



# Victory Lap

CSE 312 Summer 21  
Lecture 25

# Content

## Combinatorics (*fancy* counting)

Permutations, combinations, inclusion-exclusion, pigeonhole principle

## Formal definitions of Probability

Probability space, events, conditional probability, independence, expectation, variance

## Common patterns in probability

Equations and inequalities, common random variables, tail bounds

## Continuous Probability

Density, sample distributions, Central Limit Theorem, Estimating probabilities

## Applications

Across CS

# Themes

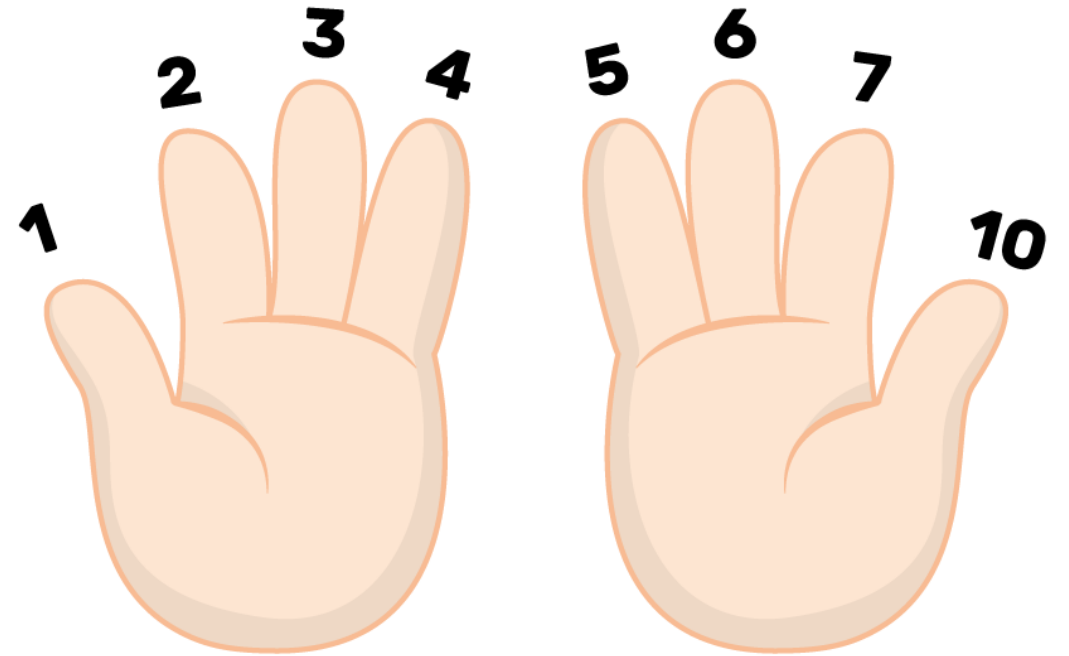
Precise mathematical communication  
Both reading and writing dense statements.

Probability in the “real world”  
A mix of CS applications  
And some actual “real life” ones.

Refine your intuition  
Most people have some base level feeling of what the chances of some event are.  
We’re going to train you to have better gut feelings.

