Intro to Digital Design

Combinational Logic

Instructor: Justin Hsia

Teaching Assistants:

Emilio Alcantara	Eujean Lee
Naoto Uemura	Pedro Amarante
Wen Li	

Introducing Your Course Staff

- Your Instructor: just call me Justin
  - CSE Associate Teaching Professor
  - From California (UC Berkeley and the Bay Area)
  - Raising a toddler takes up energy and dictates my schedule
- TAs:


Emilio


Eujean


Naoto


Pedro


Wen

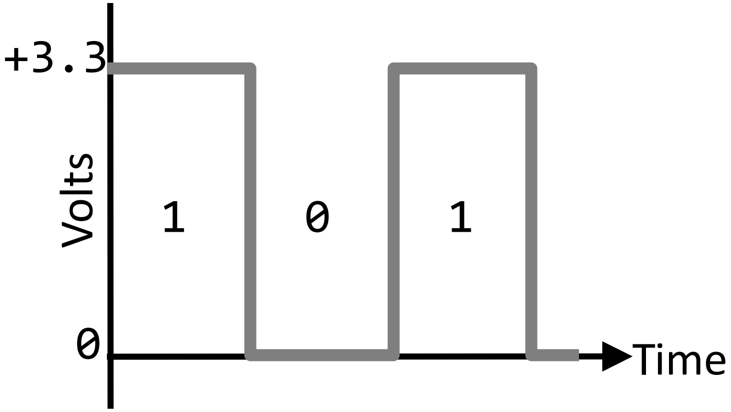
  - Available in labs, office hours, and on Ed discussion
  - An invaluable source of information and help
- Get to know us – we are here to help you succeed!

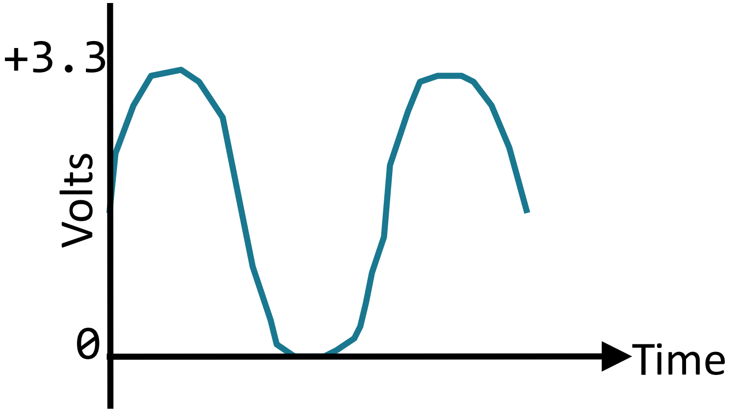
Course Motivation

- Electronics an increasing part of our lives
  - Computers & phones
  - Vehicles (cars, planes)
  - Robots
  - Portable & household electronics

- An *introduction* to digital logic design
  - **Lecture:** How to think about hardware, basic higher-level circuit design techniques – preparation for EE/CSE469
  - **Lab:** Hands-on FPGA programming using Verilog – preparation for EE/CSE371

Digital vs. Analog





Digital:

Discrete set of possible values

Binary (2 values):

On, 3.3 V, high, TRUE, "1"
Off, 0 V, low, FALSE, "0"

Analog:

Values vary over a continuous range

Digital vs. Analog Systems

- Digital systems are more reliable and less error-prone
  - Slight errors can cascade in Analog system
  - Digital systems reject a significant amount of error; easy to cascade
- Computers use digital circuits internally
  - CPU, memory, I/O
- Interface circuits with “real world” often analog
  - Sensors & actuators

This course is about logic design,
not system design (processor architecture),
and not circuit design (transistor level)

Digital Design: What's It All About?

- Come up with an implementation using a set of primitives given a functional description and constraints
- Digital design is in some ways more art than a science
    - The creative spirit is in combining primitive elements and other components in new ways to achieve a desired function
- However, unlike art, we have objective measures of a design (*i.e.*, constraints):
    - Performance
    - Power
    - Cost

Digital Design: What’s It All About?

- How do we learn how to do this?
  - Learn about the primitives and how to use them
  - Learn about design representations
  - Learn formal methods and tools to manipulate representations
  - Look at design examples
  - Use trial and error – CAD tools and prototyping (practice!)

Lecture

apply

Lab

Lecture Outline

- **Course Logistics**
- Combinational Logic Review
- Combinational Logic in the Lab

Bookmarks

- Website: https://courses.cs.washington.edu/courses/cse369/24sp/
  - Schedule (lecture slides, lab specs), weekly calendar, other useful documents
- Ed Discussion: https://edstem.org/us/courses/56771/
  - Announcements made here
  - Ask and answer questions – staff will monitor and contribute
- Gradescope: https://www.gradescope.com/courses/746339/
  - Lab submissions, Quiz grades, regrade requests
- Canvas: https://canvas.uw.edu/courses/1718545/
  - Grade book, Zoom links, lecture recordings

Grading

- Labs (66%)
  - 6 regular labs – 1 week each
    - Labs 3-4: 60 points each, Labs 1&2, 5-7: 100 points each
  - 1 “final project” – 2 weeks
    - Lab 8 Check-In: 10 points, Lab 8: 150 points
- 3 Quizzes (no final exam)
  - Quiz 1 (10%): 20 min in class on April 23
  - Quiz 2 (10%): 30 min in class on May 14
  - Quiz 3 (14%): 60 min in class on May 28
- This class uses a straight scale (≥ 95% → 4.0)
  - Extra credit points count the same as regular points

