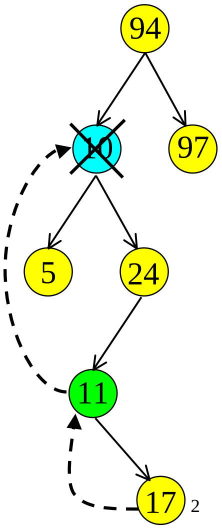
CSE 326 Lecture 7: More on Search Trees

- Today’s Topics:
  - Lazy Operations
  - Run Time Analysis of Binary Search Tree Operations
  - Balanced Search Trees
    - AVL Trees and Rotations
- Covered in Chapter 4 of the text

From Last Time: Remove (Delete) Operation

- Removing a node containing X:
  1. Find the node containing X
  2. Replace it with:
     If it has no children, with NULL
     If it has 1 child, with that child
     If it has 2 children, with the node with the smallest value in its right subtree, (or largest value in left subtree)
  3. Recursively remove node used in 2 and 3
- Worst case: Recursion propagates all the way to a leaf node – time is O(depth of tree)



Laziness in Data Structures

- A “lazy” operation is one that puts off work as much as possible in the hope that a future operation will make the current operation unnecessary

Lazy Deletion

- Idea: Mark node as deleted; *no need to reorganize tree*
  - Skip marked nodes during Find or Insert
  - Reorganize tree only when number of marked nodes exceeds a percentage of real nodes (e.g. 50%)
  - Constant time penalty only due to marked nodes – depth increases only by a constant amount if 50% are marked undeleted nodes (N nodes max N/2 marked)
- Modify Insert to make use of marked nodes whenever possible e.g. when deleted value is re-inserted
- Can also use lazy deletion for Lists

Run Time Analysis of BST operations

- All BST operations (except MakeEmpty) are O(d), where d is the depth of the accessed node in the tree
  - MakeEmpty takes O(N) for a tree with N nodes – frees all nodes
- We know: $\log N \leq d \leq N-1$ for a binary tree with N nodes
  - What is the best case tree? What is the worst case tree?
- Best Case Running Time of Insert/Remove/etc. = ?
- Worst Case Running Time = ?
- Average Case Running Time = ?

The best, the worst, and the average…

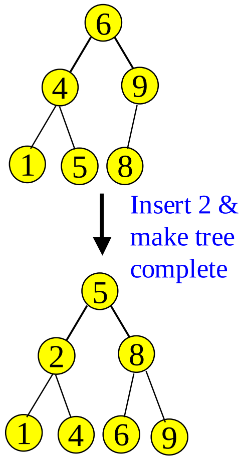
- For a binary tree with N nodes, depth d of any node satisfies: $\log N \leq d \leq N-1$
- So, best case running time of BST operations is **O(log N)**
- Worst case running time is **O(N)**
- Average case running time = O(average value of d) = **O(log N)**
  - Can prove that average depth over all nodes = O(log N) if all insertion sequences equally likely.
  - See Chap. 4 in textbook for proof

Can we do better?

- Worst case running time of BST operations is **O(N)**
- E.g. What happens when you Insert elements in ascending (or descending) order?
  - Insert 2, 4, 6, 8, 10, 12 into an empty BST
- Problem: Lack of "balance" – Tree becomes highly asymmetric
- Idea: Can we restore balance by re-arranging tree according to depths of left and right subtrees?
  - Goal: Get depth down from O(N) to O(log N)

Idea #1: Achieving the perfect balance…

- First try at balancing trees: Perfect balance
  - Re-arrange to get a complete tree after every operation
- Recall: A tree is complete if there are no "holes" when scanning from top to bottom, left to right
- Problem: Too expensive to re-arrange
  - E.g. Insert 2 in the example shown
- Need a looser constraint…

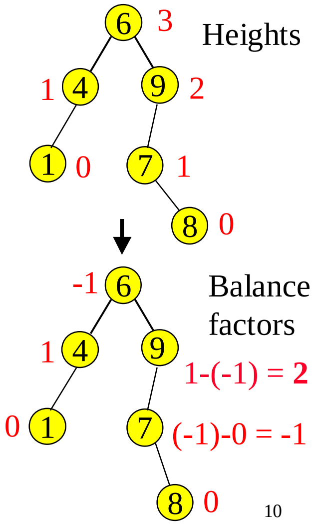


Idea #2: Leave it to the professionals…

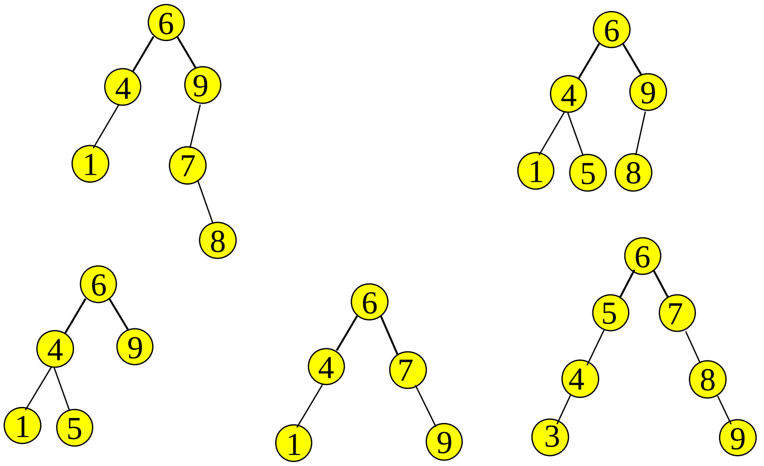
- Many efficient algorithms exist for balancing trees in order to achieve faster running times for the BST operations
    - Adelson-Velskii and Landis (AVL) trees (1962)
    - Splay trees and other self-adjusting trees (1978)
    - B-trees and other multiway search trees (1972)