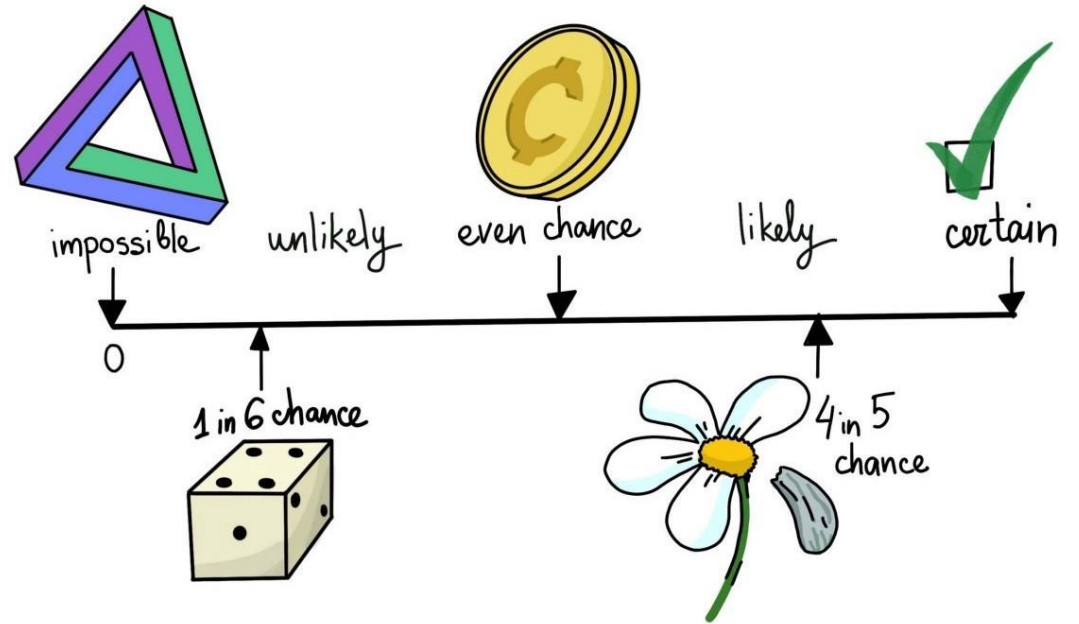
# Combinatorics

- Sum and Product Rule
- Permutations and Combinations
- Complementary Counting
- Binomial Coefficients
- Principle of Inclusion-Exclusion
- Pigeonhole Principle
- Stars and Bars
- Combinatorial Proofs



# Discrete Probability

- Axioms of Probability
- Equally Likely Outcomes
- Conditional Probability
- Bayes Theorem
- Law of Total Probability
- Chain Rule
- Independence
- Conditional Independence



