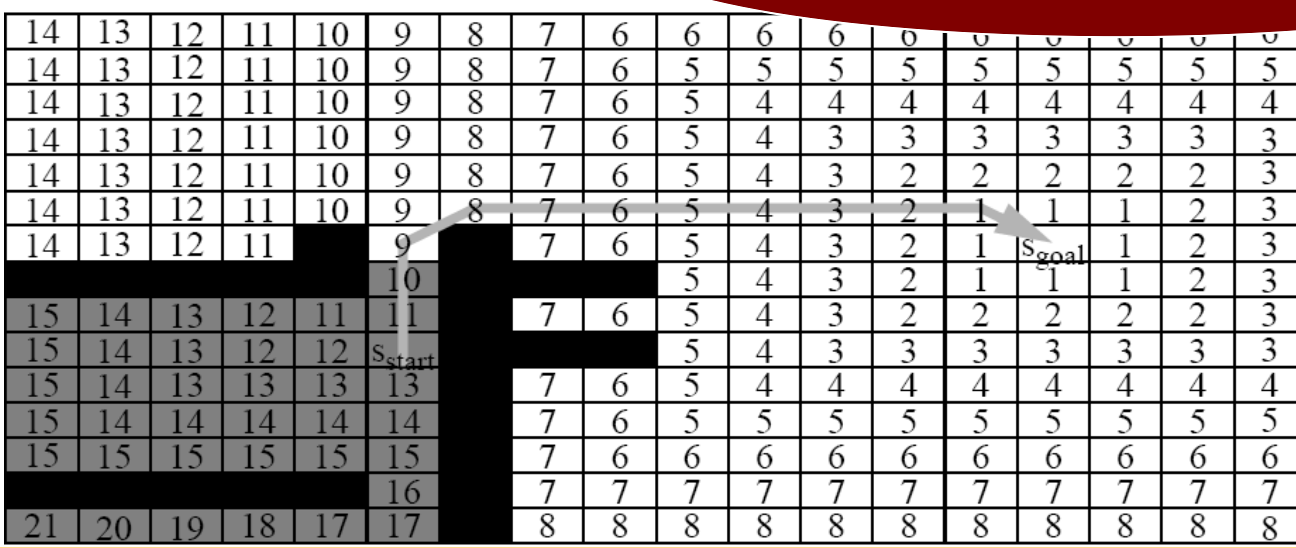
- Reuse state values from previous searches

cost of least-cost paths to s_{goal} initially

These costs are optimal g-values if search is done backwards

cost of least-cost paths to s_{goal} …

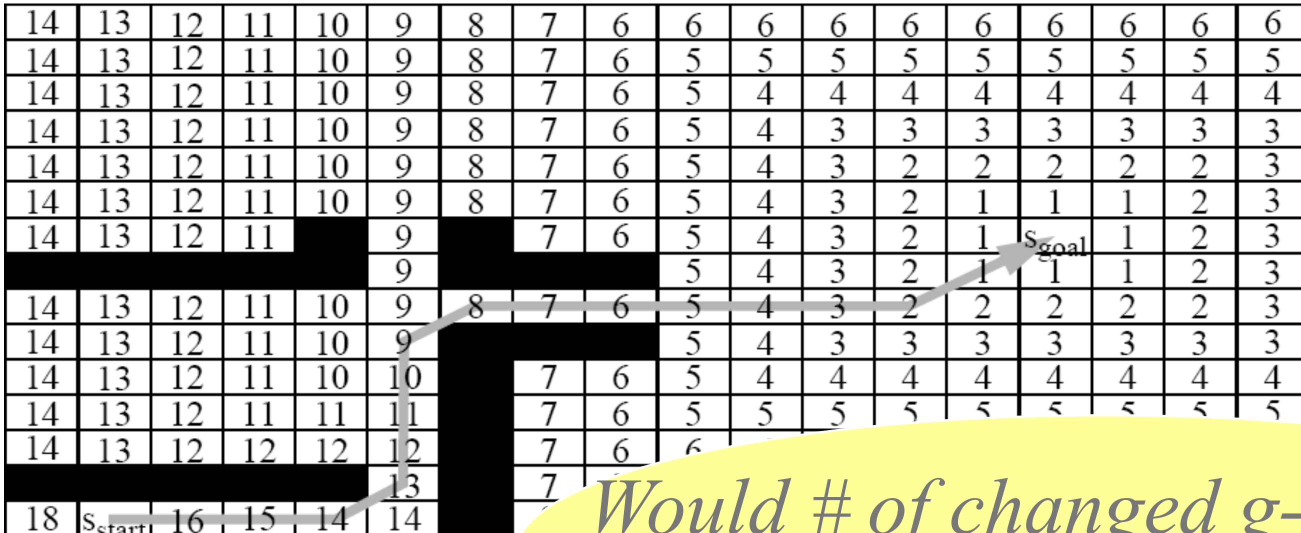
*How to reuse these g-values from one search to another? – incremental A**



Motivation for Incremental Version of A*

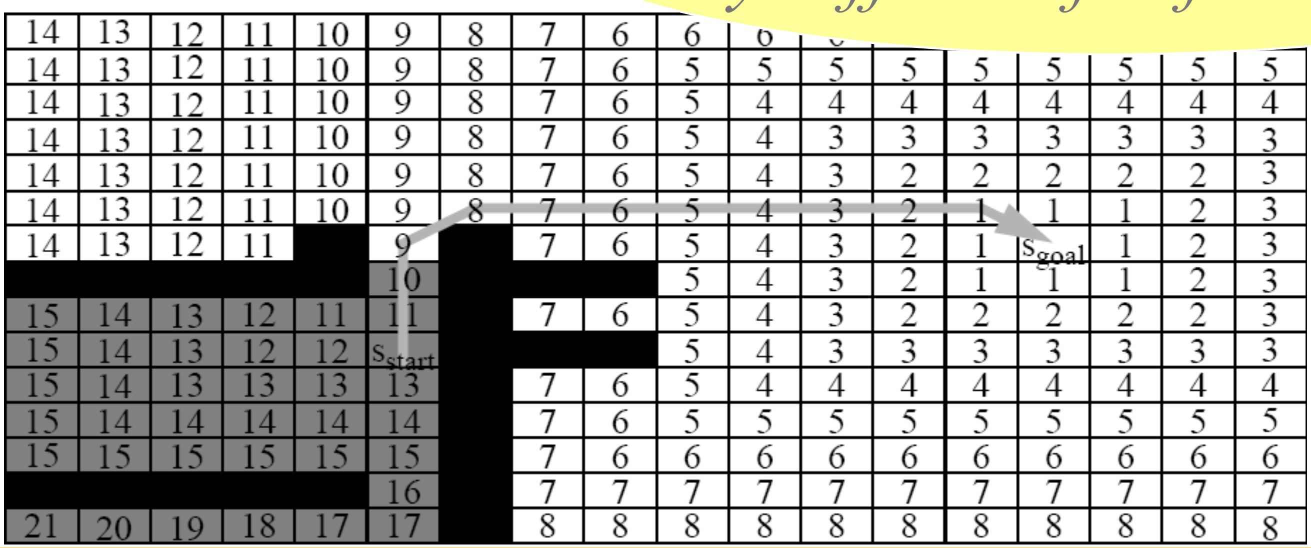
- Reuse state values from previous searches

