Plan for Today

- Continue with auctions
  - Sponsored Search
  - The VCG auction
  - The FCC Incentive Auction

Last time

$v_1, v_2, \ldots$ are private values

1st price auction: winner is highest bidder
winner pays her bid

2nd price auction: winner is highest bidder
winner pays 2nd highest bid

How to bid?
Model: each bidder bids to maximize
(expected) utility, given beliefs about others

$u_i := \underbrace{v_i}_{\text{value}} \mathbb{1}_{\text{receives item}} - \underbrace{p_i}_{\text{payment}}$ utility of bidder i

Vickrey auction

2nd price auction truthful
also called "strategyproof"
"dominant strategy incentive compatible"
DSIC

$\equiv$ it is a dominant strategy to bid your value also "individually rational" IR

$\equiv$ bidder's utility guaranteed to be nonnegative

maximizes "welfare"
also called "surplus"

$\equiv$ total happiness of all participants.

$\equiv$ sum of utilities

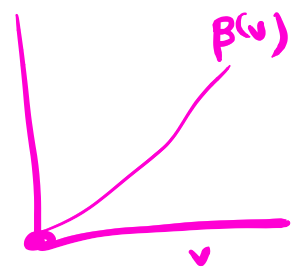
$$= \underbrace{\sum_{i=1}^{n} \left(v_i \mathbb{1}_{i \text{ wins item}} - p_i \right)}_{\text{utility of bidder } i} + \underbrace{\sum_{i=1}^{n} p_i}_{\text{utility of auctioneer.}}$$

$$= \sum_{i=1}^{n} v_i \mathbb{1}_{i \text{ wins}}$$

1^{st} price auction: no dominant strategies
analyzed by assuming each player's value was drawn from distribution F

Showed that if F is Uniform$[0, 100]$ then $\beta(v) = \frac{v}{2}$ is a Bayes-Nash equilibrium for $n=2$

$$E[\text{utility of bidding } \beta(v_1) \mid \text{other player bids } \beta(V_2)]$$
$$\geq E[\text{utility of bidding } w \mid \text{other player bids } \beta(V_2)]$$



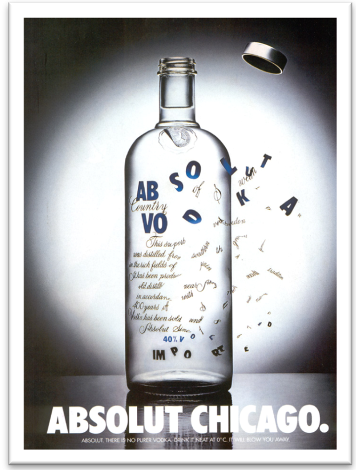
Revenue Equivalence Thm

If $V_i \sim F$ $\forall i$, then for any auction s.t. in equilibrium, item allocated to bidder w/ highest value and payment of a player with value 0 is 0 has same expected revenue.

Advertising – how it used to be

Posters

newspapers



magazines

Billboards

television

Pay-per-impression

Price depends on how many people your advertisement is shown to.

(whether or not they look at it, or care about it)

"Half the money I spend on advertising is wasted; the trouble is, I don't know which half"

Andrew Wanamaker, advertising pioneer

How is the price determined?

Complicated negotiations with high monthly premiums,

forms a barrier to entry for
small advertisers.

Modern Advertising

On the web,
Many different kinds of ads …

Sponsored Search Ads

Google

mesothelioma

All · News · Images · Videos · Books · More · Settings · Tools · SafeSea

About 7,290,000 results (0.61 seconds)

Ad · www.nationalmesotheliomaclaims.com/

Mesothelioma Claim Center | Get Asbestos Cancer Payments

Learn how the $30B Asbestos Trust Fund may pay for your asbestos-caused cancer.

Mesothelioma Claims
Asbestos Trust Fund Claims
This $30 Billion Trust Can Help

Free Info Package
Covers what you need to know
about medical and financial options

Ad · www.mesotheliomabook.com/ (888) 637-6234

Family Hurt by Mesothelioma? | We're Here to Help You

We are the largest firm devoted only to **Mesothelioma** in the US, visit us today. We come to...

Ad · www.mesothelioma-survivor.com/mesothelioma/treatment (800) 336-0086

I Survived Mesothelioma Cancer - Learn How She Beat the Odds

Learn About Heather's Successful Surgery from Preparation to the Procedure & Recovery.