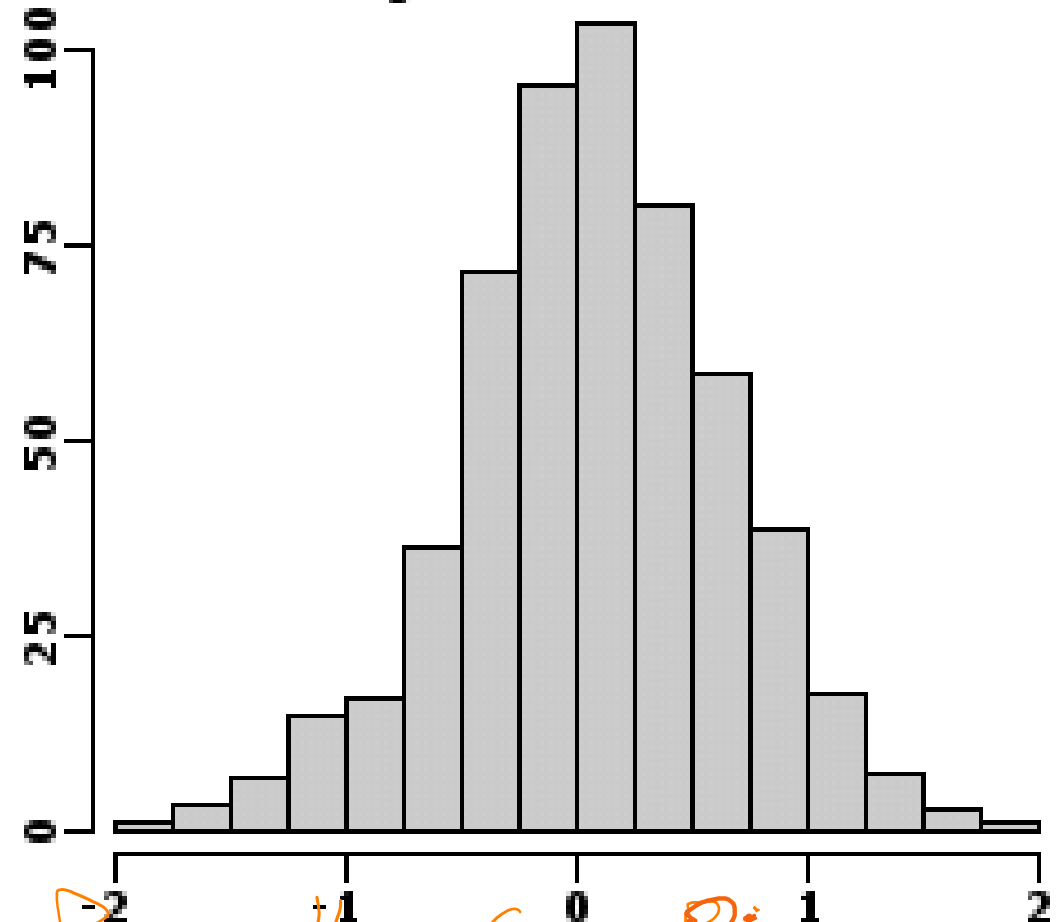
# Discrete Distributions

- Probability Mass Function
- Expectation
- Indicator Random Variables
- Linearity of Expectation
- Variance and Standard Deviation
- LOTUS (Expectation of a function  $g(X)$ )
- Independence of RVs
- Zoo of Distributions

*Ber, Unif, Geo, Neg Bin, Poi, HyperGeo, Bin*

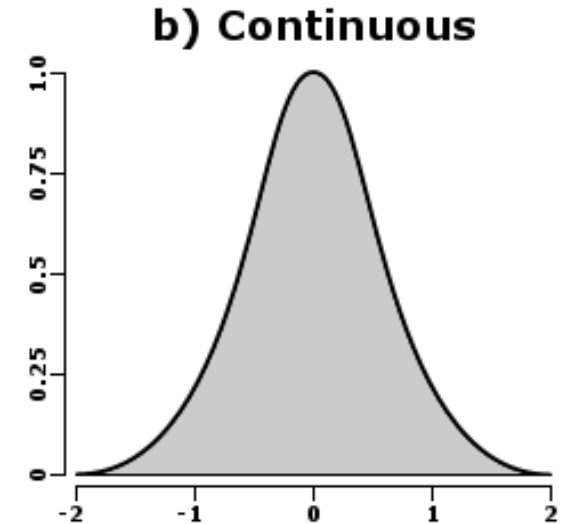
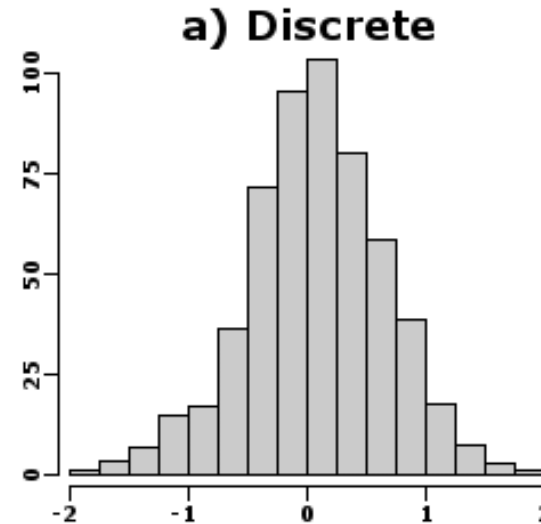
$$\binom{3}{2} \quad \binom{10}{2,1,3}$$