cost of least-cost paths to s_{goal} initially



Would # of changed g-values be very different for forward A?*

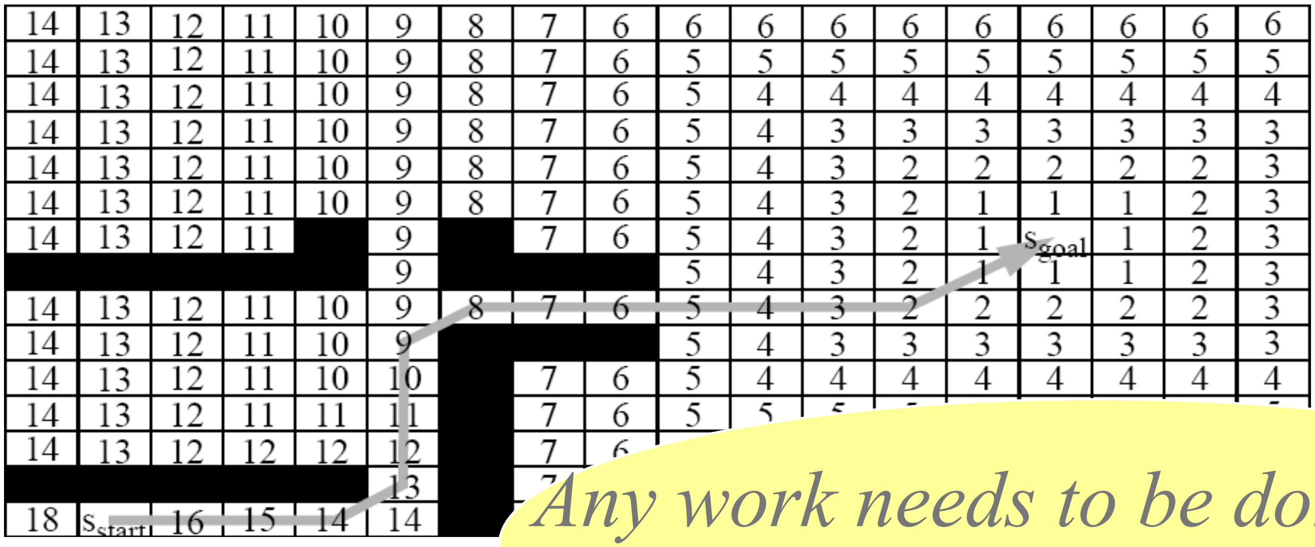
cost of least-cost paths to s_{go}…



Motivation for Incremental Version of A*

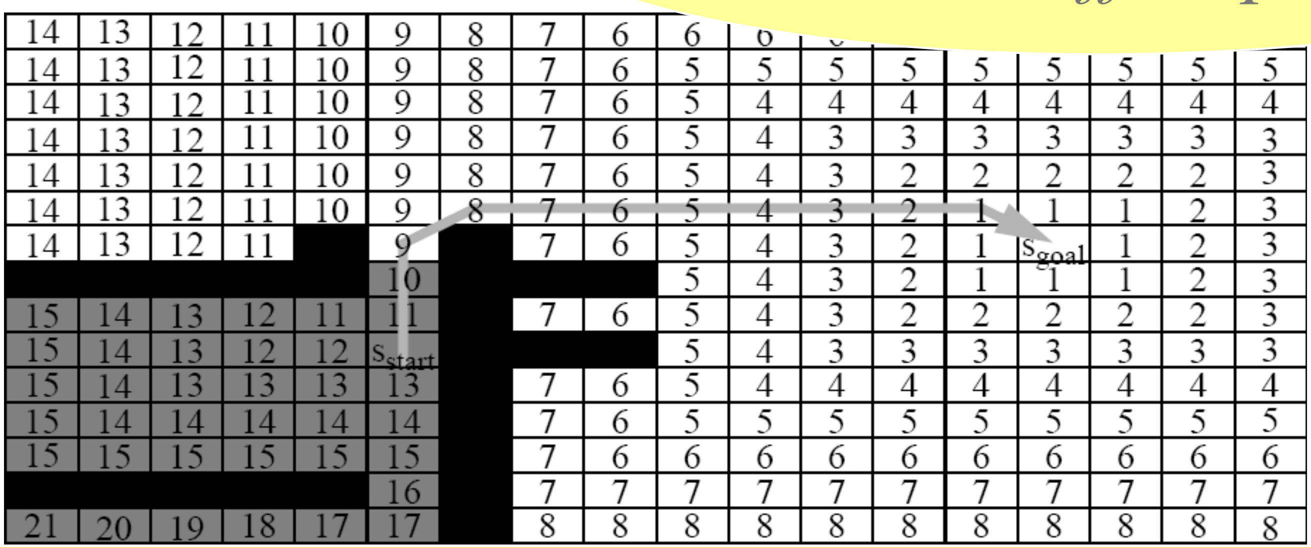
- Reuse state values from previous searches

cost of least-cost paths to s_{goal} initially



Any work needs to be done if robot deviates off its path?

