Great Ideas: Algorithm Implementation

Richard Anderson
University of Washington

Theory and Practice

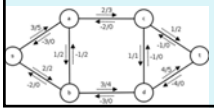
- What is the connection between theory and practice

Algorithm Implementation Challenge

- Solve large instances of problems on real world data

Network Flow

- DIMACS Network Flow Challenge
- Compare standard flow algorithms on different data sets
- Dramatic speedups over simpler algorithms
- Not all theoretical algorithms were good



N-Body Problem

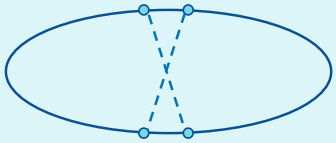
- Simulate large scale phenomena
- $O(n^2)$ algorithm reduces to O(n log n) algorithm
- Interesting questions – what types of spatial decomposition trees to use

Speeding up algorithms

- Approximation algorithms
- TSP with triangle inequality
  - Dist(a, c) ≤ Dist(a, b) + Dist(b, c)
- Theorem 1:
  - A minimum spanning tree gives a 2 approximation of the TSP
- Theorem 2 (Christophides) :
  - There exists a 3/2 approximation algorithm for the TSP
    - Construct matching of vertices of odd degree to get an eulerian tour

Local Improvement

- Improve solution until a local maximum is reached
- Two-Opting



Simulated Annealing

- Local improvement algorithm that is allowed to make things worse
- Over time the "temperature" is lowered, reducing variation

Brute Force Solution

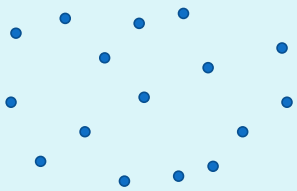
- Look at all possible tours
- n = 10 takes 1 second
- n = 12
- n = 14
- n = 16
- n = 18
- n = 20

Dynamic programming algorithm

- Reduces complexity from n! to $n^2 2^n$

Euclidean TSP

- Cities on a map
- Distance function is Euclidean distance



Exact Solution of Euclidean TSP

Year	Team	Size
1954	Dantzig, Fulkerson	49
1971	Held, Karp	64
1975	Cemerini	67
1977	Grotschell	120
1980	Crowder	318
1987	Padberg	532
1987	Grotschell	666
1987	Padberg	2392
1994	Applegate et al.	7397
1998	Applegate et al.	13509
2001	Applegate et al.	15112
2004	Applegate et al.	24978

Sweden Tour

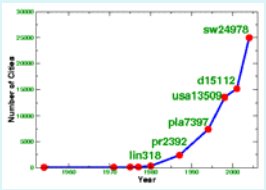
- Applegate et al.
- 24,978 cities

Finding an optimal tour
Sweden Tour Log

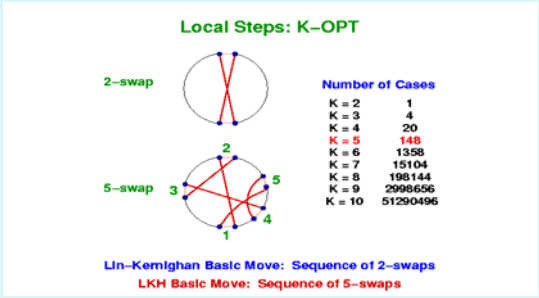
Date	Gap	Lowerbound	Upperbound
24.8.01			855683
26.8.01	.041 %	855331	
4.9.01	.034%		855618
20.9.01	.033%		855612
30.9.01	.033%		855610
16.3.03	.032%		855602
18.3.03	.031%		855597
24.3.03	.012%	855493	
2.6.03	.009%	855528	
	.001%	855595	
20.5.04	Optimal	855577	

TSP Solving

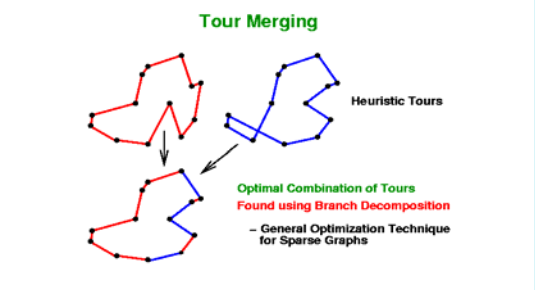
- Finding good tours
- Proving Lower Bounds



Local Improvement



Tour Merging

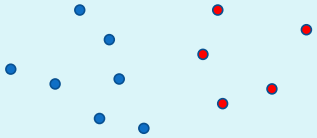


Lower Bound

- Optimal solution to an easier problem gives a lower bound
- Assignment problem
  - Select one outgoing edge from each vertex
  - Select one incoming edge to each vertex
  - Minimize the cost of the selected edges

Making a TSP out of an Assignment Problem

- Need to make sure edge cross every cut



- This adds many constraints
- To get lower bounds we progressively add more cuts

Summary for TSP Solution

- Heuristics to find good tours
- Improve lower bounds by adding cutting plans
- Know we have the optimal when the lower bound meets the upper bound