CS-XXX: Graduate Programming Languages

Lecture 2 — Syntax

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2012

Finally, some formal PL content

For our first *formal language*, let's leave out functions, objects, records, threads, exceptions, ...

What's left: integers, mutable variables, control-flow

(Abstract) syntax using a common *metalanguage*:

"A program is a statement s, which is defined as follows"

$$\begin{array}{lcl} s & ::= & \textbf{skip} \mid x := e \mid s; s \mid \textbf{if}\ e\ s\ s \mid \textbf{while}\ e\ s \\ e & ::= & c \mid x \mid e + e \mid e * e \\ (c & \in & \{\ldots, -2, -1, 0, 1, 2, \ldots\}) \\ (x & \in & \{\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{y}_1, \mathrm{y}_2, \ldots, \mathrm{z}_1, \mathrm{z}_2, \ldots, \ldots\}) \end{array}$$

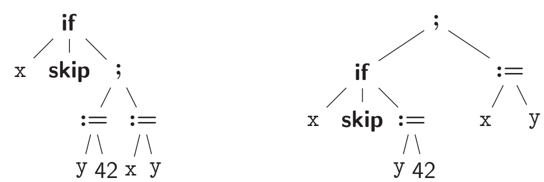
Syntax Definition

$$\begin{array}{lcl} s & ::= & \textbf{skip} \mid x := e \mid s; s \mid \textbf{if}\ e\ s\ s \mid \textbf{while}\ e\ s \\ e & ::= & c \mid x \mid e + e \mid e * e \\ (c & \in & \{\ldots, -2, -1, 0, 1, 2, \ldots\}) \\ (x & \in & \{\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{y}_1, \mathrm{y}_2, \ldots, \mathrm{z}_1, \mathrm{z}_2, \ldots, \ldots\}) \end{array}$$

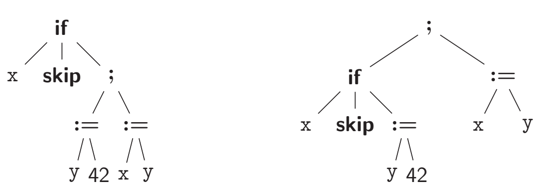
- Blue is metanotation: $::=$ for "can be a" and $\mid$ for "or"
- *Metavariables* represent "anything in the *syntax class*"
- By *abstract syntax*, we mean that this defines a set of *trees*
  - Node has some label for "which alternative"
  - Children are more abstract syntax (subtrees) from the appropriate syntax class

Examples

$$\begin{array}{lcl} s & ::= & \textbf{skip} \mid x := e \mid s; s \mid \textbf{if}\ e\ s\ s \mid \textbf{while}\ e\ s \\ e & ::= & c \mid x \mid e + e \mid e * e \end{array}$$



Comparison to ML



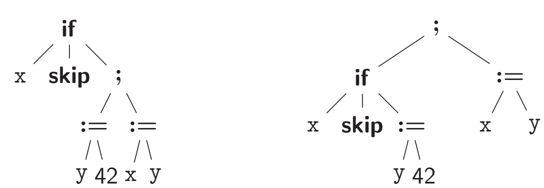

```
type exp = Const of int | Var of string
         | Add of exp * exp | Mult of exp * exp
type stmt = Skip | Assign of string * exp | Seq of stmt * stmt
          | If of exp * stmt * stmt | While of exp * stmt

If(Var("x"),Skip,Seq(Assign("y",Const 42),Assign("x",Var "y")))
Seq(If(Var("x"),Skip,Assign("y",Const 42)),Assign("x",Var "y"))
```

Very similar to trees built with ML datatypes

- ML needs "extra nodes" for, e.g., "e can be a c"
- Also pretending ML's `int` is an integer

Comparison to strings



We are used to writing programs in *concrete syntax*, i.e., strings

That can be *ambiguous*: $\textbf{if}\ \mathrm{x}\ \textbf{skip}\ \mathrm{y} := 42 \,;\, \mathrm{x} := \mathrm{y}$

Since writing strings is such a convenient way to represent trees, we allow ourselves parentheses (or defaults) for disambiguation

- Trees are our "truth" with strings as a "convenient notation"

$\textbf{if}\ \mathrm{x}\ \textbf{skip}\ (\mathrm{y} := 42 \,;\, \mathrm{x} := \mathrm{y})$ versus $(\textbf{if}\ \mathrm{x}\ \textbf{skip}\ \mathrm{y} := 42) \,;\, \mathrm{x} := \mathrm{y}$

Last word on concrete syntax

Converting a string into a tree is *parsing*

Creating concrete syntax such that parsing is unambiguous is one challenge of *grammar design*

- Always trivial if you require enough parentheses or keywords
  - Extreme case: LISP, 1960s; Scheme, 1970s
  - Extreme case: XML, 1990s
- Very well studied in 1970s and 1980s, now typically the least interesting part of a compilers course

For the rest of this course, we start with abstract syntax

- Using strings only as a convenient shorthand and asking if it's ever unclear what tree we mean

Inductive definition

$$\begin{array}{lcl} s & ::= & \textbf{skip} \mid x := e \mid s; s \mid \textbf{if}\ e\ s\ s \mid \textbf{while}\ e\ s \\ e & ::= & c \mid x \mid e + e \mid e * e \end{array}$$

This grammar is a finite description of an infinite set of trees

The apparent self-reference is not a problem, provided the definition uses well-founded induction

- Just like an always-terminating recursive function uses self-reference but is not a circular definition!

Can give precise meaning to our metanotation & avoid circularity:

- Let $E_0 = \emptyset$
- For $i > 0$, let E_i be E_{i-1} union "expressions of the form c, x, $e_1 + e_2$, or $e_1 * e_2$ *where* $e_1, e_2 \in E_{i-1}$"
- Let $E = \bigcup_{i \geq 0} E_i$

The set E is what we mean by our compact metanotation

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To get it: What set is E_1? E_2?
Could explain statements the same way: What is S_1? S_2? S?

Proving Obvious Stuff

All we have is syntax (sets of abstract-syntax trees), but let's get the idea of proving things carefully...

Theorem 1: There exist expressions with three constants.

Our First Theorem

There exist expressions with three constants.

Pedantic Proof: Consider $e = 1 + (2 + 3)$. Showing $e \in E_3$ suffices because $E_3 \subseteq E$. Showing $2 + 3 \in E_2$ and $1 \in E_2$ suffices...

PL-style proof: Consider $e = 1 + (2 + 3)$ and definition of E.

Theorem 2: All expressions have at least one constant or variable.

Our Second Theorem

All expressions have at least one constant or variable.

Pedantic proof: By induction on i, for all $e \in E_i$, e has ≥ 1 constant or variable.

- Base: $i = 0$ implies $E_i = \emptyset$
- Inductive: $i > 0$. Consider *arbitrary* $e \in E_i$ by cases:
  - $e \in E_{i-1}$ ...
  - $e = c$ ...
  - $e = x$ ...
  - $e = e_1 + e_2$ where $e_1, e_2 \in E_{i-1}$ ...
  - $e = e_1 * e_2$ where $e_1, e_2 \in E_{i-1}$ ...

A “Better” Proof

All expressions have at least one constant or variable.

PL-style proof: By *structural induction* on (rules for forming an expression) e. Cases:

- c ...
- x ...
- $e_1 + e_2$ ...
- $e_1 * e_2$ ...

Structural induction invokes the induction hypothesis on *smaller* terms. It is equivalent to the pedantic proof, and more convenient in PL