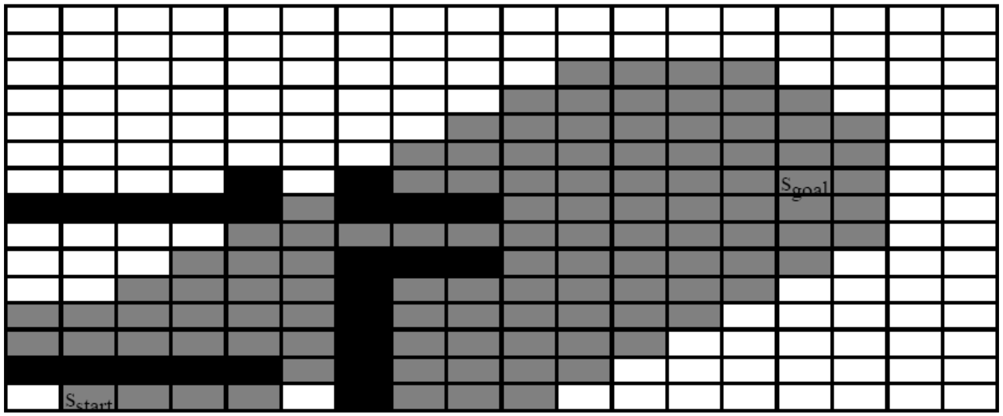
cost of least-cost paths to s_{go}



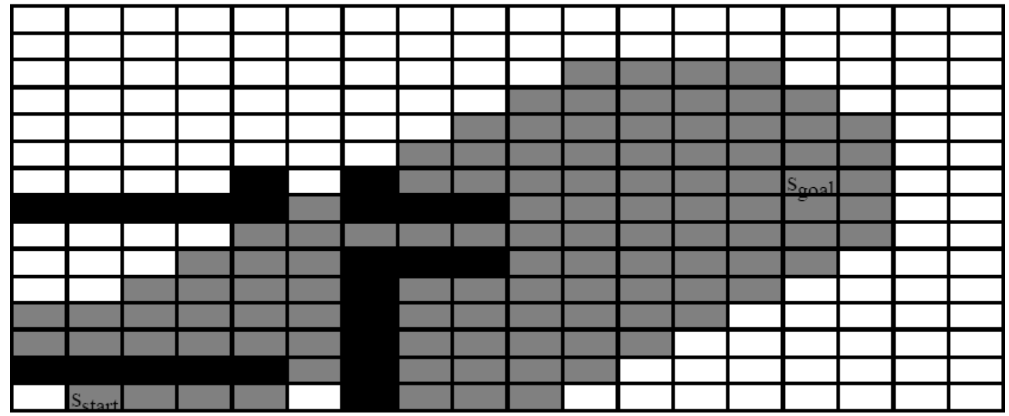
Incremental Version of A*

- Reuse state values from previous searches

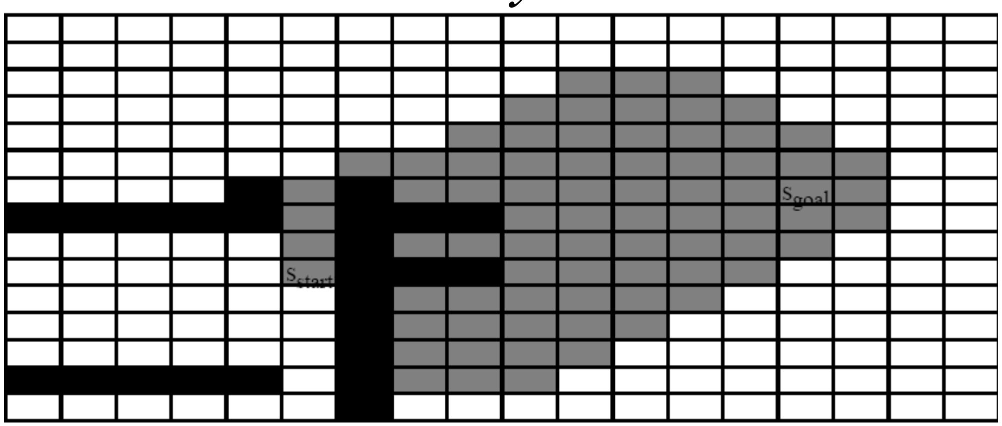
*initial search by backwards A**



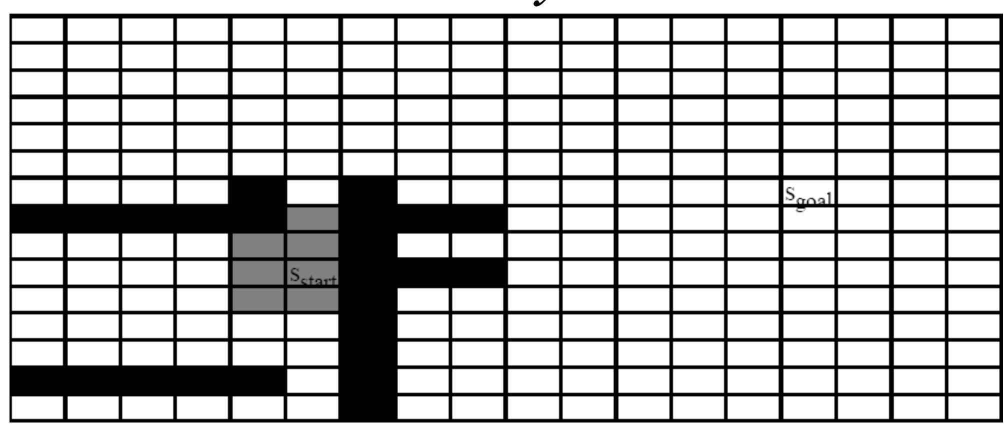
initial search by D Lite*