Labs

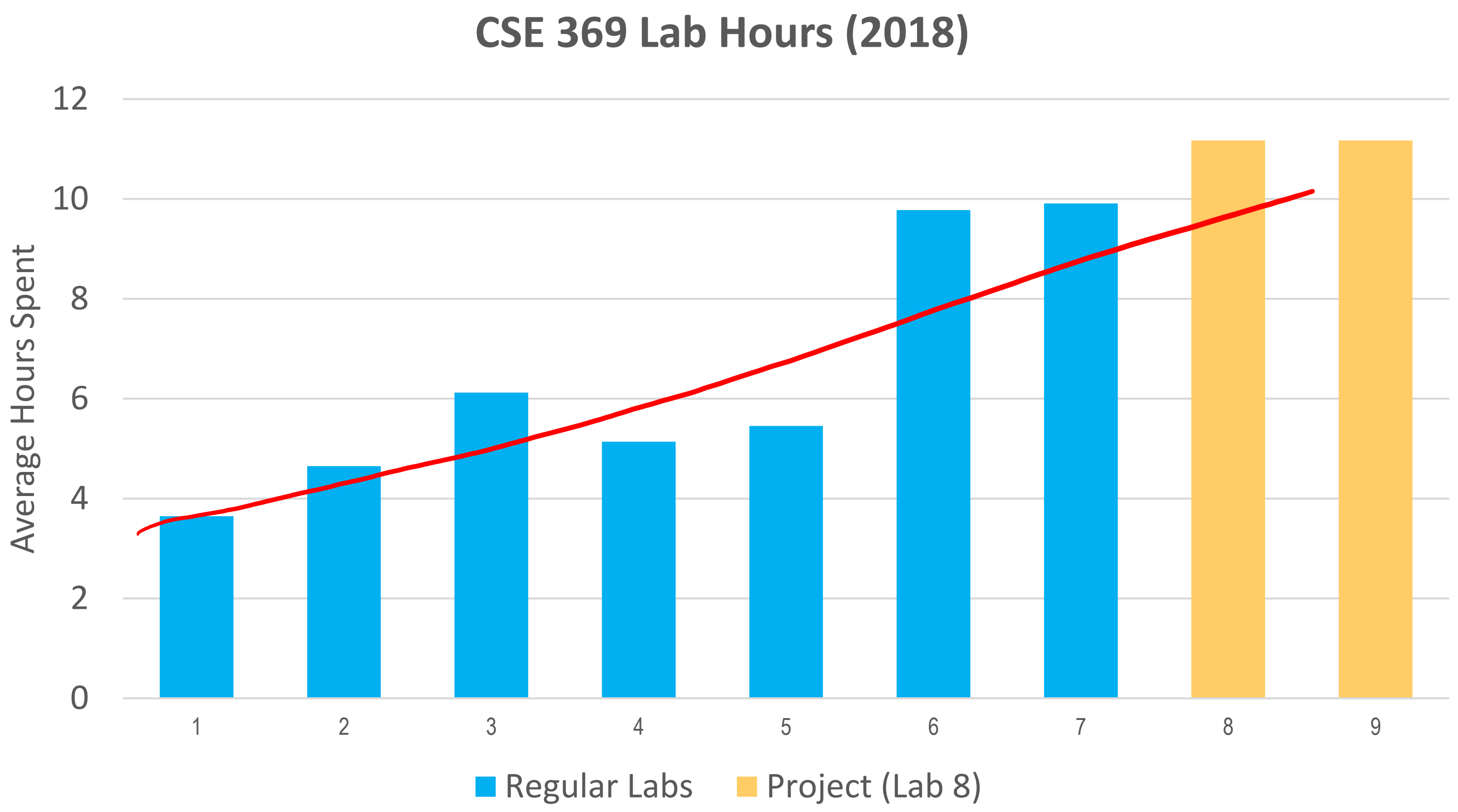
- Lab Hours: Wed & Thu 2:30-5:20 pm (CSE 003)
- Each student will get a lab kit for the quarter
  - Lab kit picked up from CSE 003 during labs/OHs this week
  - Install software on laptop (Windows or VM)
- Labs are combination of report + demo
  - Submit via Gradescope Wednesdays before 2:30 pm
  - 10-minute demos done in lab sections (sign-up process)
- Late penalties:
  - No lab report can be submitted more than two days late
  - 5 late day tokens to prevent penalties, 10%/day after that
  - No penalties on lab demos, but must be done by EOD Friday

Collaboration Policy

- Labs and project are to be completed *individually*
  - Goal is to give every student the hands-on experience
  - Violation of these rules is grounds for failing the class
- **OK:**
  - Discussing lectures and/or readings, studying together
  - *High-level* discussion of general approaches
  - Help with debugging, tools peculiarities, etc.
- **Not OK:**
  - Developing a lab together
  - Giving away solutions or having someone else do your lab for you

Course Workload

- The workload (3 credits) ramps up significantly towards the end of the quarter:

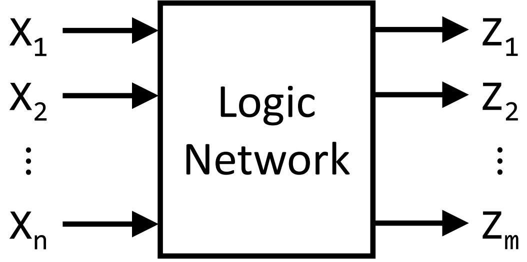


Lecture Outline

- Course Logistics
- **Combinational Logic Review**
- Combinational Logic in the Lab

