# Introduction to LS/LS+

LS+

LS

#### Laura Elisa Celis

University of Washington

February 21, 2007

University of Washington

- 4 同 6 4 日 6 4 日 6

### Cutting Planes Methods

Given a 0-1 integer program:

• Consider feasibility instead of optimization.

## Cutting Planes Methods

Given a 0-1 integer program:

- Consider feasibility instead of optimization.
- Relax given integer linear program and consider polytope satisfying constraints.

くほう くうり くうり

## Cutting Planes Methods

Given a 0-1 integer program:

- Consider feasibility instead of optimization.
- Relax given integer linear program and consider polytope satisfying constraints.
- Transform through "cuts" to integral hull of valid solutions.

・ 同 ト ・ ヨ ト ・ ヨ ト

Introduction	LS	LS+	Separation Oracles
	• 000	000	000
	• 00000	0000000	00
Review			

A cone K is a subset of  $\mathbb{R}^n$  where  $x \in K$  if and only if  $cx \in K$  for any constant  $c \ge 0$ .

University of Washington

- 4 回 > - 4 回 > - 4 回 >

Introduction	LS ●000 ○○○○○○	LS+ 000 0000000	Separation Oracles
Review			

A cone K is a subset of  $\mathbb{R}^n$  where  $x \in K$  if and only if  $cx \in K$  for any constant  $c \ge 0$ .

#### Definition

K is the cone of feasible points given by the linear program.

・ 同 ト ・ ヨ ト ・ ヨ ト

Introduction	LS	LS+	Separation Oracles
	●000	000	000
	○00000	0000000	00
Review			

A cone K is a subset of  $\mathbb{R}^n$  where  $x \in K$  if and only if  $cx \in K$  for any constant  $c \ge 0$ .

#### Definition

K is the cone of feasible points given by the linear program.

#### Goal:

Determine if the set of integer points in K is nonempty.

・ 同 ト ・ ヨ ト ・ ヨ ト

# Cone Formulation of LS

#### Definition

 $x \in N(K)$  if and only if there is a matrix Y such that:

LS

0000

- The first row of Y is x.
- The diagonal of Y is x (corresponds to  $x_i^2 = x_i$ ).
- Y is symmetric  $(Y_{ij} = Y_{ji} \text{ corresponds to } x_i x_j = x_j x_i)$ .
- All rows of Y are in K (corresponds to multiplying by  $x_i$ ).
- All  $Y_0 Y_i$  are in K (corresponds to multiplying by  $1 x_i$ ).

LS+

- 4 同 6 4 日 6 4 日 6

# Cone Formulation of LS

#### Definition

 $x \in N(K)$  if and only if there is a matrix Y such that:

LS

0000

- The first row of Y is x.
- The diagonal of Y is x (corresponds to  $x_i^2 = x_i$ ).
- Y is symmetric  $(Y_{ij} = Y_{ji} \text{ corresponds to } x_i x_j = x_j x_i)$ .
- All rows of Y are in K (corresponds to multiplying by  $x_i$ ).
- All  $Y_0 Y_i$  are in K (corresponds to multiplying by  $1 x_i$ ).

LS+

#### Goal:

Show that N(K) is stronger than K.

- 4 同 6 4 日 6 4 日 6

Validity of Cuts

Review

# Theorem If $K^0$ is convex hull of 0-1 vectors in K, then $K^0 \subseteq N(K) \subseteq K$ .

Validity of Cuts

Review

# Theorem If $K^0$ is convex hull of 0-1 vectors in K, then $K^0 \subseteq N(K) \subseteq K$ .

Proof Trivially,  $N(K) \subseteq K$ .

-

- 4 同 ト 4 ヨ ト 4 ヨ ト

Validity of Cuts

Review

#### Theorem

If  $K^0$  is convex hull of 0-1 vectors in K, then  $K^0 \subseteq N(K) \subseteq K$ .

#### Proof

Trivially,  $N(K) \subseteq K$ . If x is a 0-1 vector in  $K^0$ , the matrix  $Y = xx^T$  satisfies all necessary constraints.