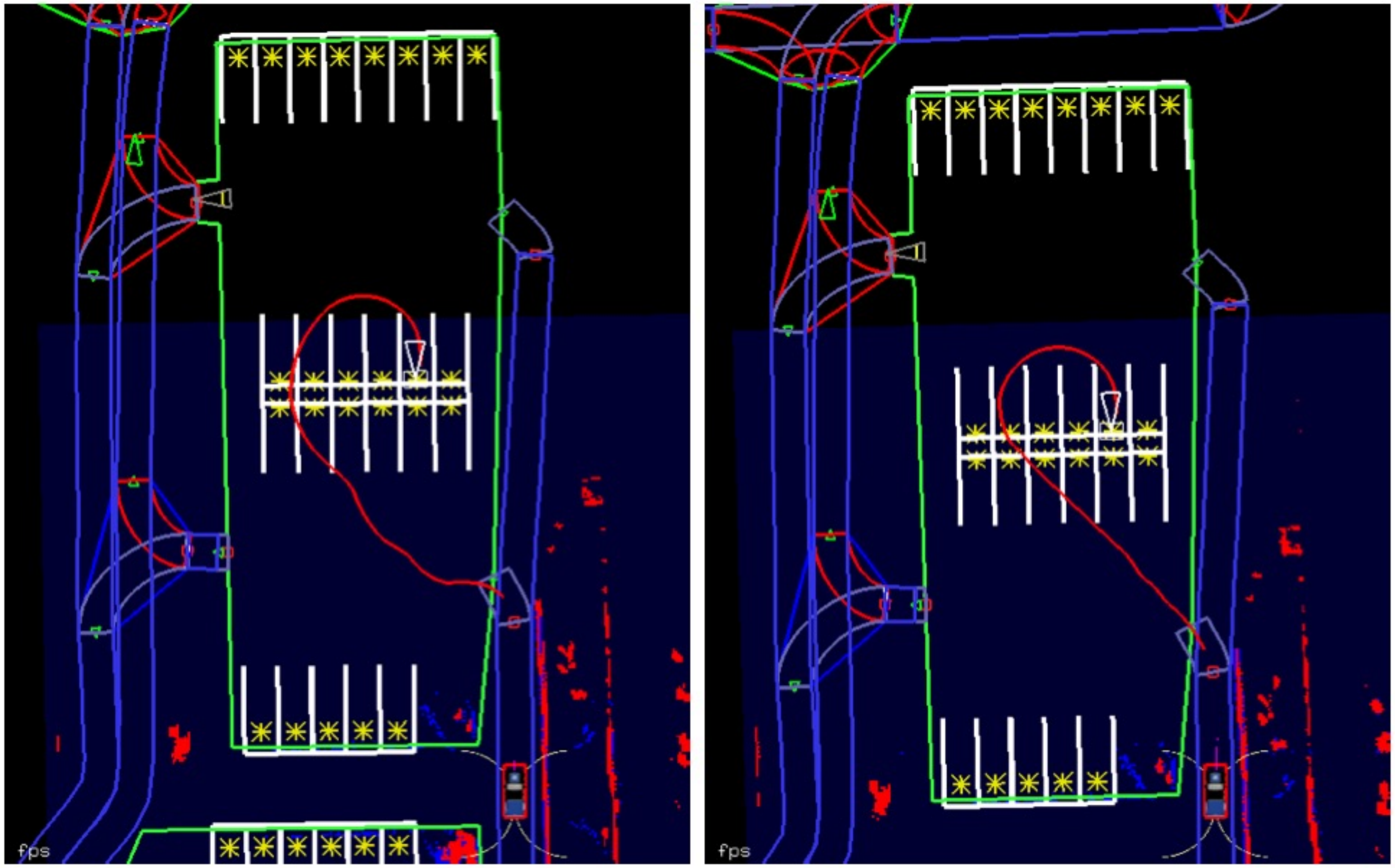
*second search by backwards A**



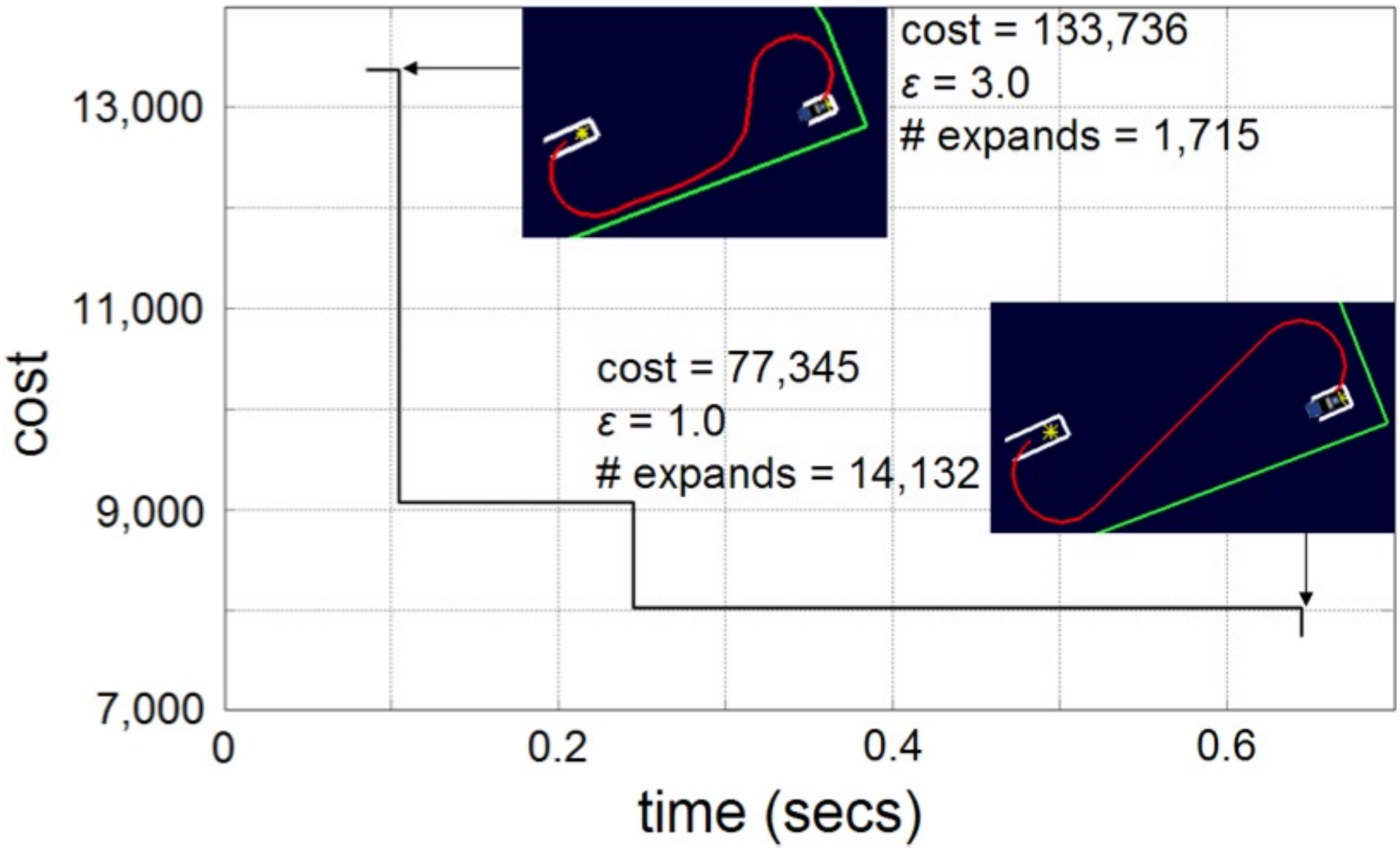
second search by D Lite*



Anytime Aspects

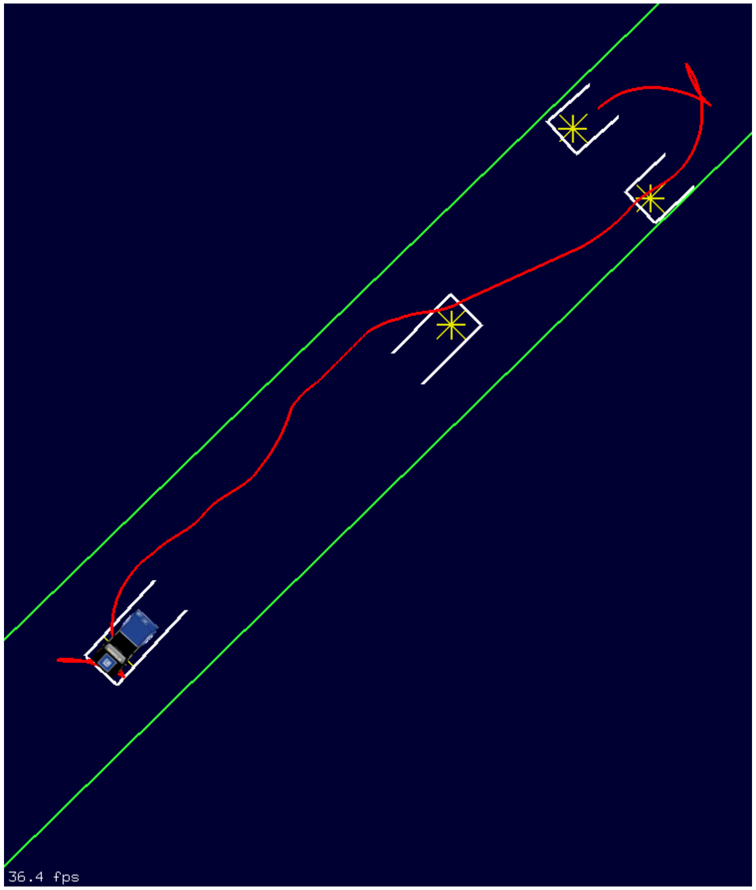


Anytime Aspects

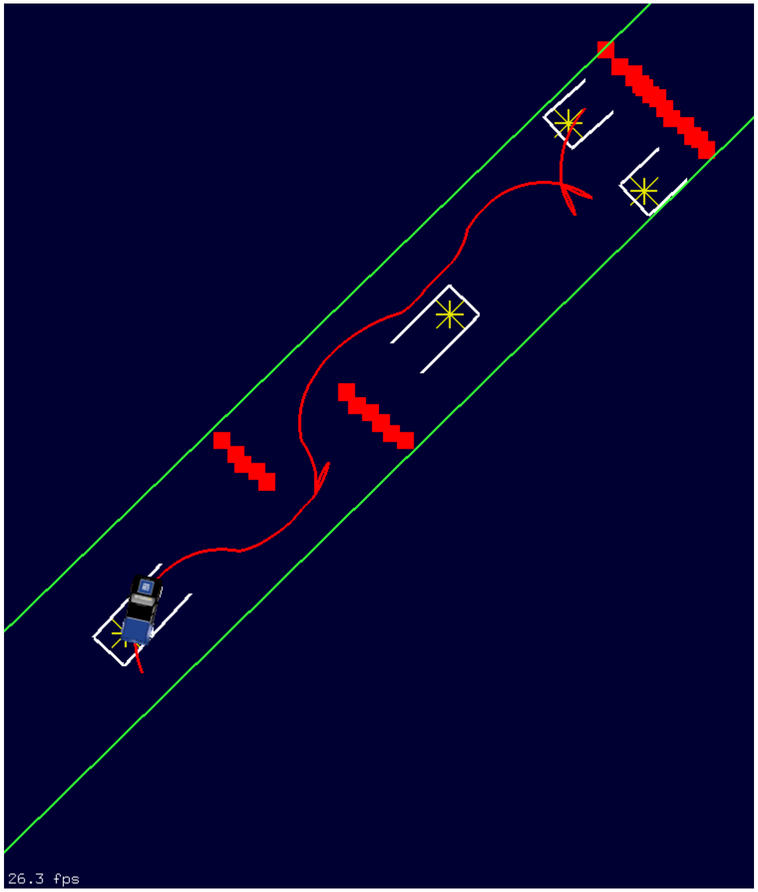


Searching the Graph

- Incremental behavior of Anytime D*:



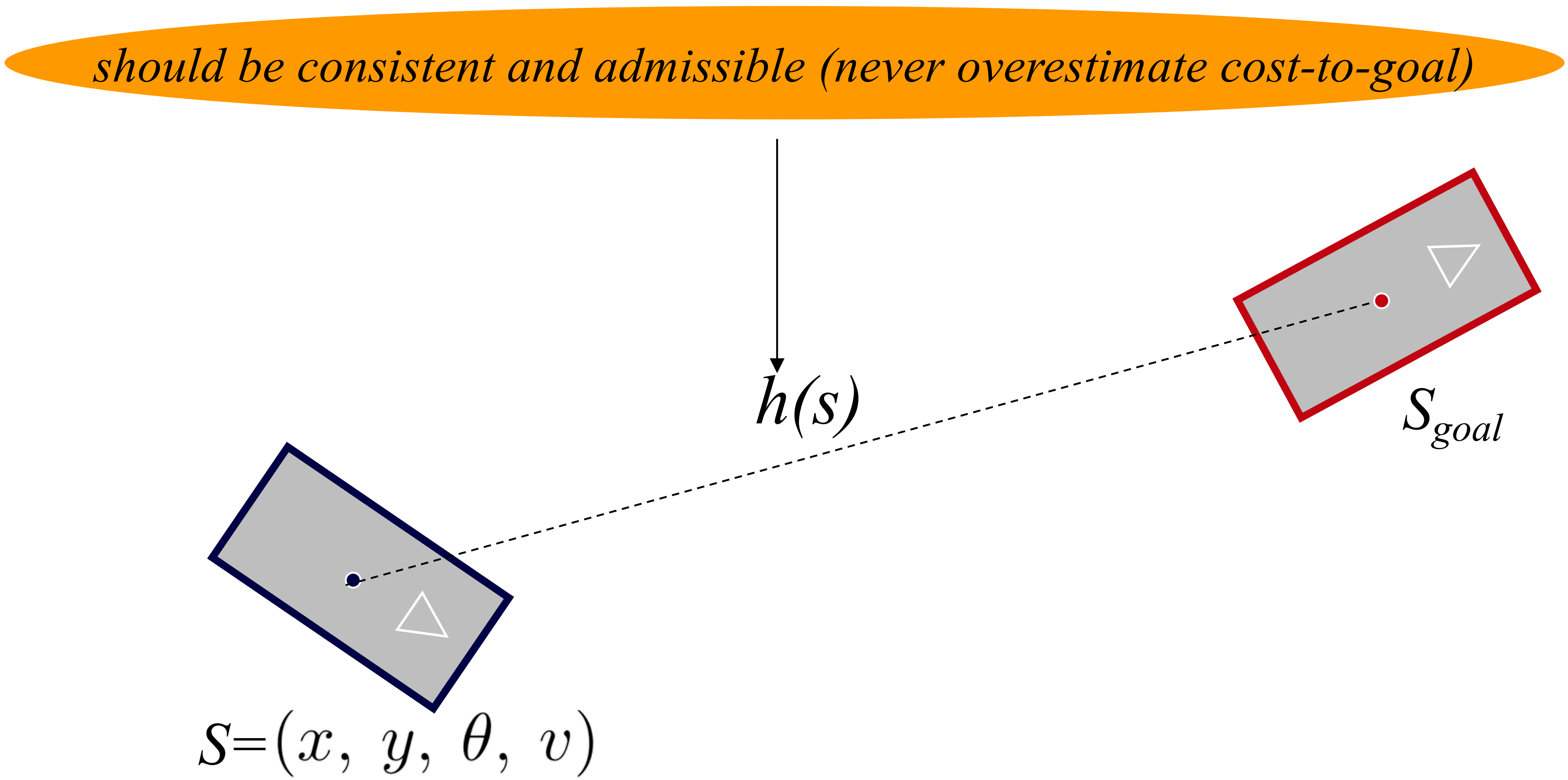
initial path



a path after re-planning

Searching the Graph

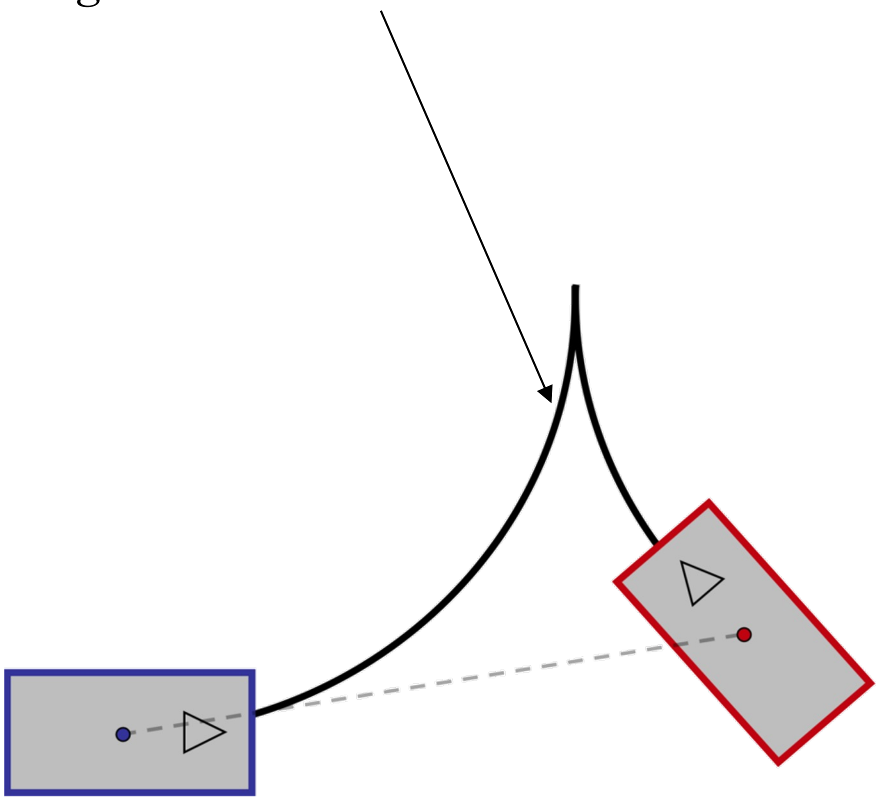
- Performance of Anytime D* depends strongly on heuristics *h(s)*: estimates of cost-to-goal



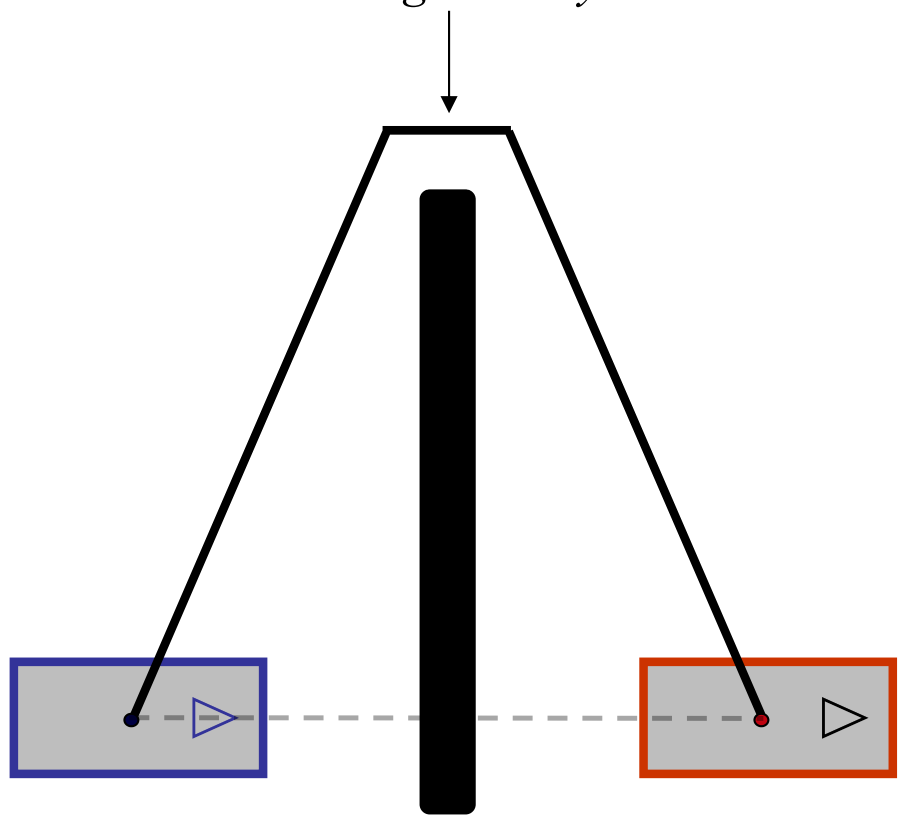
Searching the Graph

- In our planner: $h(s) = max(h_{mech}(s), h_{env}(s))$, where
  - $h_{mech}(s)$ – mechanism-constrained heuristic
  - $h_{env}(s)$ – environment-constrained heuristic

