CSE 326 Data Structures

Dave Bacon

Dijkstra, Dijkstra, Dijkstra
Minimal Spanning Trees
Fun With Tablets

Logisitics

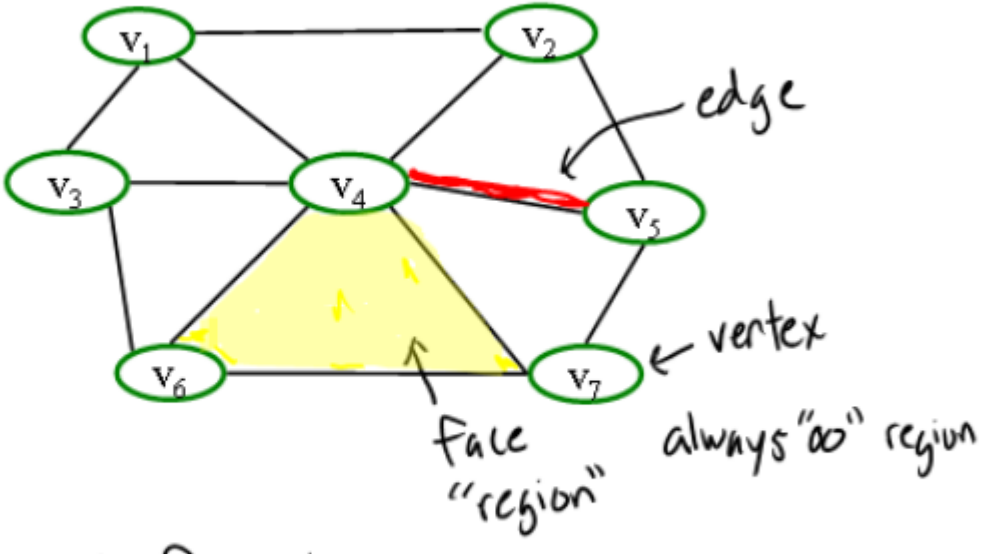
- Homework 7 will be due Wed, March 7

- Read Chapter 9 of Weiss

- Monday: Huffman Coding, Class Evaluations
- Wednesday: Final Review
- Friday: Games and NP completeness

- **Final:** Thursday March 15, 8:30-10:20 MGH 231

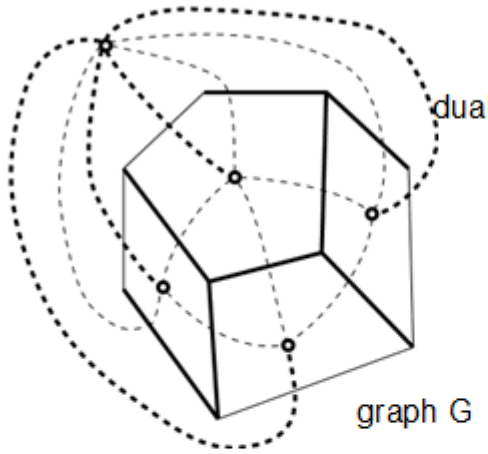
Euler's Formula (Planar Graphs)



Euler's formula:

$$|V| - |E| + |F| = 2$$

Euler's Formula (Planar Graphs)



Any cycle in G disconnects G* and vice versa

A acyclic subgraph of G does not disconnect G*

Pick spanning tree of G, call it S. (G-S) is G with S's edges removed.

Edges of (G-S) form a subgraph of dual graph G*, call it R

This subgraph is connected (since G is acyclic) and acyclic (since G is connected)

Number edges in S = |V|-1. Number of edges in R = |F|-1. But this should equal the total number of edges!