・ 同 ト ・ ヨ ト ・ ヨ ト

Introduction	LS 000● 000000	LS+ 000 0000000	Separation Oracles 000 00

# Proof

$$Y = xx^{T} = \begin{pmatrix} x_{0}x_{0} & \cdots & x_{0}x_{i} & \cdots & x_{0}x_{n} \\ \vdots & \ddots & & \vdots \\ x_{i}x_{0} & & x_{i}x_{i} & & x_{i}x_{n} \\ \vdots & & \ddots & \vdots \\ x_{n}x_{0} & \cdots & x_{n}x_{i} & \cdots & x_{n}x_{n} \end{pmatrix}$$

Ξ.

イロン イロン イヨン イヨン

Introduction	LS	LS+	Separation Oracles
	0000 000000	000 0000000	000

### Proof

$$Y = xx^{T} = \begin{pmatrix} x_{0}x_{0} \cdots x_{0}x_{i} \cdots x_{0}x_{n} \\ \vdots & \ddots & \vdots \\ x_{i}x_{0} & x_{i}x_{i} & x_{i}x_{n} \\ \vdots & \ddots & \vdots \\ x_{n}x_{0} & \cdots & x_{n}x_{i} & \cdots & x_{n}x_{n} \end{pmatrix}$$

• The first row is x

#### Introduction to LS/LS+

#### University of Washington

э

<ロ> <同> <同> < 回> < 回>

### Proof

$$Y = xx^{T} = \begin{pmatrix} x_{0}x_{0} & \cdots & x_{0}x_{i} & \cdots & x_{0}x_{n} \\ \vdots & \ddots & & \vdots \\ x_{i}x_{0} & & x_{i}x_{i} & & x_{i}x_{n} \\ \vdots & & \ddots & \vdots \\ x_{n}x_{0} & \cdots & x_{n}x_{i} & \cdots & x_{n}x_{n} \end{pmatrix}$$

• The first row is x

• The diagonal is 
$$x (x_i^2 = x_i)$$

イロト イポト イヨト イヨト

э

### Proof

$$Y = xx^{T} = \begin{pmatrix} x_{0}x_{0} & \cdots & x_{0}x_{i} & \cdots & x_{0}x_{n} \\ \vdots & \ddots & & \vdots \\ x_{i}x_{0} & & x_{i}x_{i} & & x_{i}x_{n} \\ \vdots & & \ddots & \vdots \\ x_{n}x_{0} & \cdots & x_{n}x_{i} & \cdots & x_{n}x_{n} \end{pmatrix}$$

LS

0000

• The first row is x

LS+

• The diagonal is  $x (x_i^2 = x_i)$ 

(日) (同) (三) (三)

• Symmetric  $(x_i x_j = x_j x_i)$ 

## Proof

$$Y = xx^{T} = \begin{pmatrix} x_{0}x_{0} & \cdots & x_{0}x_{i} & \cdots & x_{0}x_{n} \\ \vdots & \ddots & & \vdots \\ x_{i}x_{0} & & x_{i}x_{i} & & x_{i}x_{n} \\ \vdots & & \ddots & \vdots \\ x_{n}x_{0} & \cdots & x_{n}x_{i} & \cdots & x_{n}x_{n} \end{pmatrix}$$

LS

0000

• The first row is x

LS+

- The diagonal is  $x (x_i^2 = x_i)$
- Symmetric  $(x_i x_j = x_j x_i)$
- Each row is in  $K(x_i \cdot x)$

・ロト ・同ト ・ヨト ・ヨト

## Proof

$$Y = xx^{T} = \begin{pmatrix} x_{0}x_{0} & \cdots & x_{0}x_{i} & \cdots & x_{0}x_{n} \\ \vdots & \ddots & & \vdots \\ x_{i}x_{0} & & x_{i}x_{i} & & x_{i}x_{n} \\ \vdots & & \ddots & \vdots \\ x_{n}x_{0} & \cdots & x_{n}x_{i} & \cdots & x_{n}x_{n} \end{pmatrix}$$

LS

0000

• The first row is x

LS+

- The diagonal is  $x (x_i^2 = x_i)$
- Symmetric  $(x_i x_j = x_j x_i)$
- Each row is in  $K(x_i \cdot x)$

- 4 同 6 4 日 6 4 日 6

• 
$$Y_0 - Y_i$$
 is in  $K$   
 $((1 - x_i) \cdot x)$ .