$h_{mech}(s)$ – considers only dynamics constraints and ignores environment



$h_{env}(s)$ – considers only environment constraints and ignores dynamics



Searching the Graph

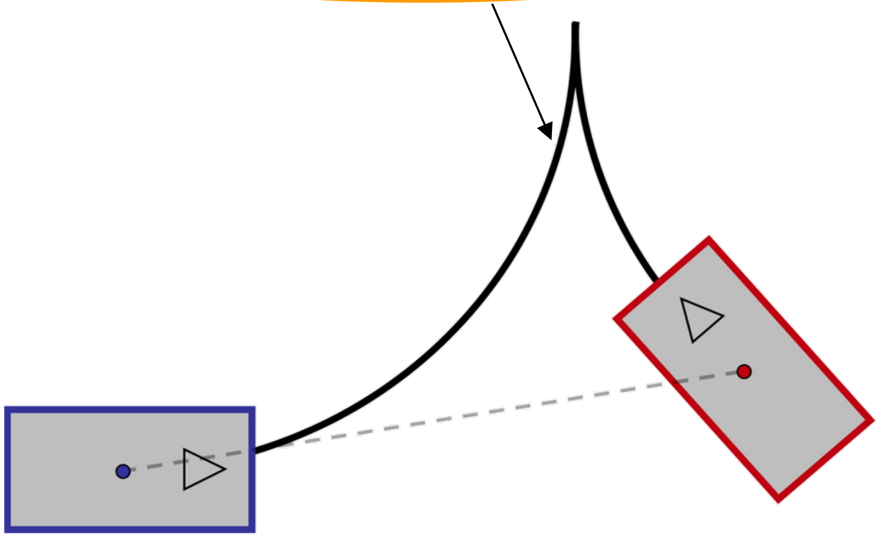
- In our planner: $h(s) = max(h_{mech}(s), h_{env}(s))$, where
  - $h_{mech}(s)$ – mechanism-constrained heuristic
  - $h_{env}(s)$ – environment-constrained heuristic

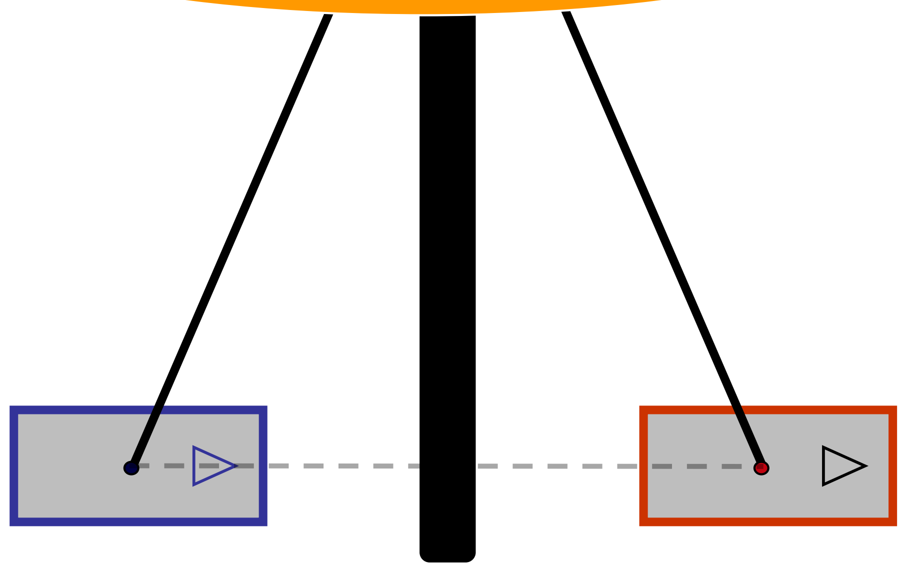
$h_{mech}(s)$ – considers only dynamics constraints and ignores environment

pre-computed as a table lookup for high-res. lattice

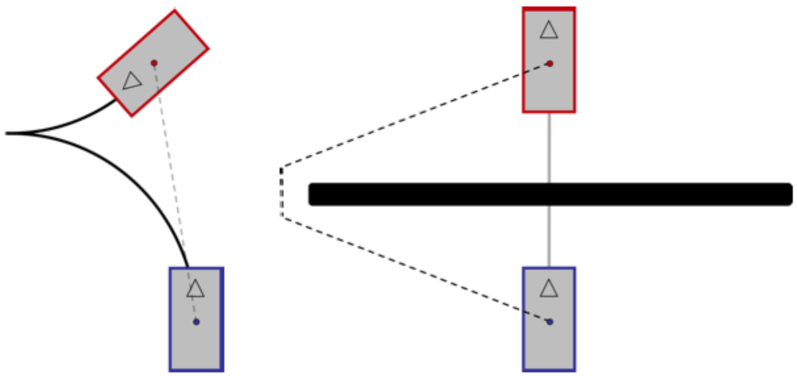
$h_{env}(s)$ – considers only environment constraints and ignores dynamics

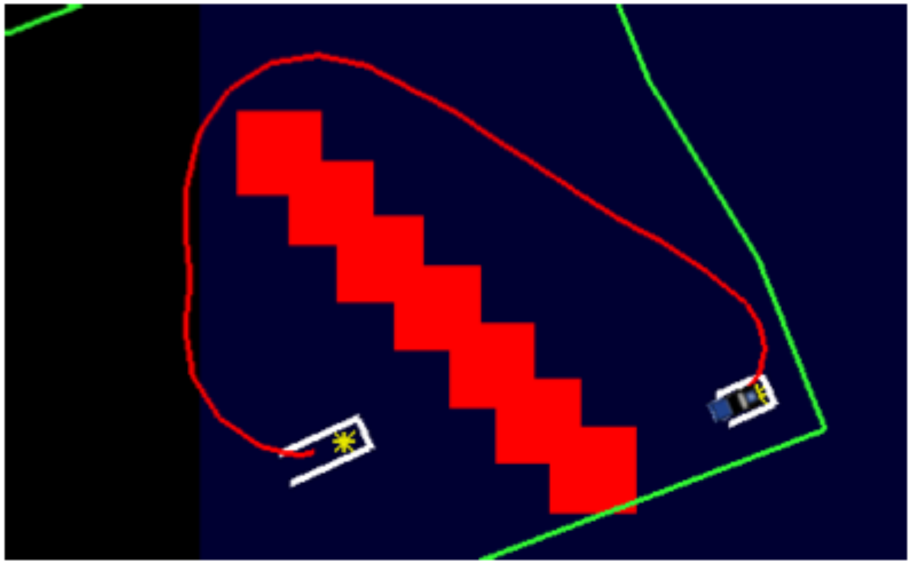
computed online by running a 2D A with late termination*





