Plan for Today

- Strategic behavior related to transaction fees in Bitcoin
- Revisiting basics of equilibria
- Best response dynamics and potential games
- Online learning as a way to play games

Transaction fees in Bitcoin

Q: what will happen when block rewards are negligible & all reward is transaction fees.

Model

- transactions arrive at const rate
- in an interval of t units of time total amt fees in is $c \cdot t$ $(c=1)$
- blocks are created at const rate
- R transaction fees available
  miner can put any fraction into block
- miners have enough space to include everything.

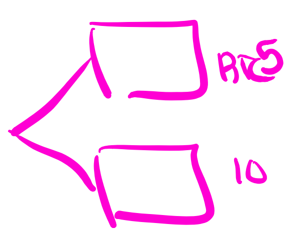
Strategic decisions

- which block to extend?
- how much of outstanding transactions to include
- when to publish found blocks

Protocol (honest miner)

- mine on longest chain (tie breaking for what heard about first)
- Include all transactions you know about
- publish found blocks immediately

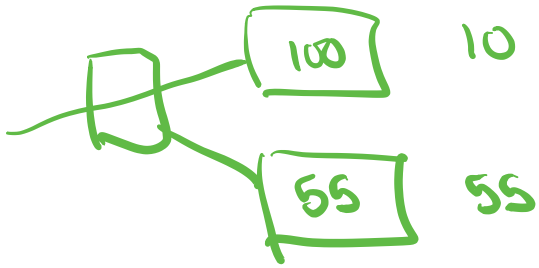
Petty Compliant Strategy

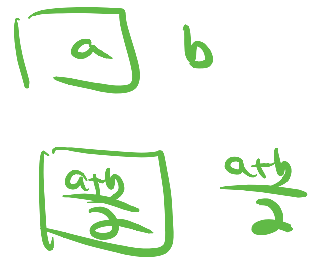


break ties for longest chain w/ most leftover fees

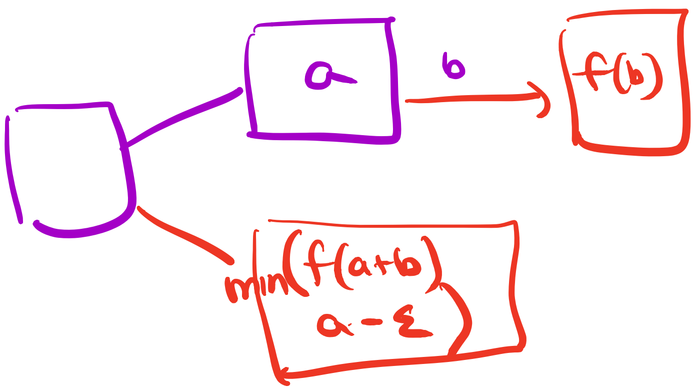
Lazy undercutting.

Suppose $a > b$





Function-Fork(f)



$f(x) = kx \qquad 0 < k < 1$

Theorem 5.1. *For any constant $y \leq 1/2$ such that $2y - \ln(y) \geq 2$,*[5] *define:*

$$f(x) = x, \quad \forall\, x \leq y \tag{1}$$

$$f(x) = -W_0(-ye^{x-2y}), \quad \forall\, y < x < 2y - \ln(y) - 1 \tag{2}$$

$$f(x) = 1, \quad \forall\, x \geq 2y - \ln(y) - 1 \tag{3}$$

Then it is an equilibrium for every miner to use the strategy FUNCTION-FORK(f) *as long as:*

- *Every miner is* non-atomic.
- *Miners may only mine on top of chains of length* H *or* H − 1.

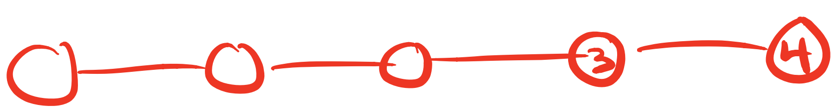
Furthermore, in any such equilibrium, the expected number of backlogged transactions after n time steps is $\Theta(\sqrt{n})$.

[5] Such y exist. This range is $(0, \approx 0.2]$.

Selfish mining

goal: to trick other miners into creating blocks that will be orphaned

- creating private chain



Equilibria.

Cost minimization games.