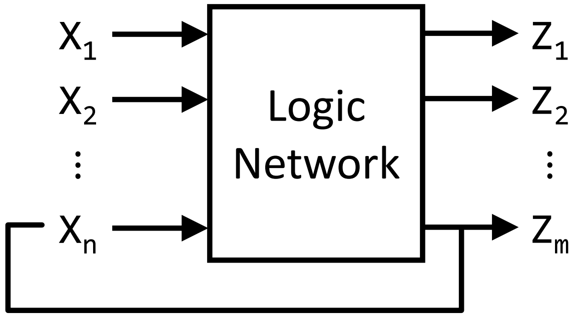
Combinational vs. Sequential Logic

Combinational Logic (CL)



- Network of logic gates without feedback
- Outputs are functions only of inputs

Sequential Logic (SL)



- The presence of feedback introduces the notion of “state”
- Circuits that can “remember” or store information

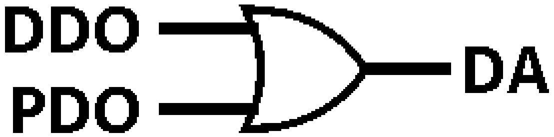
Representations of Combinational Logic

- 1 Text Description
- 2 Circuit Description
  - ~~Transistors~~ Not covered in 369
  - Logic Gates
- 3 Truth Table
- 4 Boolean Expression

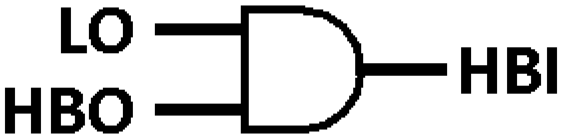
- *All are equivalent!*

Example: Simple Car Electronics

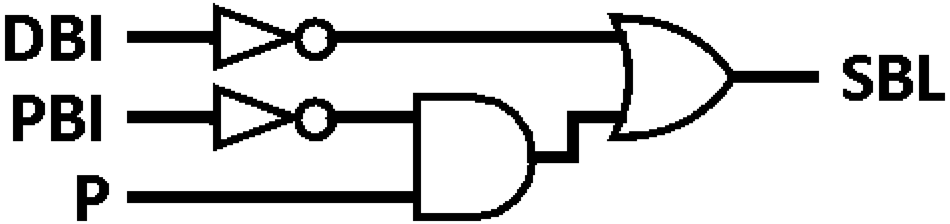
- Door Ajar (DriverDoorOpen, PassengerDoorOpen)
  - $\text{DA} = \text{DDO} + \text{PDO}$



- High Beam Indicator (LightsOn, HighBeamOn)
  - $\text{HBI} = \text{LO} \cdot \text{HBO}$

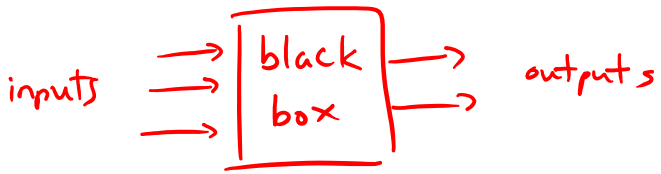


- Seat Belt Light (DriverBeltIn, PassengerBeltIn, Passenger)
  - $\text{SBL} = \overline{\text{DBI}} + (\text{P} \cdot \overline{\text{PBI}})$



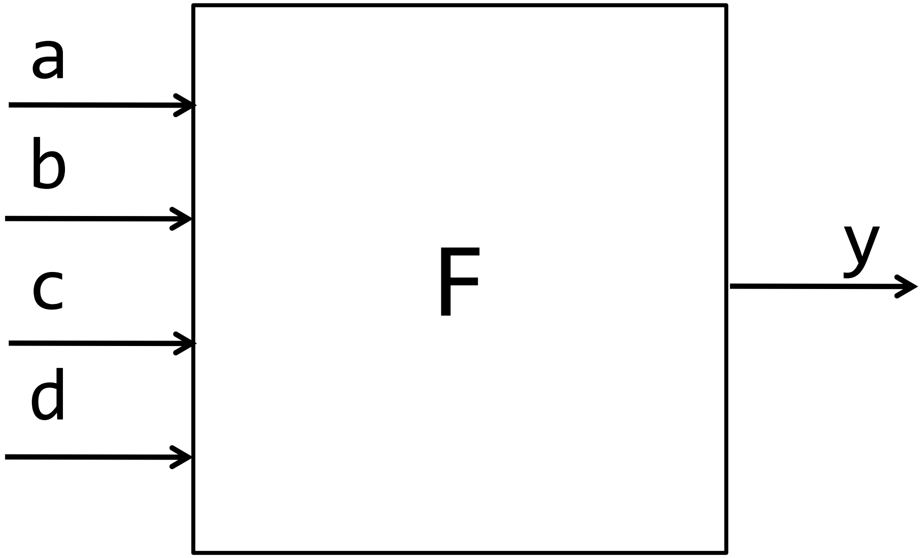
Truth Tables

- Table that relates the inputs to a combinational logic (CL) circuit to its output
  - Output *only* depends on current inputs
  - Use abstraction of 0/1 instead of high/low voltage
  - Shows output for every possible combination of inputs (“black box” approach)



- How big is the table?
  - 0 or 1 for each of N inputs 2^N rows
  - Each output is a separate function of inputs, so don’t need to add rows for additional outputs

CL General Form



If we have N inputs, how many distinct functions F do we have?