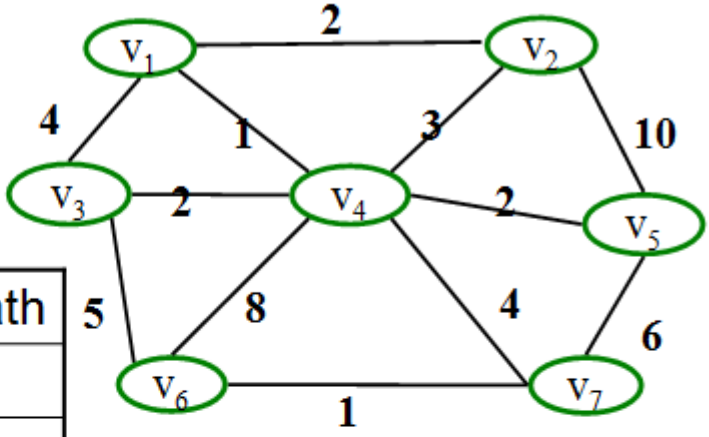
Dijkstra's Alg: Implementation Optimized

Initialize the cost of each node to ∞

Initialize the cost of the source to 0

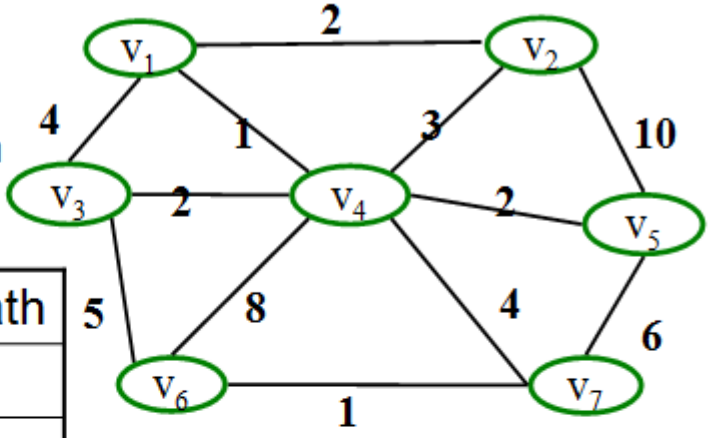
While there are unknown nodes left in the graph

- Select the unknown node *b* with the lowest cost
- Mark *b* as known
- For each node *a* adjacent to *b*
  - *a*'s cost = min(*a*'s old cost, *b*'s cost + cost of (*b*, *a*))



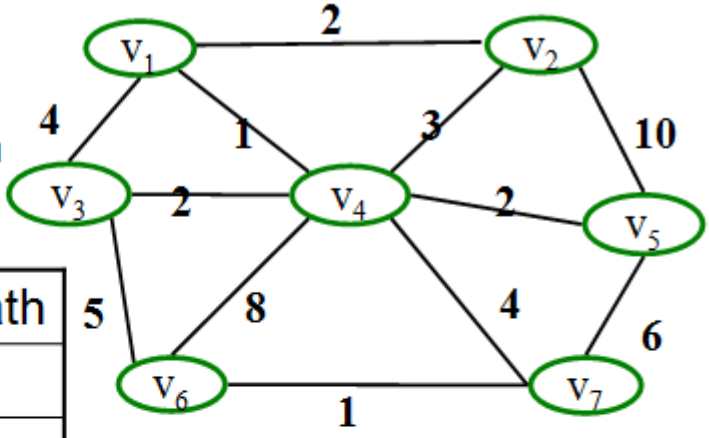
V	Kwn	Distance	path
v1	F	0	0
v2	F	∞	0
v3	F	∞	0
v4	F	∞	0
v5	F	∞	0
v6	F	∞	0
v7	F	∞	0

After v_1 is declared known



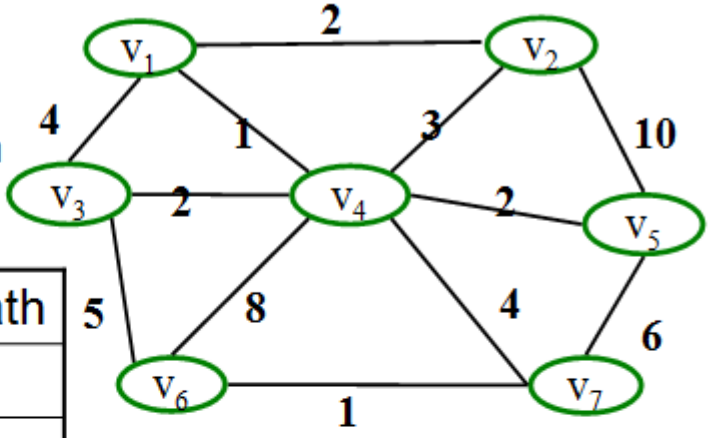
V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