University of Washington

-

Introduction	LS	LS+	Separation Oracles
	0000	000	000
	000000	0000000	00

### Strength of Cuts

#### Theorem

*n* rounds of *LS* give the convex hull of 0-1 solutions (i.e.  $K^0 = N^n(K)$ ).

Introduction to LS/LS+

University of Washington

- A - E - N

Image: A = A

Introduction	LS	LS+	Separation Oracles
	0000	000	000
	000000	0000000	00
Strength Result			

#### . . . .

### Strength of Cuts

#### Theorem

*n* rounds of *LS* give the convex hull of 0-1 solutions (i.e.  $K^0 = N^n(K)$ ).

# Proof $(K^0 \subseteq N^n(K))$ From before, $K^0 \subseteq N(K) \subseteq K$ .

< A > < B

Introduction	LS	LS+	Separation Oracles
	0000	000	000
	000000	0000000	00

## Strength of Cuts

#### Theorem

*n* rounds of *LS* give the convex hull of 0-1 solutions (i.e.  $K^0 = N^n(K)$ ).

#### Proof $(K^0 \subseteq N^n(K))$

From before,  $K^0 \subseteq N(K) \subseteq K$ . Hence,  $K^0 \subseteq N^k(K)$  for all k.

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○●○○○○	0000000	00

## Strength of Cuts

・ロト・西ト・西ト・西ト・日・ のへの

Introduction to LS/LS+

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○●○○○○	0000000	00
Strength Result			

#### Theorem

- $v \in H_i$  if and only if  $v_i = 0$
- $v \in G_i$  if and only if  $v_i = v_0$
- $F_i = H_i \cup G_i$

Then,  $N(K) \subseteq \operatorname{cone}(K \cap F_i)$ .

- A - E - N

◆ 同 ♪ ◆ 三 ♪

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○●○○○○	0000000	00
Strength Result			

#### Theorem

- $v \in H_i$  if and only if  $v_i = 0$
- $v \in G_i$  if and only if  $v_i = v_0$
- $F_i = H_i \cup G_i$

Then,  $N(K) \subseteq \operatorname{cone}(K \cap F_i)$ .

#### Proof

Let  $x \in N(K)$ , with corresponding Y. Let  $Y_i$  be the *i*th row of Y.

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○○●○○○	0000000	00
Strength Result			

### Proof (cont.)

(	$Y_{00}$		<i>Y</i> <sub>0<i>i</i></sub>		$Y_{0n}$
	÷	••.			÷
	$Y_{i0}$		Y <sub>ii</sub>		Yin
	÷			·	÷
	$Y_{n0}$		Y <sub>ni</sub>		$Y_{nn}$

э

-∢ ≣ →

▲ 同 ▶ → ● 三

Introduction	LS	LS+	Separation Oracles
	0000	000 0000000	000

Strength of Cuts

Proof (cont.)

(	$Y_{00}$		<i>Y</i> <sub>0<i>i</i></sub>		$Y_{0n}$
	÷	•••			÷
	$Y_{i0}$		Y <sub>ii</sub>		Y <sub>in</sub>
	÷			·	÷
	$Y_{n0}$		Y <sub>ni</sub>		$Y_{nn}$

• 
$$Y_{ii} = Y_{i0}$$
 implies  $Y_i \in G_i$ .

<ロ> <同> <同> < 回> < 回>

Introduction to LS/LS+

University of Washington

э

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○○●○○○	0000000	00
Strength Result			

Proof (cont.)