AVL Trees

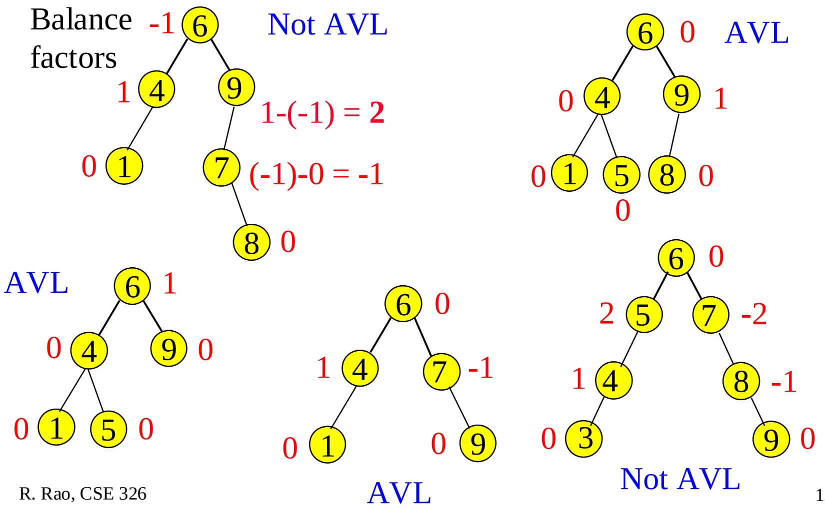
- AVL trees are height-balanced binary search trees
- Balance factor of a node = height(left subtree) - height(right subtree)
- An AVL tree can only have balance factors of 1, 0, or –1 at every node
    - For every node, heights of left and right subtree differ by no more than 1
    - Height of an empty subtree = -1
- Implementation: Store current heights in each node



Which of these are AVL trees?

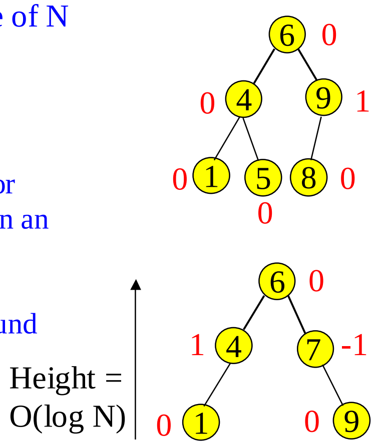


AVL Trees: Examples and Non-Examples



The good news about AVL Trees

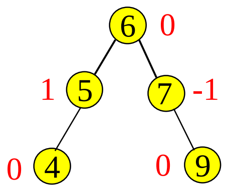
- Can prove: Height of an AVL tree of N nodes is always O(log N)
- How? Can show:
  - Height h ≤ 1.44 log(N+2)-0.328
  - Prove using recurrence relation for minimum number of nodes S(h) in an AVL tree of height h:
    S(h) = S(h-1) + S(h-2) + 1
  - Use Fibonacci numbers to get bound on S(h) bound on height h
  - See textbook for details



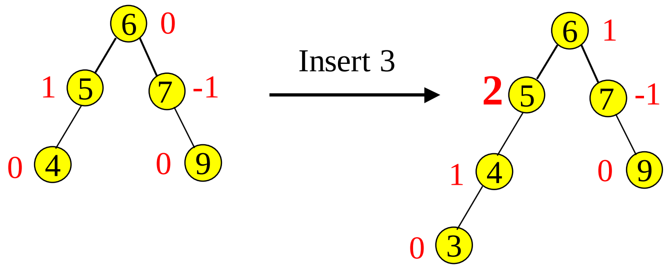
The really good news about AVL Trees

- Can prove: Height of an AVL tree of N nodes is always O(log N)
- All operations (e.g. Find, Remove using lazy deletion, etc.) on an AVL tree are O(log N)…
- …except Insert
  - Why is Insert different?

Insert 3



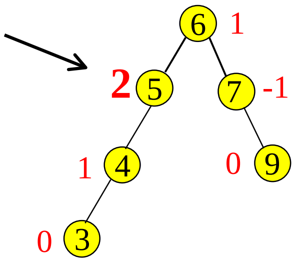
The bad news about AVL Trees



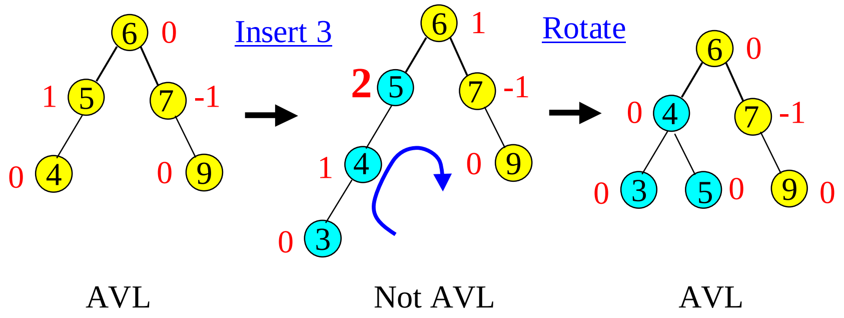
No longer an AVL tree
(i.e. not balanced anymore)

Restoring Balance in (the life of) an AVL Tree

- Problem: Insert may cause balance factor to become 2 or –2 for some node on the path from insertion point to root node
- Idea: After Inserting the new node,
  1. Back up to root updating heights along the access path
  2. If Balance Factor = 2 or –2, adjust tree by rotation around deepest such node.



Rotating to restore Balance: A Simple Example



Next Class:

Rotating and Splaying for Fun and Profit

To Do:

Finish Reading Chapter 4