After v_4 is declared known



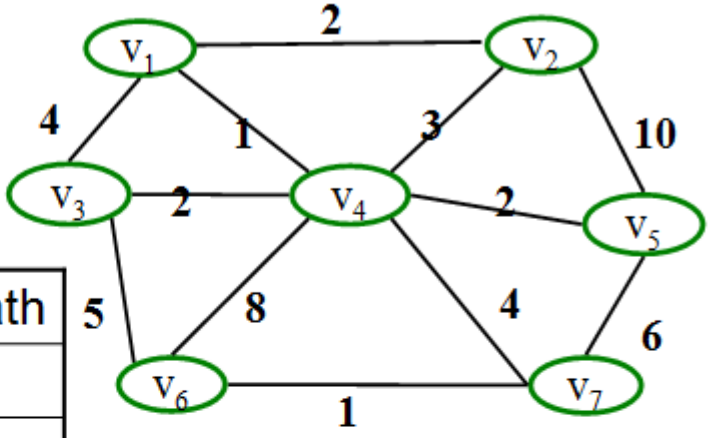
V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

After v_2 is declared known



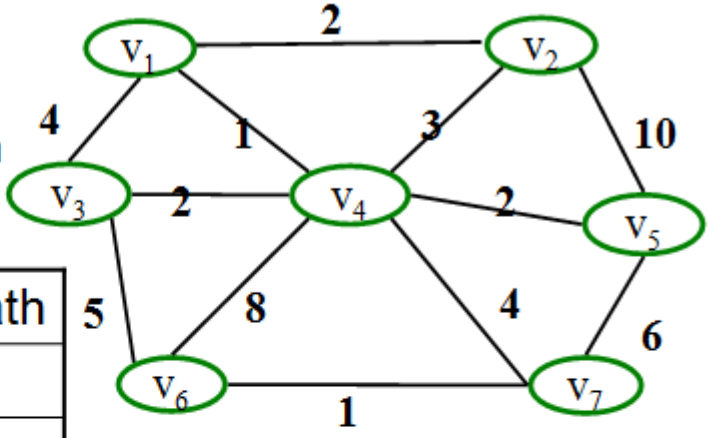
V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

After v_5 and v_3 is declared known



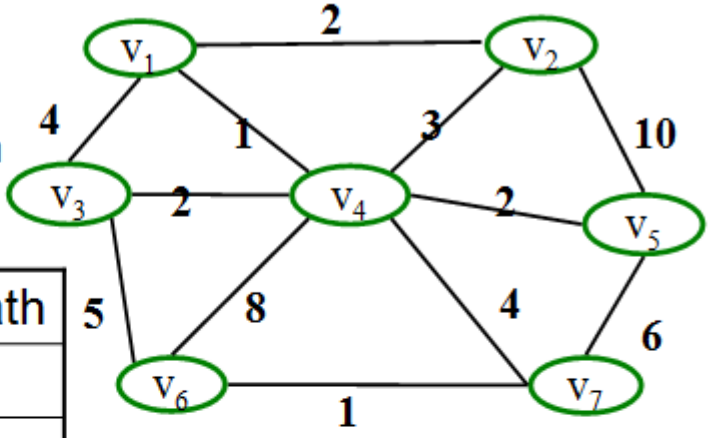
V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

