Great Ideas in Computing Average Case Analysis

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What happens “on the average”

Worst case versus average case

- T(n) = max {T(I) | I is a problem instance of size n}
- T(n) = ave {T(I) | I is a problem instance of size n}
  = sum {p(I) T(I) | I is a problem instance of size n}

Is real world data random?

- Is real world data worst case?

Average case analysis

```
max := A[0]
for i := 1 to n-1
        if A[ i ] > max
*               max := A[ i ]
```

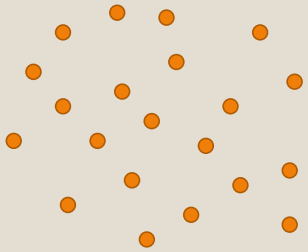
How many times is line * executed

Quicksort

- Worst case runtime is n^2
- Average case is n log n

Random Trees

Euclidean TSP



Asymmetric TSP

- Random Asymmetric TSP
  - $n \times n$ matrix with random entries in [0, 1]
- Theorem
  - There is a polynomial time algorithm that gives a very good approximation for a random ATSP

Assignment Problem

- Minimum weight perfect matching in a complete, bipartite graph
- Solvable in polynomial time
- How is the assignment problem related to the ATSP?

ATSP Algorithm

- Solve relaxed version with Assignment Problem
- Splice together cycles
- Solution to the Assignment problem is a random permutation
  - Random permutations have a small number of cycles

Random Graphs

- What is a random graph?

Standard model

- Each edge is present with probability p
- Expected degree of a vertex is pn

Does a random graph have a Hamiltonian circuit

Hamiltonian Circuit Algorithm

What does the Web Graph look like?

Degree distribution in the web graph