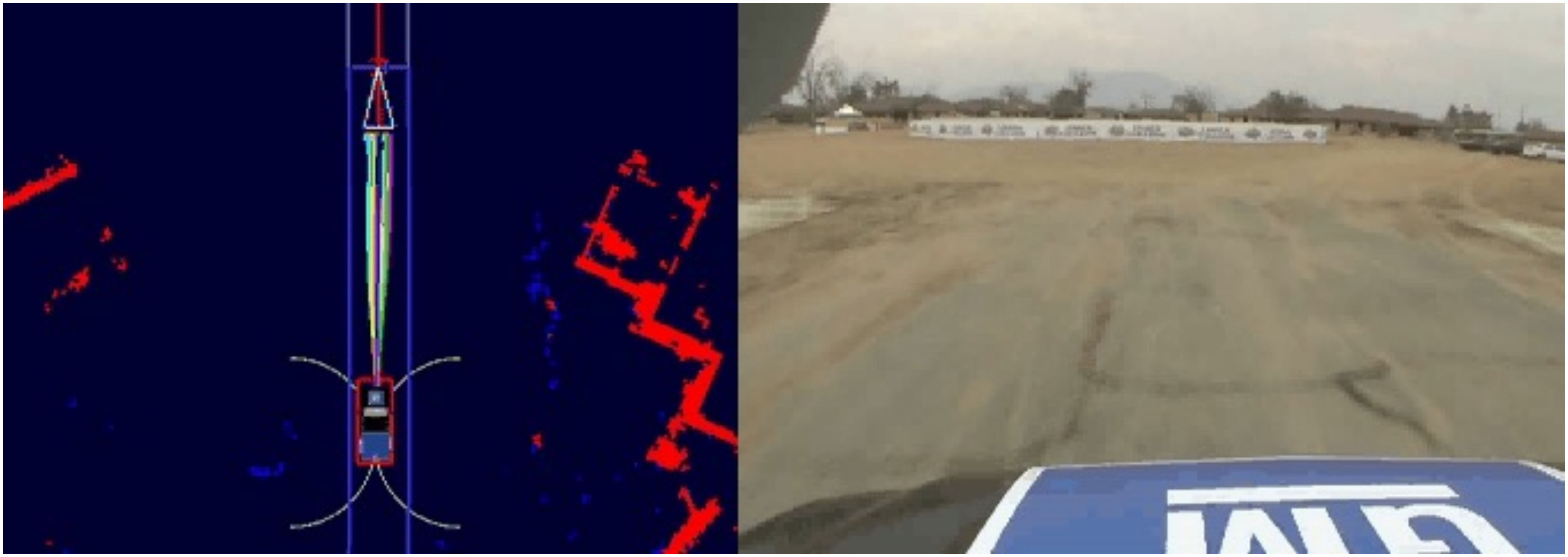
Heuristics





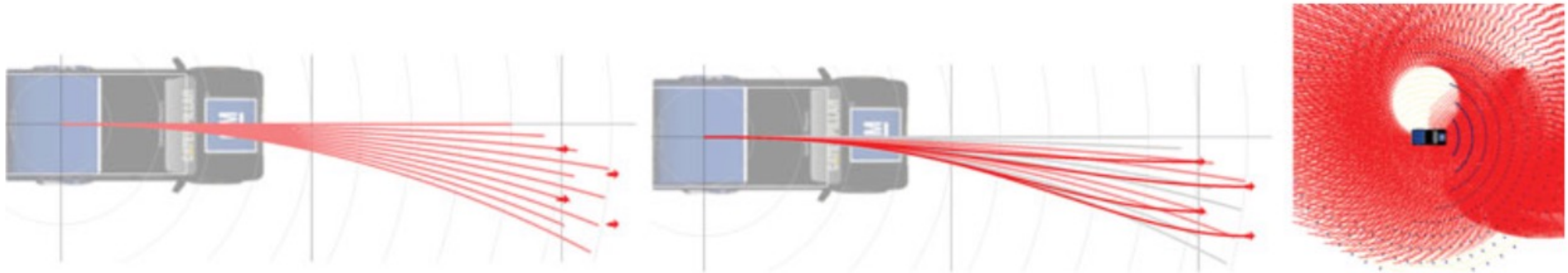
heuristic	states expanded	time (secs)
h	2,019	0.06
h_{2D}	26,108	1.30
h_{fsh}	124,794	3.49