## a) Discrete



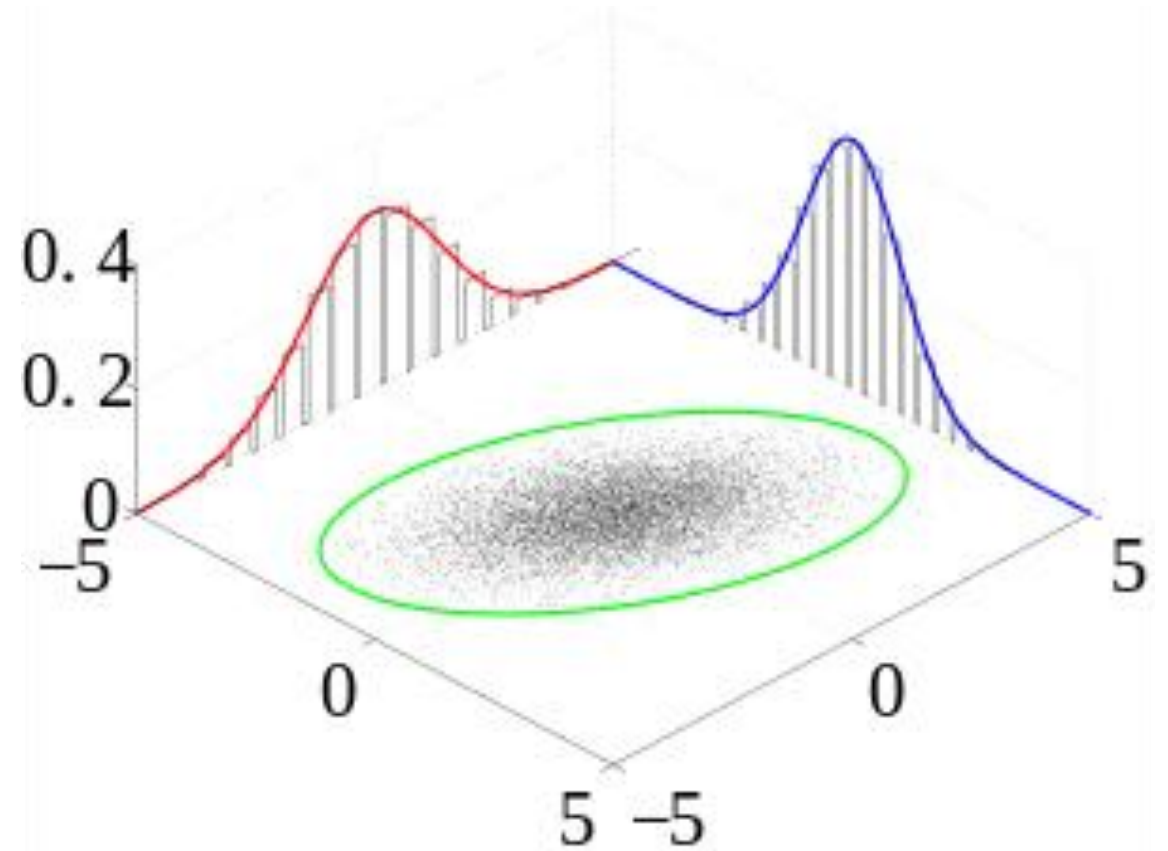
# Continuous Distributions

- Probability Density Function
- Cumulative Distribution Function
- Zoo of Continuous RVs
- Normal Distribution *Normal, Exp, Unif*
  - Standardizing
  - Linear Transformation
  - Sum of Normals  $\rightarrow$  Normal