2^N output "positions", each being 0/1

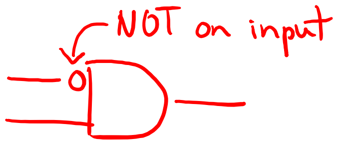
so $\boxed{2^\wedge(2^\wedge N) \text{ possible functions}}$

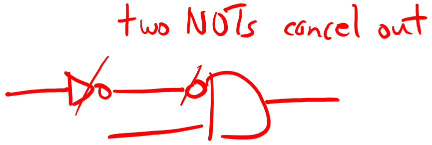
0/1

a	b	c	d	y
0	0	0	0	F(0,0,0,0)
0	0	0	1	F(0,0,0,1)
0	0	1	0	F(0,0,1,0)
0	0	1	1	F(0,0,1,1)
0	1	0	0	F(0,1,0,0)
0	1	0	1	F(0,1,0,1)
0	1	1	0	F(0,1,1,0)
1	1	1	1	F(0,1,1,1)
1	0	0	0	F(1,0,0,0)
1	0	0	1	F(1,0,0,1)
1	0	1	0	F(1,0,1,0)
1	0	1	1	F(1,0,1,1)
1	1	0	0	F(1,1,0,0)
1	1	0	1	F(1,1,0,1)
1	1	1	0	F(1,1,1,0)
1	1	1	1	F(1,1,1,1)

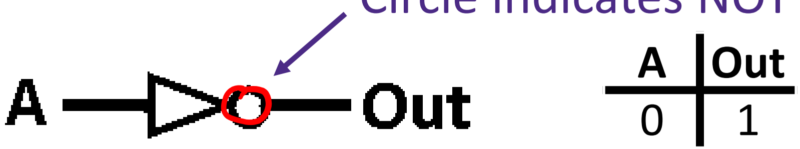
2^N rows (16)

Logic Gate Names and Symbols





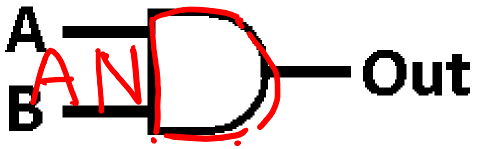
- **NOT**



Circle indicates NOT

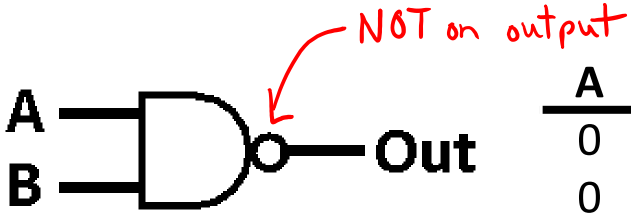
A	Out
0	1
1	0

- **AND**



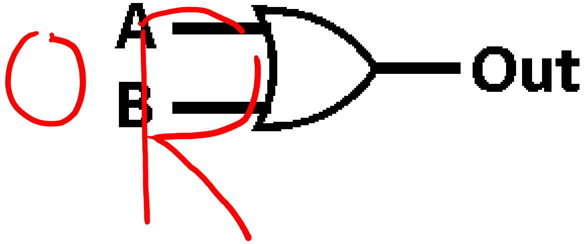
A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

- **NAND**



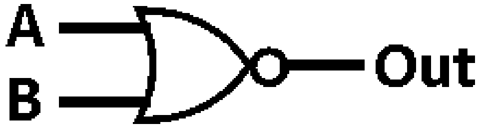
A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

- **OR**



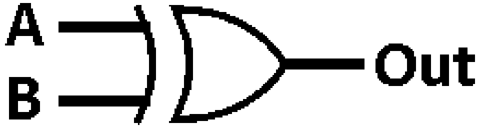
A	B	Out
0	0	0
0	1	1
1	0	1
1	1	1

- **NOR**



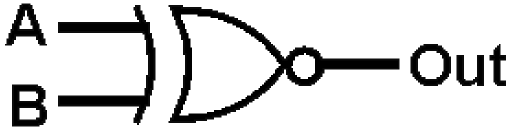
A	B	Out
0	0	1
0	1	0
1	0	0
1	1	0

- **XOR**



A	B	Out
0	0	0
0	1	1
1	0	1
1	1	0

- **XNOR**



A	B	Out
0	0	1
0	1	0
1	0	0
1	1	1

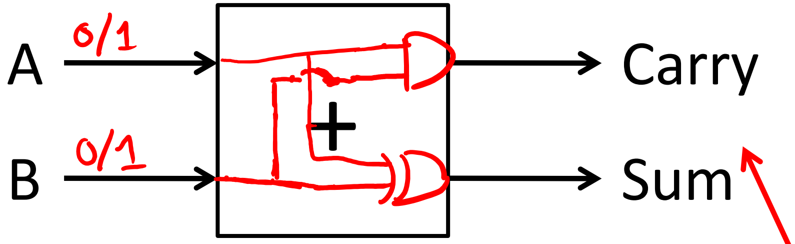
More Complicated Truth Tables

3-Input Majority

How many rows? $2^3 = 8$ rows

A	B	C	Out
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

1-bit Adder



2 separate functions (columns)

A	B	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$A \cdot B$ $A \oplus B$

Boolean Algebra

- Represent inputs and outputs as variables
  - Each variable can only take on the value 0 or 1

$\neg$
- Overbar is NOT: “logical complement”
  - If A is 0, then $\overline{\mathrm{A}}$ is 1 and vice-versa

$\vee$
- Plus $(+)$ is 2-input OR: “logical sum”

$\wedge$
- Product $(\cdot)$ is 2-input AND: “logical product”

- All other gates and logical expressions can be built from combinations of these
  - *e.g.*, $\mathrm{A\ XOR\ B} = \mathrm{A} \oplus \mathrm{B} = \overline{\mathrm{A}}\mathrm{B} + \overline{\mathrm{B}}\mathrm{A}$

Truth Table to Boolean Expression

a	b	c	row
0	0	0	1
0	1	1	2
1	0	1	3
1	1	0	4

- Read off of table
  - For 1, write variable name
  - For 0, write complement of variable
- *Sum of Products (SoP)*
  - Take rows with 1's in output column, sum products of inputs

    sets to 1 when input combination matches
  - $C = \underset{\text{row 2}}{\overline{A}B} + \underset{\text{row3}}{\overline{B}A}$
- *Product of Sums (PoS)*
  - Take rows with 0's in output column, product the sum of the *complements of the inputs*

    sets to 0 when input combination matched
  - $C = \underset{\text{row 1}}{(A + B)} \cdot \underset{\text{row 4}}{(\overline{A} + \overline{B})}$

We can show that these are equivalent!