Example, again

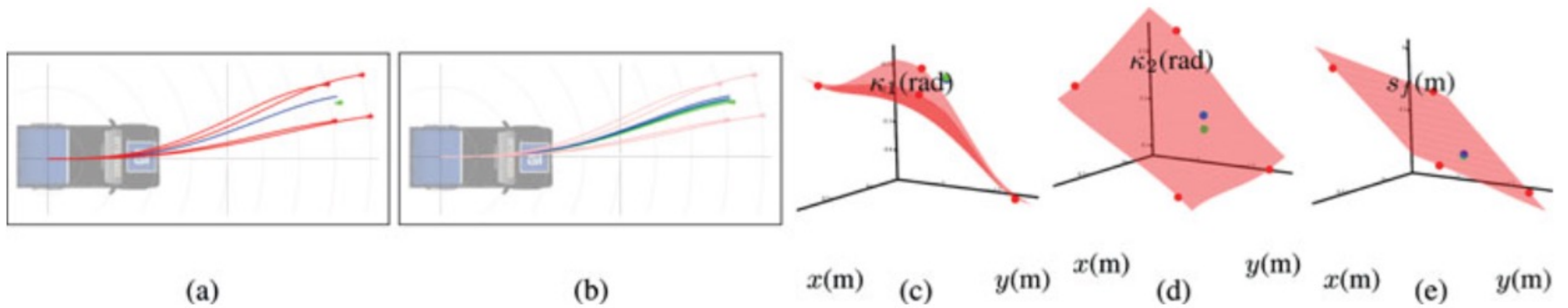


*Urban Challenge Race, CMU team, planning with Anytime D**

Trajectory Pre-Computation and Optimization

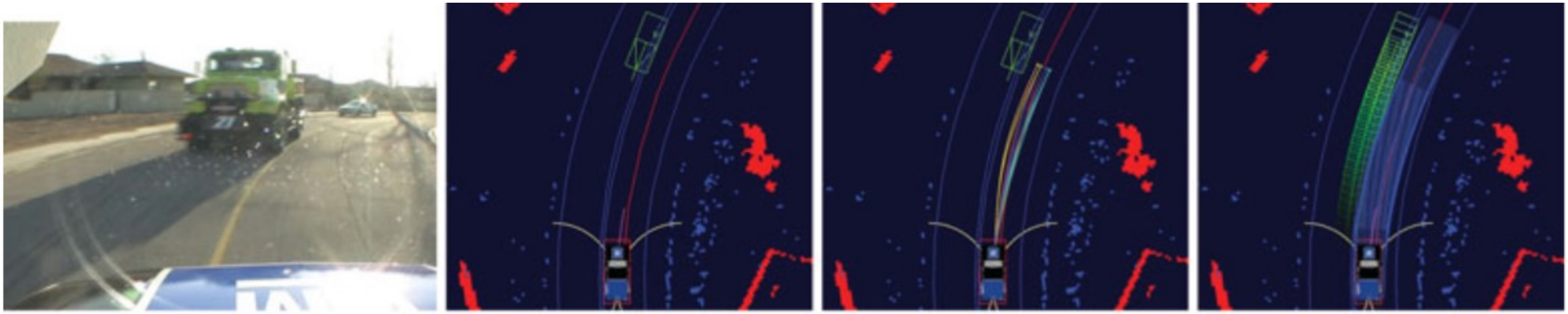


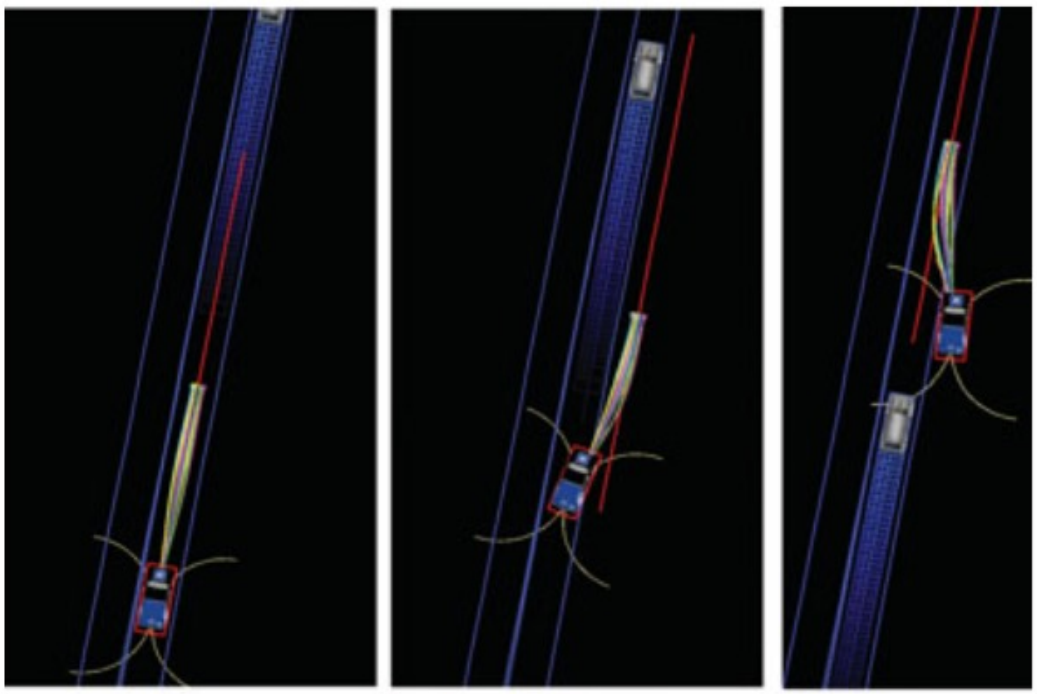
Pre-compute parameters for set of end points

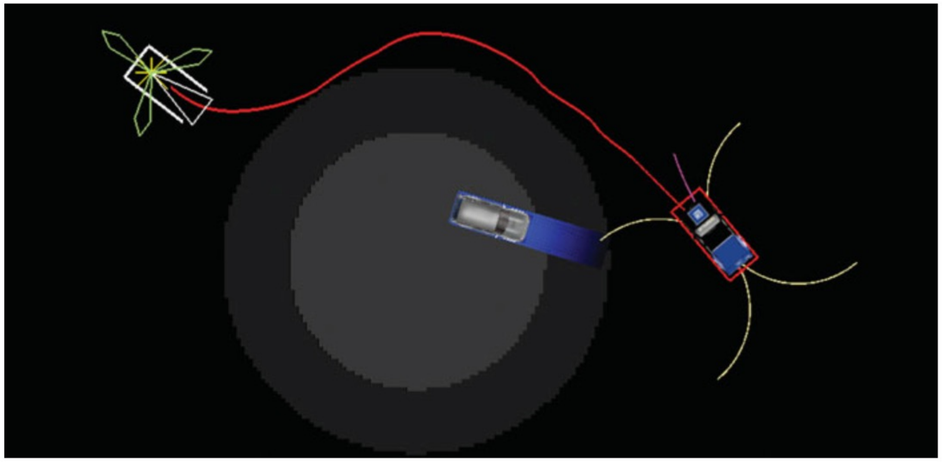


Optimize (fine-tune) parameters initialized via interpolation

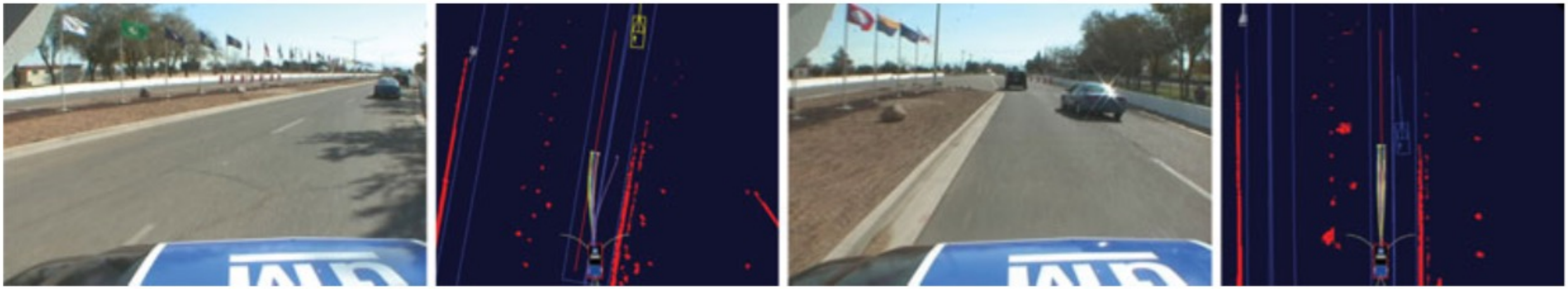
Predicting and Avoiding Other Vehicles

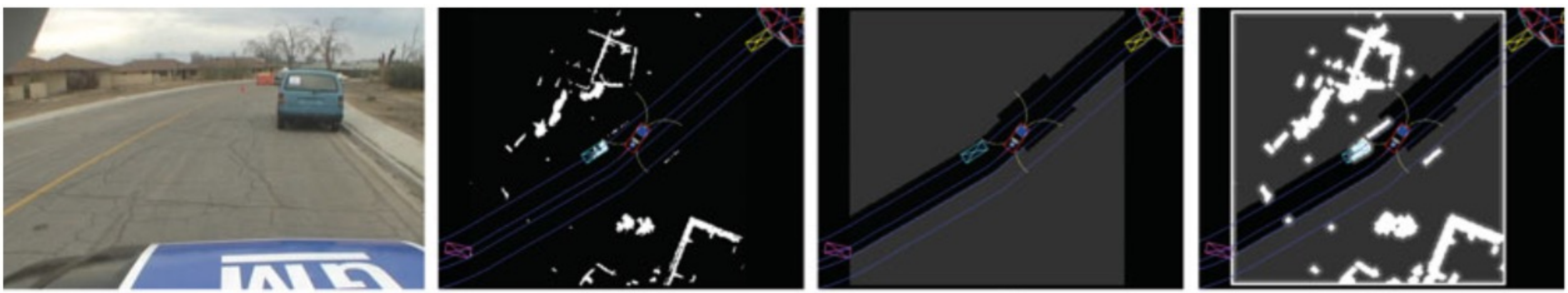






Passing and Cost





Summary

- Deterministic planning
  - constructing a graph
  - search with A*
  - search with D*

used a lot in real-time

think twice before trying to use it in real-time

- Planning under uncertainty
  - Markov Decision Processes (MDP)
  - Partially Observable Decision Processes (POMDP)

think three or four times before trying to use it in real-time

Many useful approximate solvers for MDP/POMDP exist!!

Manipulation Planning Examples