# Multiple Random Variables

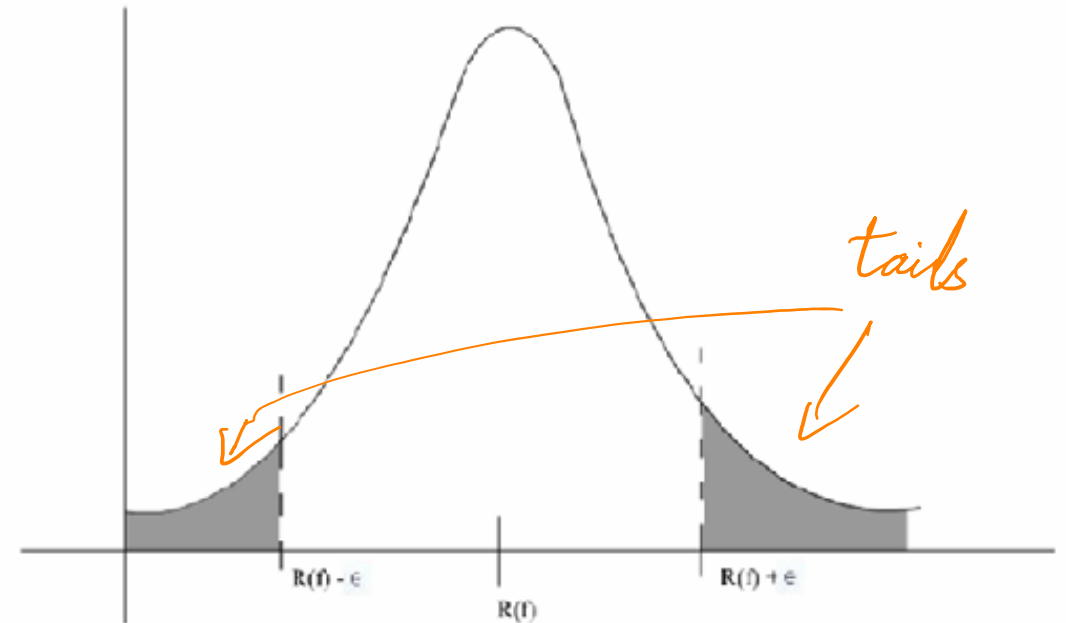
- Joint PMFs / PDFs
- Marginal PMFs / PDFs
- Conditional PMFs / PDFs
- Law of Total Expectation
- Covariance
- Central Limit Theorem





# Tail Bounds or Concentration Inequalities

- Markov's Inequality
- Chebyshev's Inequality
- Chernoff Bound
- Union Bound



# Estimation

- Likelihood
- Log-likelihood
- Maximum Likelihood Estimation
- Two Parameter Estimation
- Bias

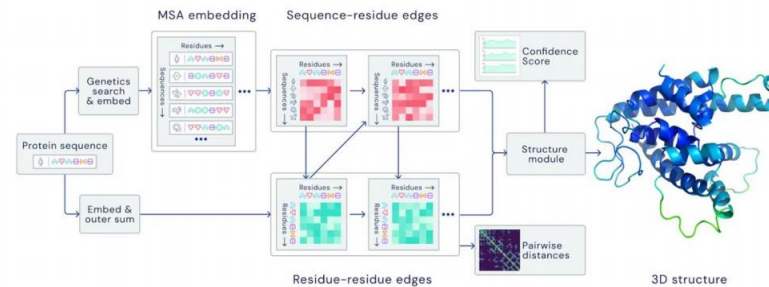
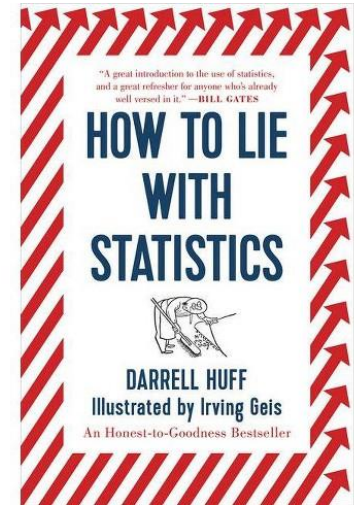
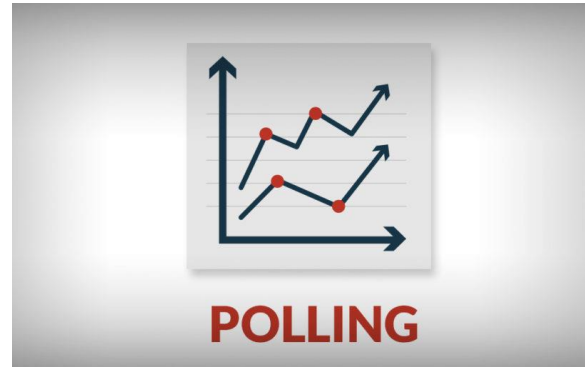
## Estimation