- k players
- A_i strategies of player i
- $\underline{c_i(\vec{s})}$ cost to player i $\qquad \boxed{\vec{s}=(s_1,s_2,\dots,s_k)}$ $\quad s_i \in A_i$

$\vec{s}$ is a pure NE if $\forall i, \ \forall s_i' \in A_i$

$$c_i(s_i', \vec{s}_{-i}) \geq c_i(s_i, \vec{s}_{-i})$$

$\vec{p}=(p_1,\dots,p_k)$ are prob distns where p_i is a prob distn over A_i

$\vec{p}$ is a mixed NE if $\forall i, \ \forall s_i'$

$$E_{\vec{s}\sim\vec{p}}\left[c_i(s_i', s_{-i})\right] \geq E_{\vec{s}\sim\vec{p}}\left[c_i(\vec{s})\right]$$

$$\begin{array}{c|c}
 & \begin{matrix} p_{21} & \cdots & p_{2r_2} \end{matrix} \\
\begin{matrix} p_{11} \\ p_{12} \\ \vdots \\ p_{1r_1} \end{matrix} & \left[\begin{matrix} \; & \; & \; \\ \; & \; & \; \\ \; & \; & \; \\ \; & \; & \; \end{matrix}\right]
\end{array}$$

Any finite game has a mixed NE.

Correlated equilibrium.

Distn $\vec{p}$ on $A_1 \times A_2 \times \ldots \times A_k$

over strategy profiles

is a correlated equilibrium if $\forall i$, $\forall s_i, s_i' \in A_i$

$$E_{\vec{s} \sim \vec{p}} [C_i(\vec{s}) \mid s_i] \leq E_{\vec{s} \sim \vec{p}} [C_i(s_i', \vec{s}_{-i}) \mid s_i]$$

Traffic Light

		N/S stop	N/S go
E/W	stop	$\frac{1}{3}$ (0,0)	$\frac{1}{3}$ (0,1)
	go	$\frac{1}{3}$ (1,0)	0 (-5,-5)

$$\begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & & & \\ & & & \\ & & & \end{bmatrix}$$

Distn $\vec{p}$ on $A_1 \times A_2 \times \ldots \times A_K$

is a coarse correlated eq CCE if δ-approx

$\forall i, \forall s_i'$

$$E_{\vec{s}\sim\vec{p}}[c_i(\vec{s})] \le E_{\vec{s}\sim\vec{p}}[c_i(s_i', s_{-i})] + \delta$$

$\sqrt{\frac{\log n}{T}}$

	R	P	S
R	(0,0)	1/6 (-1,1)	1/6 (1,-1)
P	1/6 (1,-1)	(0,0)	1/6 (-1,1)
S	1/6 (-1,1)	1/6 (1,-1)	(0,0)

Best response dynamics.

if this halts $\to$ pure NE.

not guaranteed to reach pure NE even if one exists

2,2	0,0	0,0
0,0	1,-1	-1,1
0,0	-1,1	1,-1

Potential games

$\Phi : \prod A_i \to \mathbb{R}$

$\forall \vec{s} = (s_1, \ldots, s_K) \ \forall i, \ s_i' \in A_i$

$c_i(s_i', \vec{s}_{-i}) - c_i(\vec{s}) = \Phi(s_i', \vec{s}_{-i}) - \Phi(\vec{s})$

Every finite potential game has a pure NE.