Ad · www.navy-veterans-mesothelioma.org/Financial/Benefits

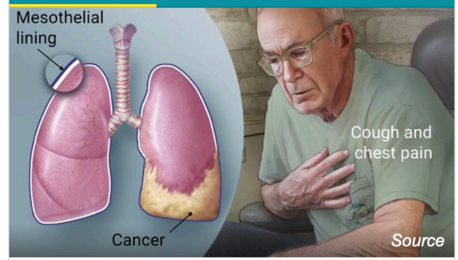
Mesothelioma & Asbestos Risk | Navy Vets Asbestos Claims

Important info for Navy Vets. Learn About **Mesothelioma** Claims

Mesothelioma is a type of cancer that develops from the thin layer of tissue that covers many of the internal organs (known as the mesothelium). The most

Mesothelioma

ABOUT · SYMPTOMS · TREATMENTS



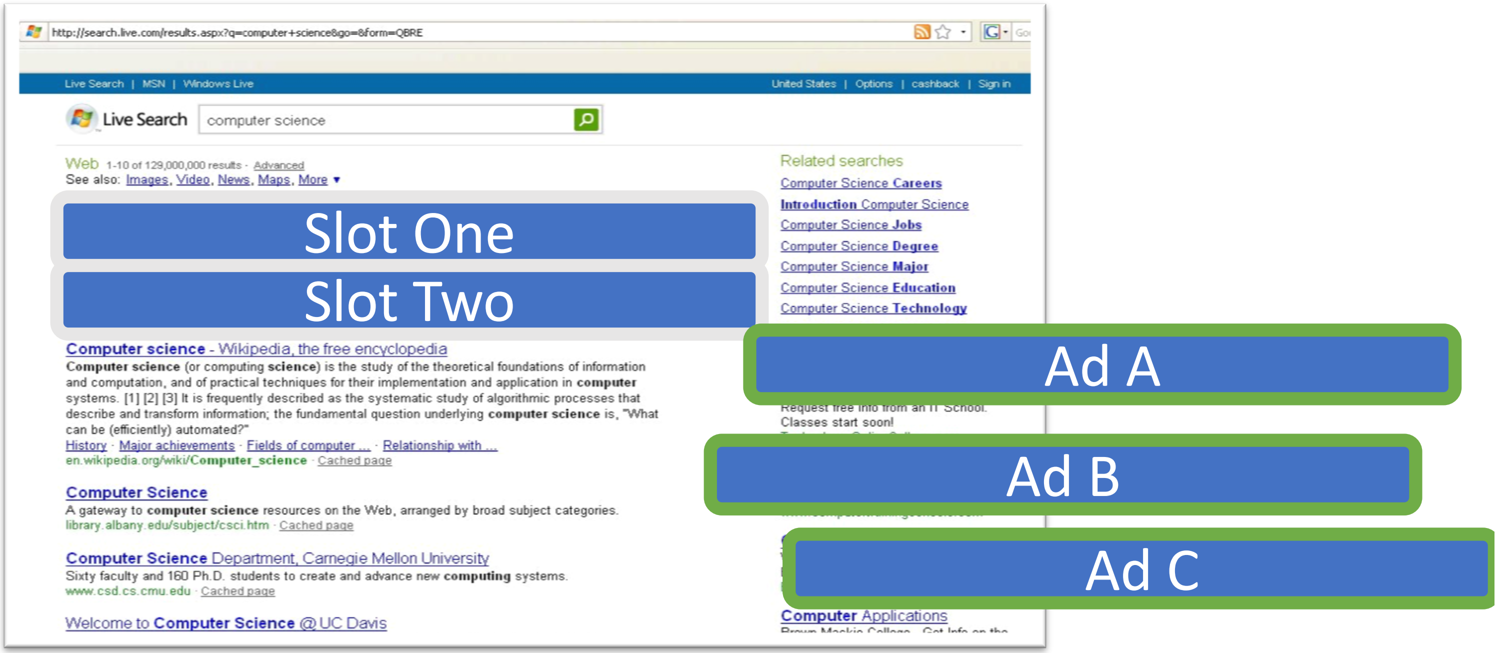
A tumor of the tissue that lines the lungs, stomach, heart, and other organs.

Very rare

Fewer than **20,000** US cases per year

- Treatment can help, but this condition can't be cured
- Requires a medical diagnosis
- Lab tests or imaging always required

Price determined by auction – per keyword



"Most people don't realize that all that money comes in pennies at a time"
Hal Varian, Google Chief Economist

How are these ads different than the ads in the offline media?

In many cases: Pay-per-click

Model: per keyword auction.

Bidders: advertisers who have a standing bid on keyword.