Basic Boolean Identities

- $X + 0 = X$
- $X + 1 = 1$
- $X + X = X$
- $X + \overline{X} = 1$
- $\overline{\overline{X}} = X$

- $X \cdot 1 = X$
- $X \cdot 0 = 0$
- $X \cdot X = X$
- $X \cdot \overline{X} = 0$

Basic Boolean Algebra Laws

- **Commutative Law:**

  $X + Y = Y + X$ $X \cdot Y = Y \cdot X$

- **Associative Law:**

  $X+(Y+Z) = (X+Y)+Z$ $X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$

- **Distributive Law:**

  $X{\cdot}(Y+Z) = X{\cdot}Y+X{\cdot}Z$ $X+YZ = (X+Y){\cdot}(X+Z)$

Advanced Laws (Absorption)

- $X + XY = X$
- $XY + X\overline{Y} = X$
- $\underline{X + \boxed{\overline{X}Y}} = X + Y$
- $X(X + Y) = X$
- $(X + Y)(X + \overline{Y}) = X$
- $X(\overline{X} + Y) = XY$

$$X + \overline{X}Y = X \cdot 1 + \overline{X}Y = X \cdot (1 + Y) + \overline{X}Y$$
$$= X + XY + \overline{X}Y$$
$$= X + Y \quad ☑$$

$$Y = \overline{X}Y + XY = Y(\overline{X} + X)$$

Practice Problem

- Boolean Function: $F = \underline{\overline{X}YZ} + \underline{\underline{XZ}}$

Truth Table:

	X	Y	Z	F
	0	0	0	0
	0	0	1	0
	0	1	0	0
→	0	1	1	1
	1	0	0	0
⇒	1	0	1	1
	1	1	0	0
⇒	1	1	1	1

Simplification:

$$= \overline{X}YZ + X\overline{Y}Z + XYZ$$

$$= \overline{X}YZ + XZ$$

distribution

$$= (\overline{X}Y + X)Z$$

absorption

$$= (X + Y)Z$$

distribute

$$= XZ + YZ$$

2 gates (1 OR, 1 AND)

Which of these is “simpler”?

3 gates (2 AND, 1 OR)

Technology Break

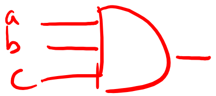
Lecture Outline

- Course Logistics
- Combinational Logic Review
- **Combinational Logic in the Lab**

Why Is This Useful?

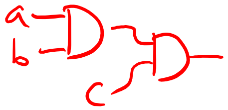
- Logic minimization: reduce complexity at gate level
  - Allows us to build smaller and faster hardware
  - Care about both # of gates, # of literals (gate inputs), # of gate levels, and types of logic gates

literals:



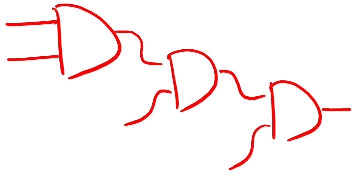
1, 3-input gate

⟺



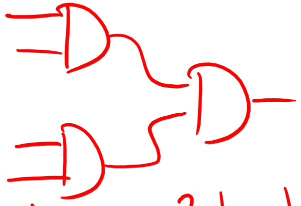
2, 2-input gates

levels:



3 gates in 3 levels

vs.



3 gates in 2 levels
(faster)

types: speed of NOT gate vs. AND gate?

Why Is This Useful?

- Logic minimization: reduce complexity at gate level
  - Allows us to build smaller and faster hardware
  - Care about both # of gates, # of literals (gate inputs), # of gate levels, and types of logic gates
- Faster hardware?
  - Fewer inputs implies faster gates in some technologies
  - Fan-ins (# of gate inputs) are limited in some technologies
  - Fewer levels of gates implies reduced signal propagation delays
  - # of gates (or gate packages) influences manufacturing costs
  - Simpler Boolean expressions → smaller transistor networks → smaller circuit delays → faster hardware

Are Logic Gates Created Equal?

- No!

2-Input Gate Type	# of CMOS transistors
NOT	2
AND	6
OR	6
NAND	4
NOR	4
XOR	8
XNOR	8

← simplest, but not too useful

useful, and simpler than alternatives

- Can recreate all other gates using only NAND or only NOR gates
  - Called "universal" gates
  - *e.g.*, A NAND A = $\overline{A}$, B NOR B = $\overline{B}$
  - DeMorgan's Law helps us here!

