

Mathematical Structure of Computation

- What is computation?
- Church's Thesis - all forms of computation are equivalent



## Combinatorial Optimization

- Ford-Fulkerson 1956



## Hard problems

- Satisfiability
- Bin Packing
- Integer Programming
- Hamiltonian Circuit
- Vertex Cover
- 3 Dimensional Matching
- Traveling Salesman Problem

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## NP Completeness

- Non-deterministic polynomial time
- Cook's theorem:
- Satisfiability is the hardest problem in NP
- Simulate a polynomial time non-deterministic computation with satisfiability formula
- Karp
- Showed that a wide range of other problems were also NP-complete
- Showed how to convert satisfiability into TSP
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## Simulation of a formula with a path in a graph

- G has a Hamiltonian Circuit if and only if F has a satisfying truth assignment
- G can be constructed "easily" from F
- CNF
- Each clause has at most 3 literals
$(x||y|| z) \& \&(!x| |!y| | \mid z) \& \&(!x| | y) \& \&(x|\mid ~!y) \& \&(y||\mid z) \& \&(!y| | z)$


## Gadgets: Truth Setting



Gadgets: Truth Testing



## Euclidean TSP

- $n$ points in a $\mathrm{R}^{\mathrm{n}}$
- Distance between a pair of points is the Euclidean distance
- Is the Euclidean TSP
 NP-complete?