(	$Y_{00}$	• • •	<i>Y</i> <sub>0<i>i</i></sub>		$Y_{0n}$
	÷	·			÷
	$Y_{i0}$		Y <sub>ii</sub>		Yin
	÷			·	÷
	$Y_{n0}$		Y <sub>ni</sub>		$Y_{nn}$

•  $Y_i \in K \cap G_i$ .

< D > < P > < P >

Introduction to LS/LS+

University of Washington

э

- ∢ ⊒ →

Introduction	LS	LS+	Separation Oracles
	0000	000 0000000	000

Strength of Cuts

Proof (cont.)

(	$Y_{00}$		<i>Y</i> <sub>0<i>i</i></sub>		$Y_{0n}$
	÷	·			÷
	$Y_{i0}$		Y <sub>ii</sub>		Yin
	÷			·	÷
	$Y_{n0}$		Y <sub>ni</sub>		$Y_{nn}$

- $Y_i \in K \cap G_i$ .
- $(Y_0 Y_i)_i = Y_{0i} Y_{ii} = 0$ implies  $(Y_0 - Y_i) \in H_i$ .

(日) (同) (三) (三)

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○○●○○○	0000000	00

Strength of Cuts

Proof (cont.)

Y<sub>0i</sub> <u>:</u>  $Y_{i0}$  $Y_{ii}$ Yin  $Y_{n0}$ 

•  $Y_i \in K \cap G_i$ .

• 
$$(Y_0 - Y_i) \in K \cap H_i$$
,

Introduction to LS/LS+

University of Washington

-

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○○○●○○	0000000	00
Strength Result			

Proof (cont.) So:

$$x = Y_0$$

University of Washington

э

イロト イポト イヨト イヨト

Introduction	LS	LS+	Separation Oracles
	0000	000	000
	000000	0000000	00
Strength Result			

Proof (cont.) So:

$$\begin{array}{rcl} x & = & Y_0 \\ & = & (Y_0 - Y_i) + Y_i \end{array}$$

Introduction to LS/LS+

University of Washington

э

< ロ > < 同 > < 回 > < 回 >

Introduction	LS	LS+	Separation Oracles
	000	000	000
	000	0000000	00

Strength of Cuts

Proof (cont.) So:

$$\begin{aligned} x &= Y_0 \\ &= (Y_0 - Y_i) + Y_i \\ &\in (K \cap H_i) + (K \cap G_i) \end{aligned}$$

Introduction to LS/LS+

University of Washington

э

<ロ> <同> <同> < 同> < 同>

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○○○●○○	0000000	00

Strength of Cuts

Proof (cont.) So:

$$\begin{array}{rcl} x & = & Y_0 \\ & = & (Y_0 - Y_i) + Y_i \\ & \in & (K \cap H_i) + (K \cap G_i) \\ & \subseteq & \operatorname{cone}(K \cap (H_i \cup G_i)) \end{array}$$

Introduction to LS/LS+

University of Washington

э

< ロ > < 同 > < 回 > < 回 >

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○○○●○○	0000000	00

Strength of Cuts

Proof (cont.) So:

$$x = Y_0$$
  
=  $(Y_0 - Y_i) + Y_i$   
 $\in (K \cap H_i) + (K \cap G_i)$   
 $\subseteq \operatorname{cone}(K \cap (H_i \cup G_i))$   
=  $\operatorname{cone}(K \cap F_i)$ 

э

<ロ> <同> <同> < 同> < 同>

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○○○○●○	0000000	00

### Example



University of Washington

э

<ロ> <同> <同> < 回> < 回>

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○○○○●○	0000000	00

### Example



University of Washington

э

<ロ> <同> <同> < 回> < 回>

Introduction	LS	LS+	Separation Oracles
	0000	000	000
	000000	0000000	00

# Example



≣ ▶ ∢ ≣ ▶ ा≣ ∽ ९ ० University of Washington

< ロ > < 同 > < 回 > < 回 >

Introduction	LS	LS+	Separation Oracles
	0000	000	000
	000000	0000000	00

# Example



University of Washington

- ∢ ⊒ →

A B > A B >

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○○○○●○	0000000	00

# Example



University of Washington

э

<ロ> <同> <同> < 回> < 回>

Introduction	LS	LS+	Separation Oracles
	○○○○	000	000
	○○○○●	0000000	00
Strength Result			