After v_7 is declared known



V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

After v_6 is declared known



V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

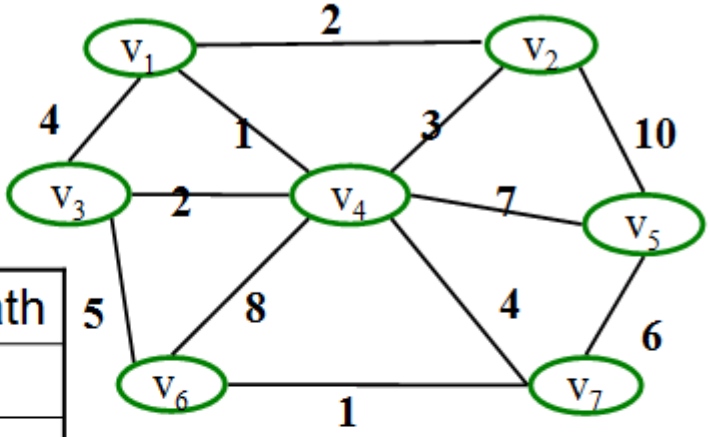
Prim's Algorithm for MST

A *node-based* greedy algorithm

Builds MST by greedily adding nodes

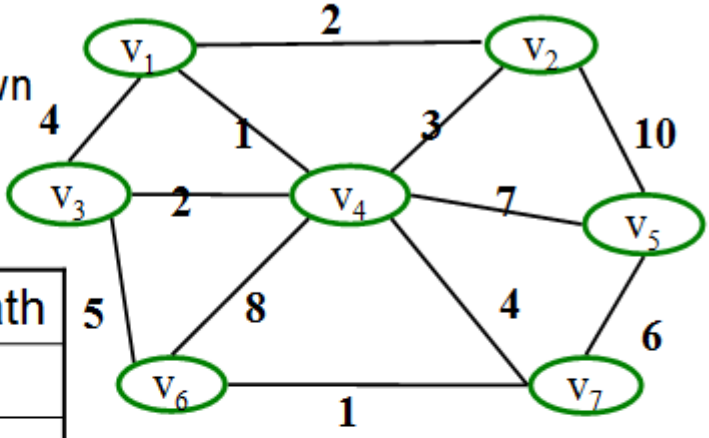
1. Select a node to be the "root"
   - mark it as known
   - Update cost of all its neighbors
2. While there are unknown nodes left in the graph
   a. Select an unknown node *b* with the smallest cost from some *known* node *a*
   b. Mark *b* as known
   c. Add (*a*, *b*) to MST
   d. Update cost of all nodes adjacent to *b*

Note: cost from some a, ***not*** from root



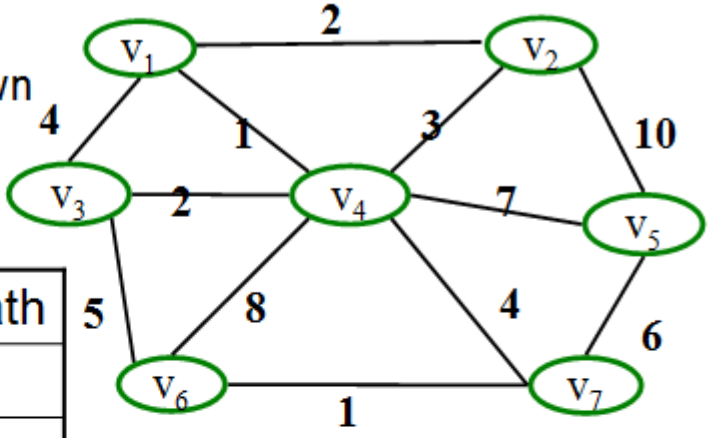
V	Kwn	Distance	path
v1	F	0	0
v2	F	∞	0
v3	F	∞	0
v4	F	∞	0
v5	F	∞	0
v6	F	∞	0
v7	F	∞	0

After v_1 is declared known



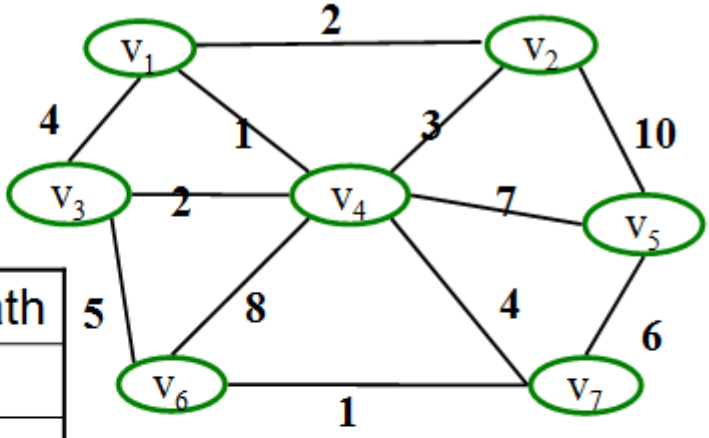
V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