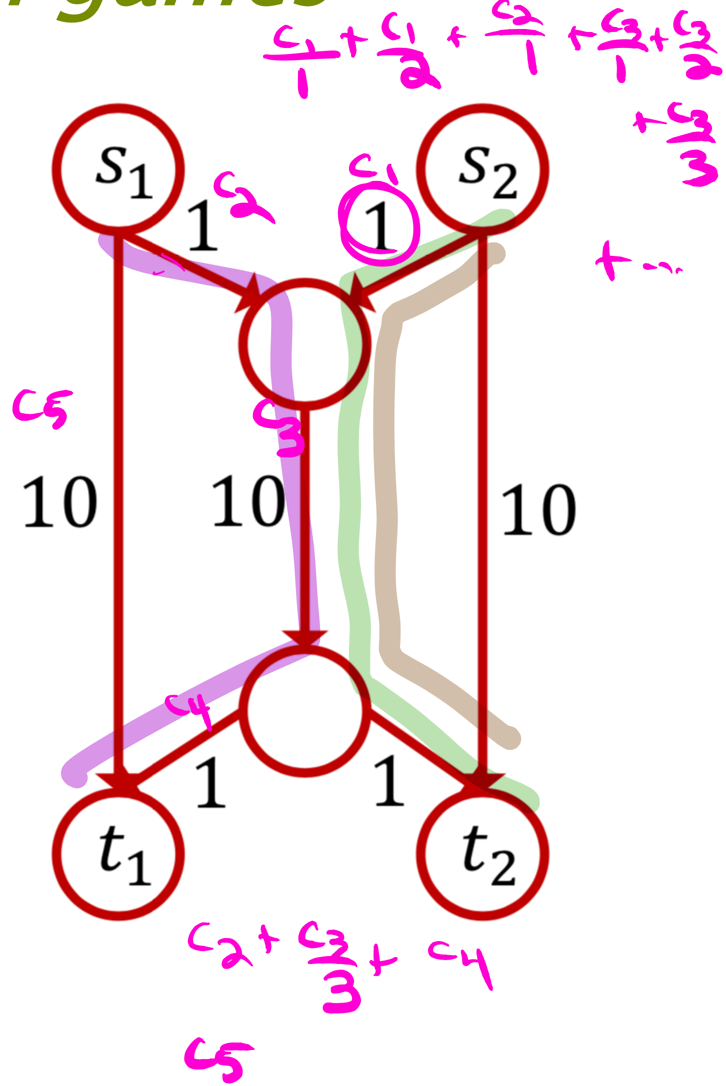
Consider min of Φ

Network formation games

$\vec{P} = (P_1, P_2, \ldots, P_k)$

$$\Phi(\vec{P}) = \sum_{e \in E(\vec{P})} \sum_{k=1}^{n_e(\vec{P})} \frac{c_e}{k}$$

$n_e(\vec{P})$ - # of paths thru edge e.



$$c_i(s_i', \vec{s}_{-i}) - c_i(\vec{s}) = \Phi(s_i', \vec{s}_{-i}) - \Phi(\vec{s})$$

Online Learning

single player game against adversary

A: set of possible actions player can take each day

$|A| = n$

for $t = 1, \ldots, T$

player picks $a \in A$

prob distn over actions

$p_1, p_2, \ldots, p_n$

$c_i \in [0,1]$

$\xleftarrow[c_1, c_2, \ldots c_n]{}$

Exp cost on day

$$= \sum_{i=1}^{n} p_i c_i$$

investment strategy

routes.

$$\xrightarrow{\vec{p}^{\,t}}$$

$$\xleftarrow{\vec{c}^{\,t}}$$

Benchmark?

$$\sum_{t=1}^{T} \min_{a \in A} c^t(a)$$

Impossible.

Ex 2 actions.

every day adv picks unif at random

1 path cost 1

other cost 0.

Exp cost of any alg $\frac{T}{2}$

The <u>regret</u> of an alg on a sequence of cost vectors

$$\frac{1}{T}\left[\sum_{t=1}^{T} \boxed{c_t(a^t)} - \underbrace{\min_{a\in A} \sum_{t=1}^{T} c_t(a)}\right]$$

$\sum_{a\in A} p^t(a)\, c^t(a)$

Goal get regret $\longrightarrow 0$
$T \to \infty$

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play actn 3 $\rightarrow$ cost 1 $\rightarrow$ actn 3
0 others.

$\Rightarrow$ alg pays T

$\exists$ actn that has cost $\le \frac{T}{n}$

n is # of actns

Best we can hope for is regret $\sqrt{\frac{\ln n}{T}}$

Adv $n=2$

$\frac{1}{2}$ $(1,0)$ $\frac{1}{2}$ $(0,1)$

Any alg has exp cost $\frac{T}{2}$

$E\left[\begin{array}{l}\min(\#H, \#T) \\ \text{in } T \text{ tosses}\end{array}\right]$

$= \frac{T}{2} - \sqrt{T}$

For each action maintain weight $w^t = (w_1^t, w_2^t, \dots w_n^t)$

initialize $w^1(a) = 1 \quad \forall a$

for $t = 1$ to T

choose a^t with prob proportional to w^t

$$p^t(a) = \frac{w^t(a)}{\sum_{\tilde{a} \in A} w^t(\tilde{a})}$$

given c^t

$\forall$ action a

$$w^{t+1}(a) = w^t(a)\,(1 - \varepsilon)^{c^t(a)}$$

ε parameter $(0, \frac{1}{2})$ to be set,

Thm This alg has regret $O\left(\sqrt{\frac{\log n}{T}}\right)$

player i $\quad A_i$

$s_i \quad c(\cdot, \vec{s}_{-i})$

Let $\vec{s}^1, \vec{s}^2, \dots \vec{s}^T$

No-regret dynamics.

Let $\vec{p}$ be the uniform distn on $\vec{s}^1, \vec{s}^2, \dots, \vec{s}^T$

$\vec{p}$ is approx CCE

regret of player i at end of rounds $= \frac{1}{T}\left[\sum_{t=1}^{T} c_i(\vec{s}^t) - \min_{s_i \in A_i} \sum_{t=1}^{T} c_i(s_i, \vec{s}_{-i}^t)\right]$