# Conclusion

In terms of cones:

$$N^{t}(K) = N(N^{t-1}(K)) \subseteq \operatorname{cone}(N^{t-1}(K) \cap F_{t})$$

Introduction to LS/LS+

University of Washington

э

イロト イポト イヨト イヨト

Introduction	LS ○○○○ ○○○○○●	LS+ 000 0000000	Separation Oracles
Strength Recult			

### Conclusion

In terms of cones:

$$N^{t}(K) = N(N^{t-1}(K)) \subseteq \operatorname{cone}(N^{t-1}(K) \cap F_{t})$$

Since  $\cap_1^n F_t$  is the vertices of hypercube,  $N^n(K) \subseteq K^0$ .

- 4 同 2 4 日 2 4 日 2

Introduction	LS 0000 000000	LS+ ••• ••••••	Separation Oracles
Introduction			

LS+

 $\mathsf{LS+}$  strengthens an  $\mathsf{LP}$  in the same way as  $\mathsf{LS},$  also adds squares of linear terms.

Introduction to LS/LS+

Introduction	LS	LS+	Separation Oracles
	0000	○●○	000
	000000	○○○○○○○	00
Introduction			

Let  $x \in N_+(K)$  if and only if there is a matrix Y such that:

- The first column of Y is x
- The diagonal of Y is x
- Y is symmetric  $Y_{ij} = Y_{ji}$
- All the rows of Y are in K
- For all i,  $Y_0 Y_i$  is in K
- Y is positive semidefinite.

Introduction	LS	LS+	Separation Oracles
	0000	○○●	000
	000000	○○○○○○○○	00
Introduction			

Recall: Y positive semidefinite means:  $v^T Y v \ge 0$  for all v.



Introduction to LS/LS+

Recall: Y positive semidefinite means:  $v^T Y v \ge 0$  for all v. Let A, B be positive semidefinite matrices.

•  $v^T(cA)v = c(v^TAv) \ge 0$ , so cA is positive semidefinite.

-

- 4 回 2 - 4 □ 2 - 4 □

Recall: Y positive semidefinite means:  $v^T Y v \ge 0$  for all v. Let A, B be positive semidefinite matrices.

- $v^T(cA)v = c(v^TAv) \ge 0$ , so cA is positive semidefinite.
- v(A+B)v = v<sup>T</sup>Av + v<sup>T</sup>Bv ≥ 0, so A + B is positive semidefinite.

(人間) (人) (人) (人) (人) (人)

Recall: Y positive semidefinite means:  $v^T Y v \ge 0$  for all v. Let A, B be positive semidefinite matrices.

- $v^T(cA)v = c(v^TAv) \ge 0$ , so cA is positive semidefinite.
- v(A + B)v = v<sup>T</sup>Av + v<sup>T</sup>Bv ≥ 0, so A + B is positive semidefinite.

 $N_+(K)$  is a convex cone.

(人間) (人) (人) (人) (人) (人)

Introduction	LS	LS+	Separation Oracles
	0000	000	000
	000000	000000	00
Semi Definite Programming			

# SDP

A semi-definite program is of the form:

 $\begin{array}{rcl} \min C \circ Y & \text{s.t.} \\ A_1 \circ Y &=& b_1 \\ &\vdots \\ A_m \circ Y &=& b_m \\ Y \succeq 0 \end{array}$ 

Where  $C, A_1, \ldots, A_m$  are symmetric matrices, and  $b_1, \ldots, b_m$  are scalars.