After v_4 is declared known



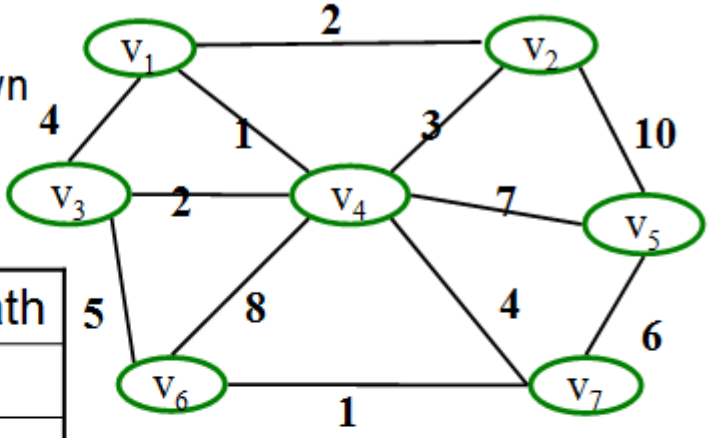
V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

After v_2 and v_3 are declared known



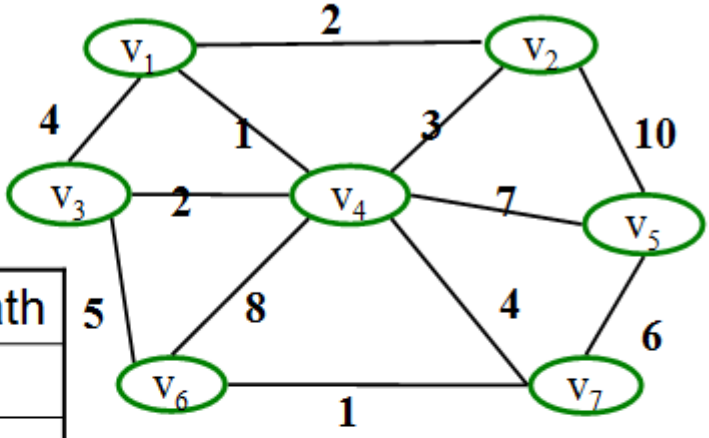
V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

After v_7 is declared known



V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

After v_6 and v_5 are declared known



V	Kwn	Distance	path
v1			
v2			
v3			
v4			
v5			
v6			
v7			

Prim's Algorithm Analysis

Running time:

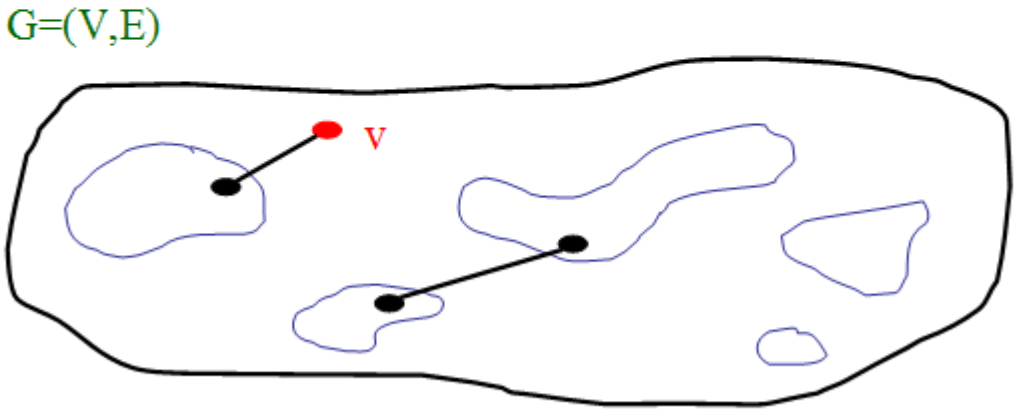
Same as Dijkstra's: $O(|E| \log |V|)$

Correctness:

Proof is similar to Dijkstra's

Kruskal's MST Algorithm

Idea: Grow a forest out of edges that do not create a cycle. Pick an ***edge with the smallest weight.***



Kruskal's Algorithm for MST

An *edge-based* greedy algorithm

Builds MST by greedily adding edges

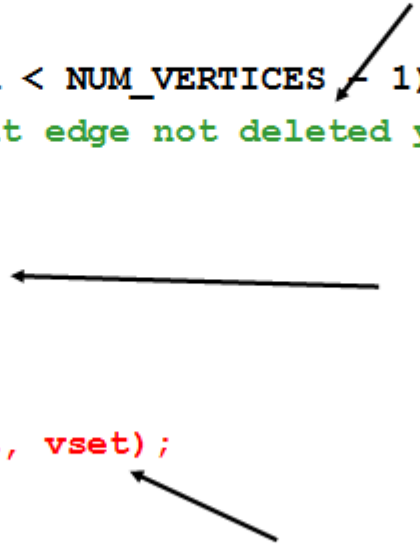
1. Initialize with
   - empty MST
   - all vertices marked unconnected
   - all edges unmarked
2. While there are still unmarked edges
   a. Pick the lowest cost edge `(u,v)` and mark it
   b. If `u` and `v` are not already connected, add `(u,v)` to the MST and mark `u` and `v` as connected to each other

Doesn't it sound familiar?

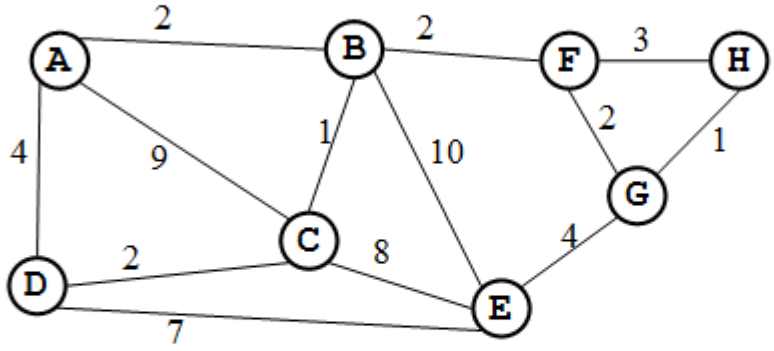
Kruskal code

```
void Graph::kruskal(){
  int edgesAccepted = 0;
  DisjSet s(NUM_VERTICES);

  while (edgesAccepted < NUM_VERTICES - 1){
    e = smallest weight edge not deleted yet;
    // edge e = (u, v)
    uset = s.find(u);
    vset = s.find(v);
    if (uset != vset){
      edgesAccepted++;
      s.unionSets(uset, vset);
    }
  }
}
```



Find MST using Kruskal's



Kruskal's Algorithm: Correctness

It clearly generates a spanning tree. Call it T_K.

Suppose T_K is *not* minimum:

- Pick another spanning tree T_{min} with *lower cost* than T_K
- Pick the smallest edge $e_1=(u,v)$ in T_K that is **not** in T_{min}
- T_{min} already has a path p in T_{min} from u to v
  - $\Rightarrow$ Adding e_1 to T_{min} will create a cycle in T_{min}
- Pick an edge e_2 in p that Kruskal's algorithm considered *after* adding e_1 (must exist: u and v unconnected when e_1 considered)
  - $\Rightarrow$ cost(e_2) $\geq$ cost(e_1)
  - $\Rightarrow$ can replace e_2 with e_1 in T_{min} without increasing cost!
- Keep doing this until T_{min} is identical to T_K
  - $\Rightarrow$ T_K must also be minimal – contradiction!