The fine art of guessing



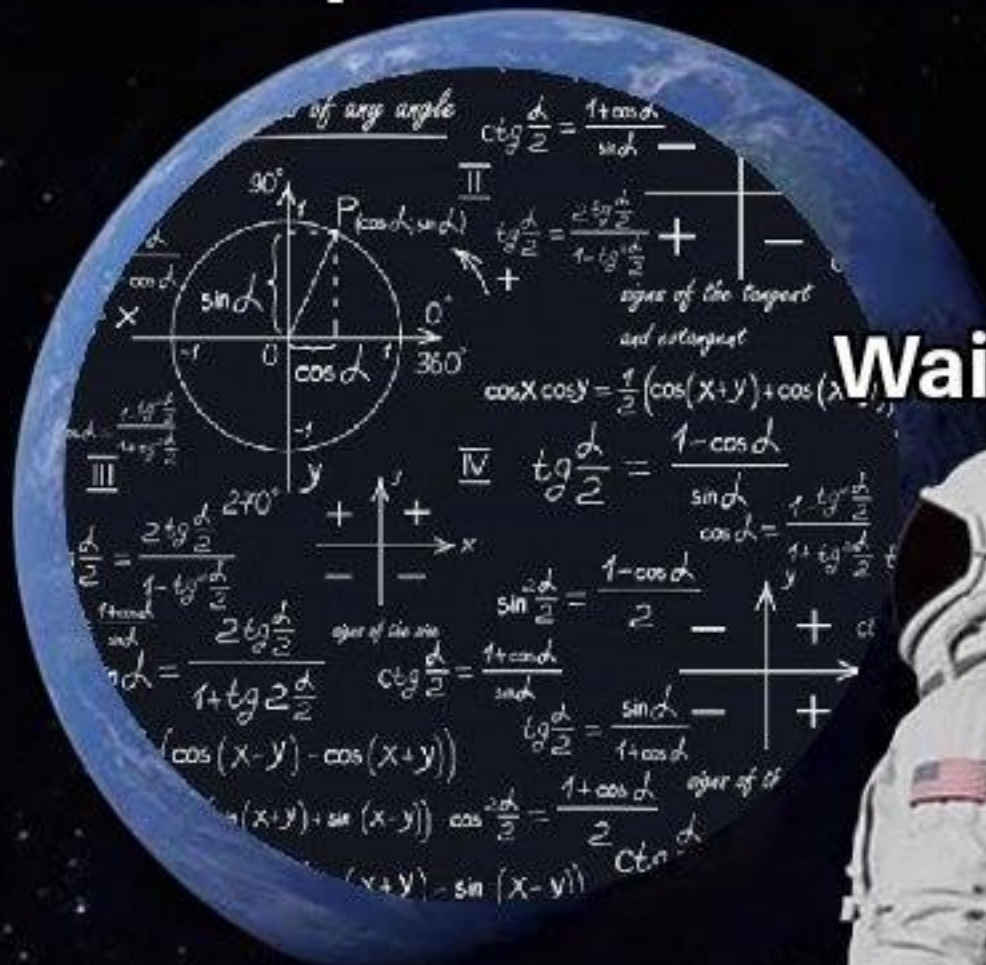
# Applications

- In Lecture
  - Polling
  - How to Lie with Statistics
  - Protein Synthesis
- Assignments
  - Spam Filtering
  - Vaccine Efficacy
  - Distinct Elements



**Computer Science**

**Always has been**



**Wait, It's all math?**



# Technical Skills Learned

- LaTeX
  - Difficult to start with
  - Very useful for Computer Scientists
  - Needed for Technical Writing and research papers
- Python
  - Understanding some libraries like numpy
  - Less wordy than Java
  - Most Data Science courses use Python

TEX



# Use Your Powers Wisely

We've seen probability can be used in the real world!

But also, that:

It can be counter-intuitive / hard to explain (Bayes Rule/Real World 1)

Probability estimates can depend on the model you're using (Real World 2)



# What to take next?

**CSE 446 ML** – using probability, linear algebra, and other techniques to extract patterns from data and make predictions.