University of Washington

-

・ロン ・四 と ・ ヨ と ・ 日 と

Introduction	LS	LS+	Separation Oracles
	0000	000	000
	000000	000000	00

Semi Definite Programming

Vertex Cover

LP:



$$egin{aligned} x_i + x_j - 1 &\geq 0 & ext{ for all } (i,j) \in E \ 0 &\leq x_i \leq 1 & ext{ for all } i. \end{aligned}$$

Introduction to LS/LS+

University of Washington

э

イロト イポト イヨト イヨト

Introduction	LS 0000 000000	LS+ ○○○ ○○●○○○○	Separation Oracles
Cardi Dafinita Daramina			

By applying LS lift rules:

$$(1-x_i)(1-x_j) \ge 0 \text{ for all } i, j.$$



э

イロト イポト イヨト イヨト

Introduction	LS	LS+	Separation Oracles
	0000	○○○	000
	000000	○○●○○○○	00
Semi Definite Programming			

By applying LS lift rules:

$$(1 - x_i)(1 - x_j) \ge 0$$
 for all  $i, j$ .  
Since  $x_i^2 = x_i$   
 $0 \le (1 - x_i)(x_i + x_i - 1)$ 

$$J \leq (1-x_i)(x_i+x_j-1)$$
  
=  $(1-x_i)(x_j-1)$  for all  $(i,j) \in E$ .

University of Washington

э

<ロ> <同> <同> < 同> < 同>

Introduction	LS	LS+	Separation Oracles
	0000	○○○	000
	000000	○○●○○○○	00
Semi Definite Programming			

By applying LS lift rules:

$$(1 - x_i)(1 - x_j) \ge 0$$
 for all  $i, j$ .  
Since  $x_i^2 = x_i$   
 $0 \le (1 - x_i)(x_i + x_j - 1)$   
 $= (1 - x_i)(x_j - 1)$  for all  $(i, j) \in E$ 

Or equivalently

$$(1-x_i)(1-x_j) \leq 0$$
 for all  $(i,j) \in E$ .

University of Washington

э

<ロ> <同> <同> < 同> < 同>

Introduction	LS	LS+	Separation Oracles
	0000 000000	000 000000	000

Semi Definite Programming

Vertex Cover

Let  $x_0 = 1$ .

$$\min \sum_{i \in V} (x_0 x_i)$$

$$(x_0 - x_i)(x_0 - x_j) = 0$$
 for all  $(i, j) \in E$   
 $(x_0 - x_i)(x_0 - x_j) \ge 0$  for all  $i, j$ .

Introduction to LS/LS+

University of Washington

э

イロト イポト イヨト イヨト

Introduction	LS	LS+	Separation Oracles
	0000	000	000
	000000	0000000	00
Semi Definite Programming			

Let  $Y = U^T U$  be LS+ lifted matrix. Let the columns of Y be  $(u_0, \ldots, u_n)$ .



▲御▶ ▲理▶ ▲理≯

Introduction to LS/LS+

Introduction	LS	LS+	Separation Oracles
	0000	○○○	000
	000000	○○○○●○○	00
Semi Definite Programming			

Let  $Y = U^T U$  be LS+ lifted matrix. Let the columns of Y be  $(u_0, \ldots, u_n)$ . Then:

$$\min\sum_{i\in V}u_0\cdot u_i$$

$$\begin{aligned} (u_0-u_i)\cdot(u_0-u_j) &= 0 & \text{ for all } (i,j) \in E \\ (u_0-u_i)\cdot(u_0-u_j) &\geq 0 & \text{ for all } i,j. \end{aligned}$$

University of Washington

→ □ → → 三 → → 三 →

Introduction	LS 0000 000000	LS+ ○○○ ○○○○○●○	Separation Oracles 000 00
Semi Definite Programming			

Let  $v_i = 2u_i - u_0$ .