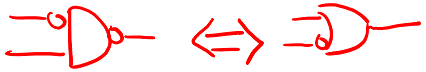
x	y	NAND	
0	0	1	0→1
0	1	1	
1	0	1	
1	1	0	1→0

DeMorgan's Law

- $\overline{X+Y} = \overline{X}\cdot\overline{Y}$
- $\overline{X\cdot Y} = \overline{X}+\overline{Y}$

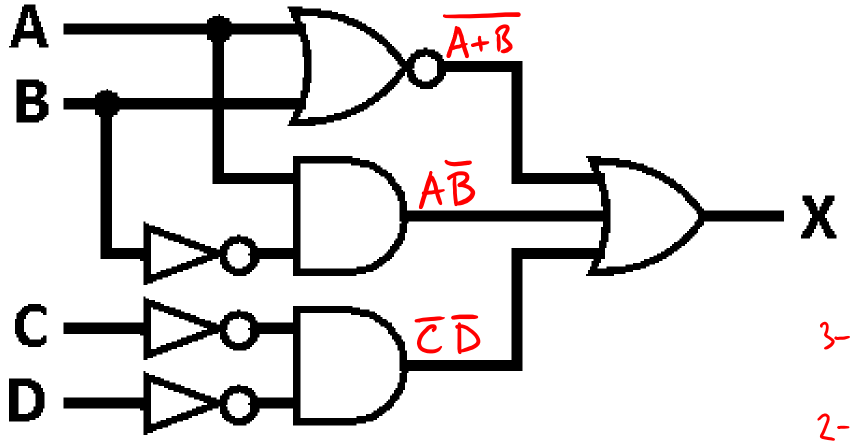
				NOR		NAND	
X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X+Y}$	$\overline{X}\cdot\overline{Y}$	$\overline{X\cdot Y}$	$\overline{X}+\overline{Y}$
0	0	1	1	1	1	1	1
0	1	1	0	0	0	1	1
1	0	0	1	0	0	1	1
1	1	0	0	0	0	0	0

- In Boolean Algebra, converts between AND-OR and OR-AND expressions
  - $Z = \overline{A}\,\overline{B}C + \overline{A}BC + A\overline{B}C$
  - $\overline{Z} = (A+B+\overline{C})\cdot(A+\overline{B}+\overline{C})\cdot(\overline{A}+B+\overline{C})$
- At gate level, can convert from AND/OR to NAND/NOR gates
  - "Flip" all input/output bubbles and "switch" gate



DeMorgan’s Law Practice Problem

- Simplify the following diagram:



$$X = \overline{A+B} + A\overline{B} + \overline{C}\,\overline{D}$$

DeMorgan's

$$X = \overline{A}\,\overline{B} + A\overline{B} + \overline{C}\,\overline{D}$$

5 gates $X = \overline{B} + \overline{C}\,\overline{D}$

DeMorgan's

3-4 gates $X = \overline{B} + \overline{C + D}$

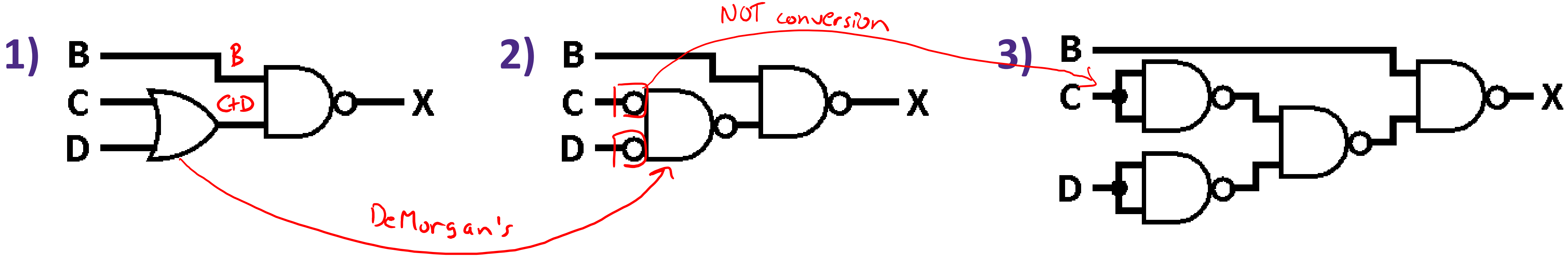
DeMorgan's

2-3 gates $X = \overline{B(C + D)}$

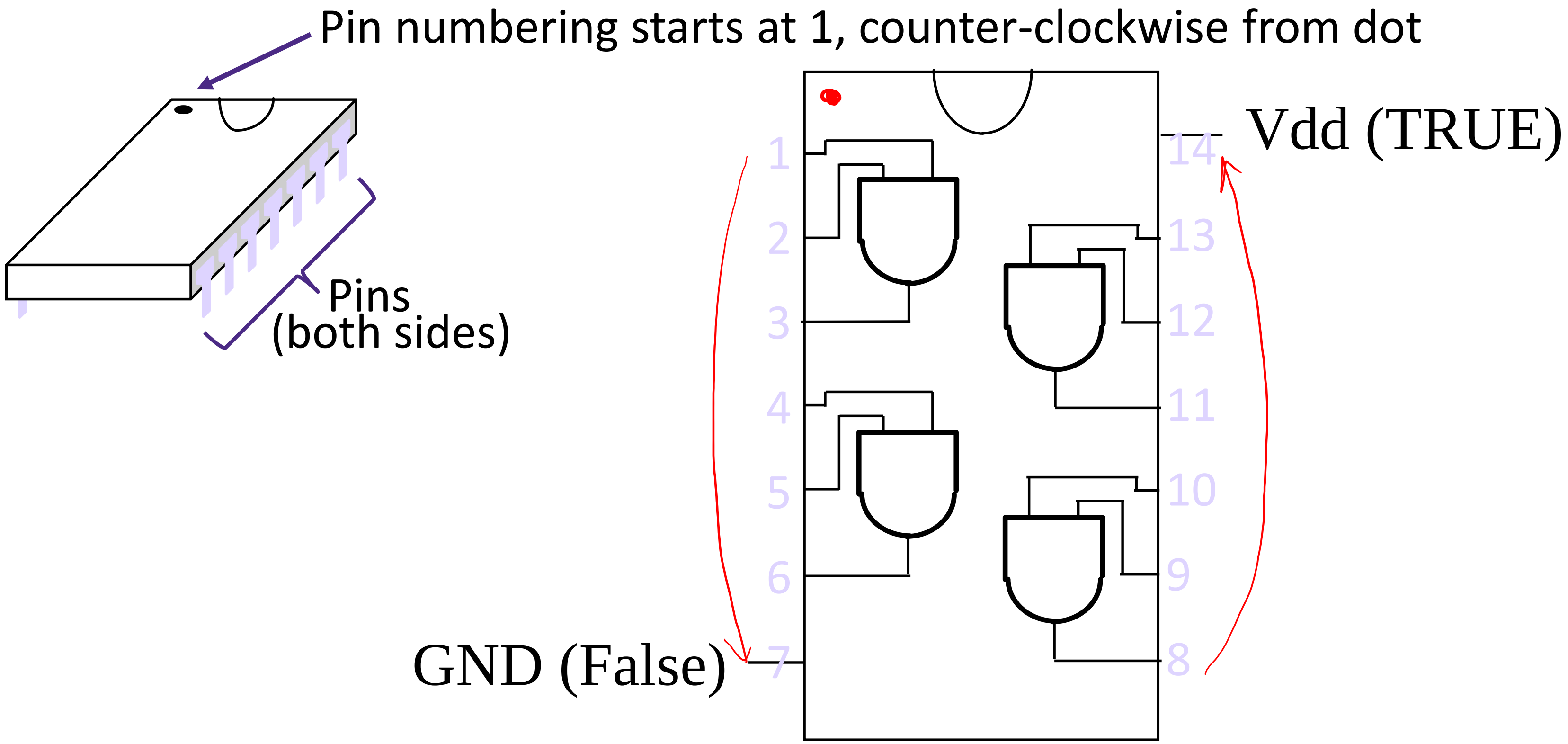
let $E = C+D$, so $X = \overline{B} + \overline{E}$

$X = \overline{BE}$

- Then implement with only NAND gates:



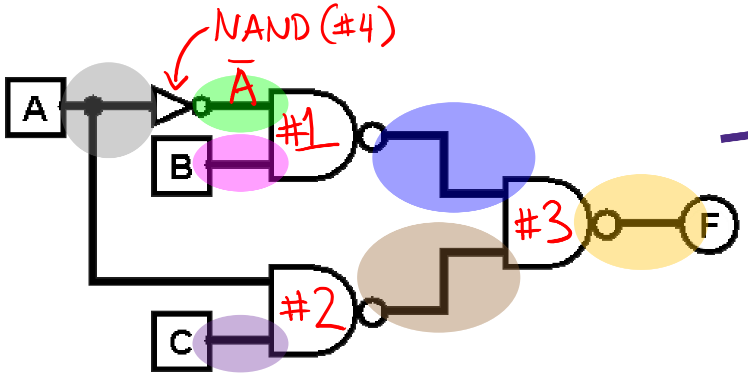
Transistor-Transistor Logic (TTL) Packages


Pin numbering starts at 1, counter-clockwise from dot

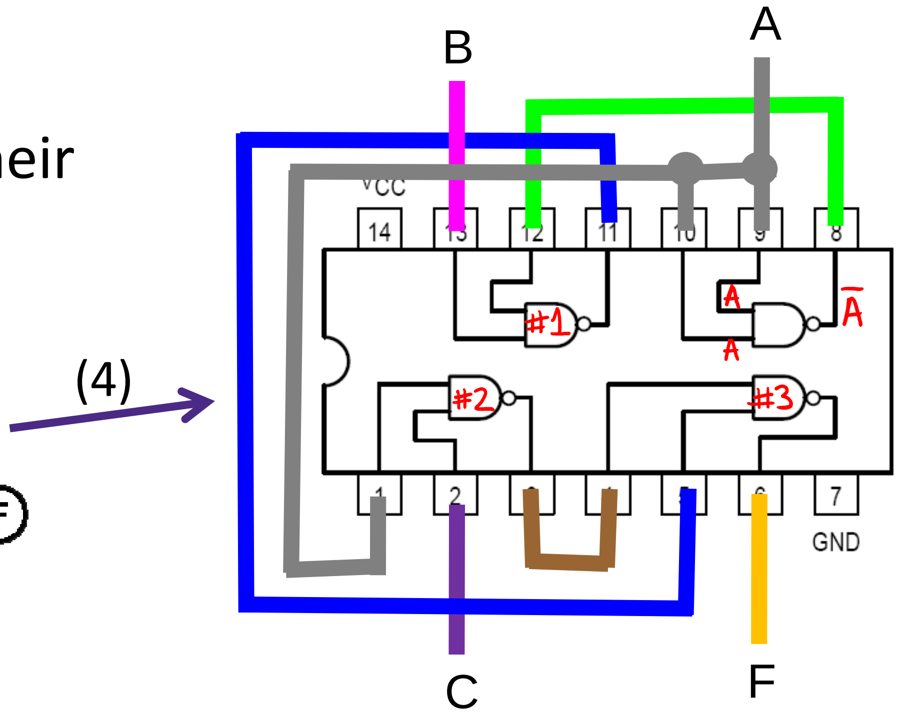
- Diagrams like these and other useful/helpful information can be found on part data sheets
  - It's really useful to learn how to read these