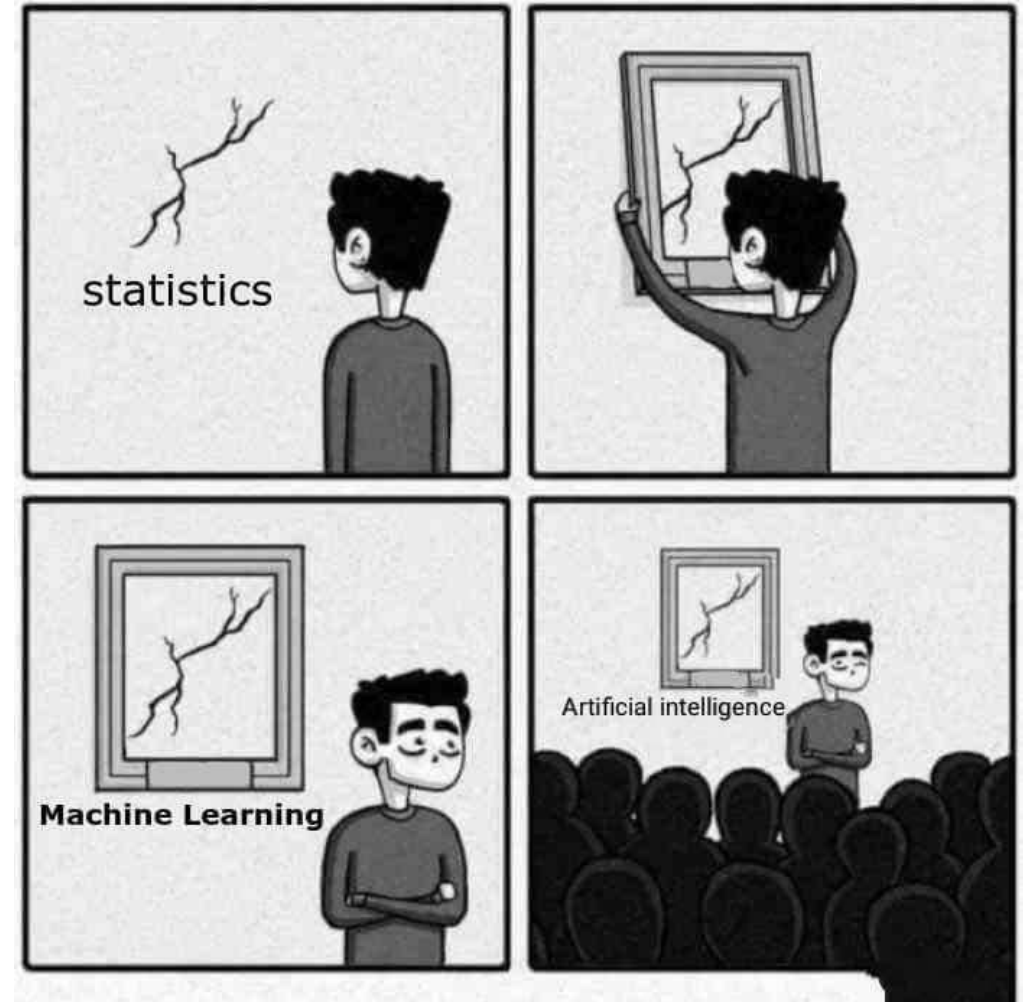
**CSE 421 designing algorithms** – very little direct probability, but the combinatorics we did at the beginning will be useful.

We also have a graduate level course in randomized algorithms which will utilize more of the content covered in 312, but it has a few more prereqs

**CSE 447 Natural Language Processing**

**CSE 490C Cryptography**

Other things!



# Parting Thoughts

This course is always a grind in the way we move from topic to topic.

You made it through over a year into a pandemic

Over zoom

Completing

- 7 Problem Sets
- 3 Review Summaries
- 2 Real World mini-projects
- 27 Concept Checks
- **And a Final!!!**



# Parting Thoughts

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# Thank you!

Thank you all for a great quarter!

This course would not have been possible without our amazing TAs!!



**IT'S BEEN AN HONOR**



**FAREWELL BUT NOT GOODBYE**