Introduction to LS/LS+

Introduction	LS	LS+	Separation Oracles
	0000	○○○	000
	000000	○○○○○●○	00

#### Semi Definite Programming

### Vertex Cover

Let 
$$v_i = 2u_i - u_0$$
.

$$||v_i||^2 = v_i \cdot v_i$$
  
=  $4u_i \cdot u_i - 4u_i \cdot u_0 + u_0 \cdot u_0$   
=  $||u_0||^2 = 1$ 

Introduction to LS/LS+

University of Washington

э

イロト イポト イヨト イヨト

Introduction	LS	LS+	Separation Oracles
	0000	000 000000●	000 00

Semi Definite Programming

Vertex Cover

SDP

$$\min\sum_{i\in V}\frac{1+v_0\cdot v_i}{2}$$

$$egin{aligned} & (v_0-v_i)\cdot(v_0-v_j)=0 & ext{ for all } & ij\in E \ & (v_0-v_i)\cdot(v_0-v_j)\geq 0 & ext{ for all } & i\in V \ & ||v_i||=1 & ext{ for all } & i\in V. \end{aligned}$$

University of Washington

э

イロト イポト イヨト イヨト

Introduction	LS	LS+	Separation Oracles
	0000	000	●○○
	000000	0000000	○○
Introduction			

# Separation Oracles

#### Definition strong separation oracle - given a point x, returns that $x \in K$ , or gives a separating hyperplane.

- A - E - N

< 🗇 🕨 < 🖻 🕨

# Ellipsoid Method for N(K)



コントロント 山田 シュール ション しょうしょう

Introduction to LS/LS+

Introduction	LS	LS+	Separation Oracles
	0000	000 0000000	

# Ellipsoid Method for N(K)



< □ > < □ > - ∢ ⊒ →

Introduction to LS/LS+

# Ellipsoid Method for N(K)

LS

LS+

000 0000000



ロマ・山マ・山マ・山、

Introduction to LS/LS+

Introduction	LS	LS+	Separation Oracles
	0000 000000	000 0000000	000

# Ellipsoid Method for N(K)



Volume of ellipse decreases by  $2^{\frac{1}{2n+2}}$ .

Introduction to LS/LS+

# Ellipsoid Method for $N_+(K)$ ?

 $N_+(K)$  not necessarily polyhedral.



Introduction to LS/LS+

# Ellipsoid Method for $N_+(K)$ ?

#### $N_+(K)$ not necessarily polyhedral.

#### Definition

weak separation oracle - given a point x and  $\epsilon > 0$ , returns that  $dist(x, K) < \epsilon$  or returns a separating hyperplane h such that  $dist(h, K^{\perp}) < \epsilon$ .

・ 同 ト ・ ヨ ト ・ ヨ ト

Introduction	LS	LS+	Separation Oracles
	0000	000	०००
	000000	0000000	●०
Separation Problem			

#### Theorem

Given a weak separation oracle for K, we can solve the weak separation problem for  $N_+(K)$  in polynomial time.



Introduction	LS	LS+	Separation Oracles
	0000	000	०००
	000000	0000000	●०
Separation Problem			

#### Theorem

Given a weak separation oracle for K, we can solve the weak separation problem for  $N_+(K)$  in polynomial time.

#### Proof

Can solve for space of Y matrices.

#### Theorem

Given a weak separation oracle for K, we can solve the weak separation problem for  $N_+(K)$  in polynomial time.

#### Proof

Can solve for space of Y matrices.

First row, diagonal, and symmetry conditions can be checked trivially.

・ 同 ト ・ ヨ ト ・ ヨ ト

#### Theorem

Given a weak separation oracle for K, we can solve the weak separation problem for  $N_+(K)$  in polynomial time.

#### Proof

Can solve for space of Y matrices.

First row, diagonal, and symmetry conditions can be checked trivially.

Positive semi-definitiveness checked by Gaussian elimination.

▲ □ ▶ ▲ □ ▶ ▲ □ ▶

### Separation Problem

#### Theorem

Given a weak separation oracle for K, we can solve the weak separation problem for  $N_+(K)$  in polynomial time.

#### Proof

Can solve for space of Y matrices.

First row, diagonal, and symmetry conditions can be checked trivially.

Positive semi-definitiveness checked by Gaussian elimination. Both row conditions given by separation oracle.

< 同 > < 回 > < 回 >

Introduction	LS	LS+	Separation Oracles
	0000	000 0000000	000

#### ▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

Introduction to LS/LS+