Auction offers same # $k > 1$ of slots.

Slots not identical

CTR click through rates

c_j: Prob of a click on the j^{th} slot. $j = 1, \ldots k$

v_i: advertiser's value for a click.

q_i: CTR rate due to quality of ad. (all qualities are same) overall: if put bidder i's ad in slot j

Prob of click $= c_j \cdot q_i$

Value to bidder i of gets his ad placed in slot j

exp $\underline{v_i} \cdot \underline{c_j}$

exp. utility $v_i \cdot c_j - p_i c_j$

Generalized Second Price GSP

- allocate to slots in order decreasing bid
- on a click to slot j, payment is next highest bid.

Model

Click-through rates	Slots	Advertisers	Value per click values. (bids)	PPC	utility
$c_1 = 1$	1	A	7	6	1
$c_2 = 0.4$	2	B	6	1	$(6-1) \cdot 0.4$
$c_3 = 0$	No slot	C	1		

Example

$c_1 = 1$

$c_2 = 0.99$

bidder	1	2	3
values	$\boxed{10}$	$\boxed{9}$	0.05

$10 \to 8$

in this example, if bidder 1 bids below 9 then his utility will be higher than it was bidding truthfully.

Truthful auction: VCG auction

chooses outcome to maximize social welfare.

$v_1 > v_2 > \dots$

social welfare of an allocation

$$= \sum_{i=1}^{k} v_i \, c_{s(i)}$$

slot i is assigned to

$c_1 > c_2 > c_3 > \dots$

Social welfare maximizing allocation

$$\sum_{i=1}^{k} v_i c_i$$

charge each bidder the "externality" her presence imposes on others

$\boxed{\text{Bidder 1:}}$

What would SW be w/o bidder 1 in auction

$$\boxed{\sum_{i=2}^{k+1} v_i c_{i-1}}$$

SW with bidder 1

$$\sum_{i=2}^{k} v_i c_i$$

VCG payment for bidder 1

charge bidder 1 loss others incur because of her

$$\sum_{i=2}^{k+1} (c_{i-1} - c_i) v_i$$

$c_{k+1} = 0$

$c_{i-1} > c_i$

VCG payment for bidder j

$$\sum_{i=j+1}^{k+1} (c_{i-1} - c_i) v_i$$

VCG auction for sponsored search

Get a bid from each bidder

Relabel bids so $b_1 > b_2 > \dots > b_n$

For $i = 1..k$ assign bidder i to i^{th} slot.

for $i = 1..k$, for each click, charge bidder i

$$p_i = \frac{1}{c_i} \sum_{j=i+1}^{k+1} b_j (c_{j-1} - c_j)$$

$$\underbrace{c_i p_i}_{} = \sum_{j=i+1}^{k+1} b_j (c_{j-1} - c_j)$$

$$b_{i+1} \left(\frac{c_i - c_{i+1}}{c_i}\right) + b_{i+2} \left(\frac{c_{i+1} - c_{i+2}}{c_i}\right)$$

PPC

Thm VCG is truthful, IR, and welfare maximizing

Proof of truthfulness

Fix other bids, let b_j denote j^{th} highest bid except me.

my value. v

slot k	$c_k b_k$
k-1	$(c_{k-1} - c_k) b_{k-1} + c_k b_k$
i	$\sum_{j=i}^{k} (c_j - c_{j+1}) b_j$

