# University of Washington - Computer Science \& Engineering 

Winter 2019 Instructor: Justin Hsia 2019-03-12


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## Please do not turn the page until 11:40.

## Instructions

- This quiz contains 4 pages, including this cover page.
- Show scratch work for partial credit, but put your final answers in the boxes and blanks provided.
- The quiz is closed book and closed notes.
- Please silence and put away all cell phones and other mobile or noise-making devices.
- Remove all hats, headphones, and watches.
- You have 50 minutes to complete this quiz.


## Advice

- Read questions carefully before starting. Read all questions first and start where you feel the most confident to maximize the use of your time.
- There may be partial credit for incomplete answers; please show your work.
- Relax. You are here to learn.

| Question | Points | Score |
| :--- | :---: | :---: |
| (1) Decoders | 12 | 12 |
| (2) Shift Registers | 10 | 10 |
| (3) Luhn Algorithm | 12 | 12 |
|  | Total: | $\mathbf{3 4}$ |
| $\mathbf{3 4}$ |  |  |

## Question 1: Decoders [12 pts]

We are building a small keypad-to-7seg decoder circuit in two parts. The keypad is 4 buttons arranged in a square. The signals $C_{1}, C_{0}, R_{1}$, and $R_{0}$ are high (1) only if a button in the corresponding column or row is being pushed. The 2-bit bus $\mathrm{K}\left(\mathrm{K}_{1}, \mathrm{~K}_{0}\right)$ represents which key is recognized (in binary). The Valid signal $(\mathrm{V})$ is high when at least one key is being pressed.

(A) Complete the truth table. We give priority to $\mathrm{C}_{1}$ and $\mathrm{R}_{1}$. [4 pt]

| $\mathbf{R}_{\mathbf{1}}$ | $\mathbf{R}_{\mathbf{0}}$ | $\mathbf{C}_{\mathbf{1}}$ | $\mathbf{C}_{\mathbf{0}}$ | $\mathbf{V}$ | $\mathbf{K}_{\mathbf{1}}$ | $\mathbf{K}_{\mathbf{0}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | $\mathbf{0}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| 0 | 0 | 0 | 1 | $\mathbf{0}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| 0 | 0 | 1 | 0 | $\mathbf{0}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| 0 | 0 | 1 | 1 | $\mathbf{0}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| 0 | 1 | 0 | 0 | $\mathbf{0}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| 0 | 1 | 0 | 1 | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| 0 | 1 | 1 | 0 | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| 0 | 1 | 1 | 1 | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| 1 | 0 | 0 | 0 | $\mathbf{0}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| 1 | 0 | 0 | 1 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| 1 | 0 | 1 | 0 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| 1 | 0 | 1 | 1 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| 1 | 1 | 0 | 0 | $\mathbf{0}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| $\mathbf{1}$ | 1 | 0 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| 1 | 1 | 1 | 0 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| 1 | 1 | 1 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |

(B) In the space below, solve for the minimal logical expression (using 2-input gates) for $\mathbf{V}$. [4 pt]

$V=R_{0} C_{0}+R_{1} C_{0}+R_{0} C_{1}+R_{1} C_{1}=\left(R_{0}+R_{1}\right)\left(C_{0}+C_{1}\right)$
(C) For the 7seg signal numbering and outputs shown above (lit/black $=1$ ), draw the minimal logic for $\mathbf{S}_{\mathbf{4}}$ in terms of $\mathrm{V}, \mathrm{K}_{1}$, and $\mathrm{K}_{0}$. All signals should be off when $V=0$. [4 pt]

| $\mathbf{K}_{1}$ | $\mathbf{K}_{\mathbf{0}}$ | $\mathbf{S}_{4}$ |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |



## Question 2: Shift Registers [10 pts]

We are using a 3 -bit LFSR as a pseudo-random number generator by connecting q1 and q0 to a 2-input NOR gate. Assume we start in state 011.

(A) Circle one: This LFSR (look at Q) is shifting bits to the LEFT)/ RIGHT. [1 pt]
(B) How many distinct states do we end up visiting? What is the length of the cycle we end up in? [4 pt]


|  |  |
| :--- | :--- |
| States visited: | 5 |
|  |  |
| Cycle length: | 3 |
|  |  |

(C) Modify the LFSR to add Enable and Load signals and draw the circuit diagram below using logic gates and routing elements discussed in class. Give priority to Load. [5 pt]

- Make sure you label the corresponding selector bits for ports of routing elements.
- Assume the clock inputs are connected properly for you.



## Question 3: Luhn Algorithm [12 pts]

The Luhn algorithm is a "mod 10 " formula used for validation of decimal numbers (like credit cards) that uses a "check digit" in the rightmost position. The validation algorithm is as follows:

1. From right to left, double every second digit.
2. Sum all of the digits.
3. The number is valid if the sum $\% 10=0$, invalid otherwise.

Example:

| Digits: | $\mathrm{d}_{3}=9$ | $\mathrm{~d}_{2}=7$ | $\mathrm{~d}_{1}=6$ | check = 1 |
| ---: | :---: | :---: | :---: | :---: |
| Double every other: | $\mathbf{1 8}$ | 7 | $\mathbf{1 2}$ | 1 |
| Sum digits: | $\mathbf{9}$ | 7 | $\mathbf{3}$ | 1 |

$=20$, so 9761 is valid $\nabla$
Here we will implement a Luhn verifier for 4 -digit numbers. You can freely add registers, constants, and the following logic blocks (with specified combinational delays):

(A) Using these parts, complete the fastest 4-digit Luhn verification circuit below. Assume the clock inputs are connected properly for you. [ 7 pts ]

(B) What is the minimum value of the bus width n for this circuit? [2 pt] max sum $=9+9+9+9=1+8+9+1+8+9=36$, ceil $\left(\log _{2} 36\right)=6$.

$$
\mathrm{n}_{\min }=6 \mathrm{bits}
$$

(C) Assume $t_{\text {hold }}=10 \mathrm{ps}, t_{\text {setup }}=40 \mathrm{ps}, t_{\text {C2Q }}=70 \mathrm{ps}$ and $t_{\text {comparator }}=$ 55 ps . How long after a clock trigger does your circuit from Part A take to compute the final value of Valid? [3 pt]

$\mathrm{t}_{\mathrm{C} 2 \mathrm{Q}}+\mathrm{t}_{\text {shift }}+\mathrm{t}_{\text {div }}+3 * \mathrm{t}_{\text {add }}+\mathrm{t}_{\text {div }}+\mathrm{t}_{\text {comparator }}=495$.