Mapping truth tables to logic gates

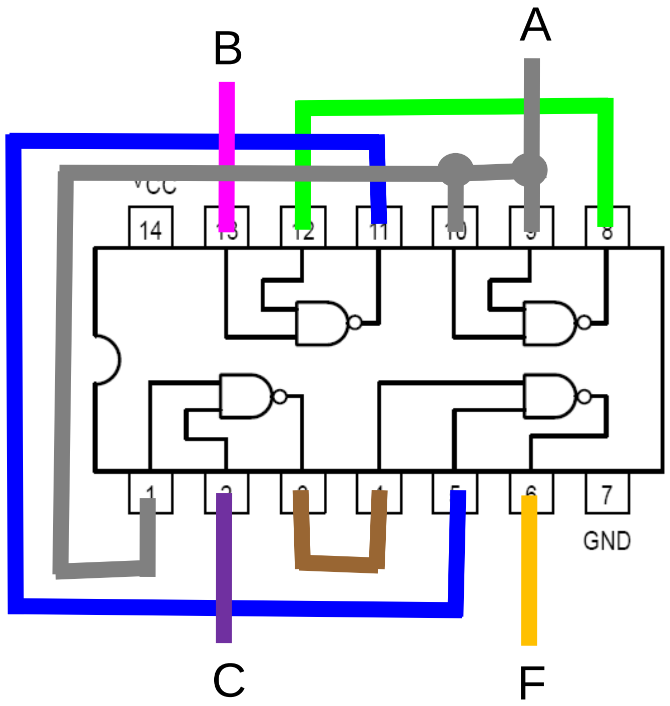
- Given a truth table:
  1) Write the Boolean expression
  2) Minimize the Boolean expression
  3) Draw as gates
  4) Map to available gates
  5) Determine # of packages and their connections

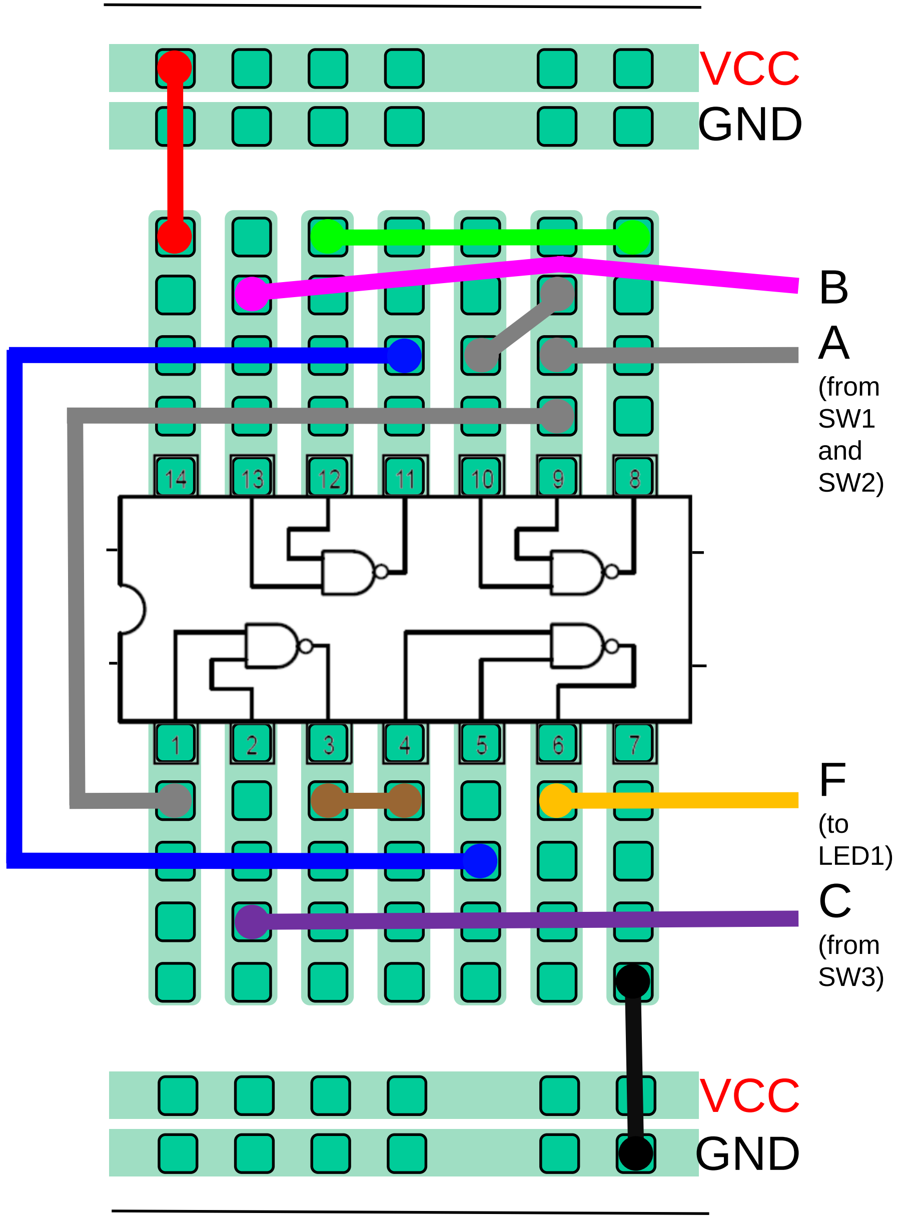


7 nets (wires) in this design



Breadboarding circuits





Summary

- Digital systems are constructed from Combinational and Sequential Logic
- Logic minimization to create smaller and faster hardware
- Gates come in TTL packages that require careful wiring