Δ value as move from i to slot i-1 $\quad (c_{i-1} - c_i) v$

Δ price $\quad$ i $\quad$ i-1 $\quad (c_{i-1} - c_i) b_{i-1}$

want to do this if $v > b_{i-1}$

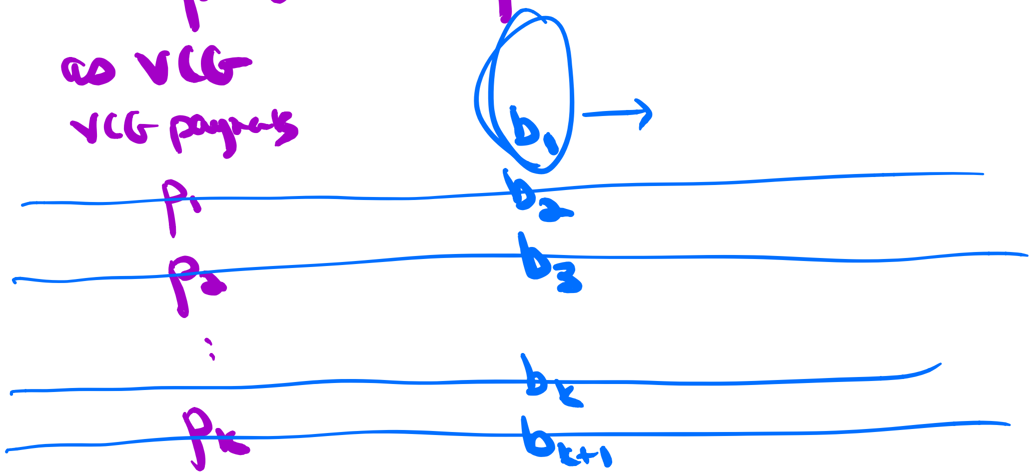
$b_i > v > b_{i+1}$

other bids

$b_1 > b_2 \geq \dots$

$\forall$ valuations & CTRs, the GSP auction has an equilibrium w/ same allocation & payments as VCG

VCG payments



no sweeping revenue comparison between GSP & VCG

Facebook

- what bidders can bid on.
  various "events"
- placement, size & format of ads
- evaluation of quality of ads

$v_i \cdot c_j$

more appropriate model,
bidder has value for each event type

difficult for advertisers to figure out CTRs

- high quality outcomes
- advertisers happy, bidding easy

VCG (fully general)

- n participants.
- finite set of outcomes of auction Ω
- each agent i has private valuation
  $v_i(w) \quad \forall w \in \Omega$
- $v_i(w) = (\text{value of some event}) \Pr(\text{event occurs in outcome } w)$

VCG choose outcome

$$\in \underset{w \in \Omega}{\text{argmax}} \sum_{i=1} b_i(w)$$

Let this outcome be w^*

$$p_i = \max_{w \in \Omega} \sum_{j \neq i} b_j(w) - \sum_{j \neq i} b_j(w^*)$$

$$u_i = v_i(w^*) - p_i$$

$$= v_i(w^*) + \sum_{j \neq i} b_j(w^*) - \underbrace{\max_{w \in \Omega} \sum_{j \neq i} b_j(w)}$$

choose $\underline{b_i(w)}$ $\forall w \in \Omega$

to maximize $\boxed{v_i(w^*) + \sum_{j \neq i} b_j(w^*)}$ $(*)$

$$w^* \in \boxed{\operatorname{argmax}_w \boxed{b_i(w) + \sum_{j \neq i} b_j(w)}}$$

User interface for bidding.

- allow bidders to bid on events they
- allow bidders to specify budget

Computational requirements

$P_a =$ total value others get if a not there − value they get with a $= 0$

5   5

$P_b = 4 - 3 = 1$

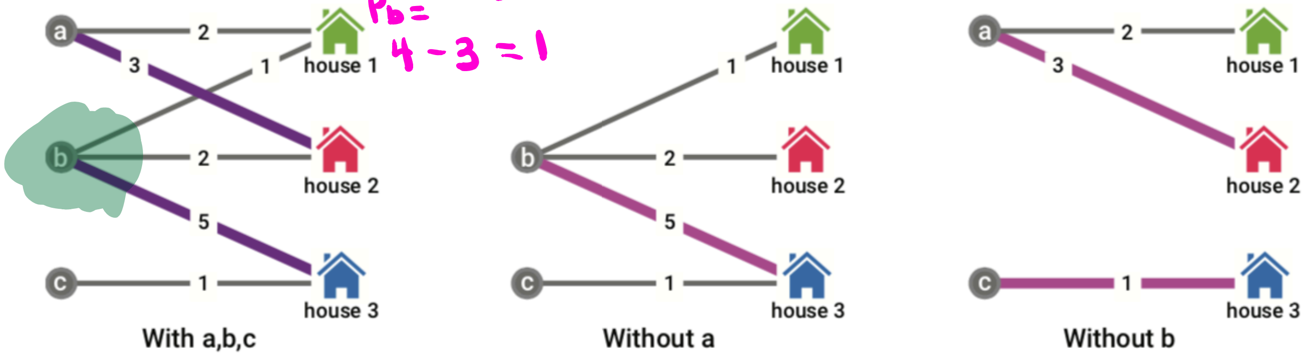


FIGURE 16.3. The label on the edge from i on the left to j on the right is the value v_{ij} that employee i has for a yearly lease of house j (say in thousands of dollars). The VCG mechanism allocates according to purple shaded edges. The payment of bidder a is 0 since in his absence house 3 is still allocated to bidder b. The payment of bidder b is 1 since in his absence the allocation is as follows: house 2 to bidder a and house 3 to bidder c, and therefore the externality he imposes is 1.