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## Please do not turn the page until 11:40.

### Instructions

- This quiz contains 4 pages, including this cover page.
- Show scratch work for partial credit, but put your final answers in the boxes and blanks provided.
- The quiz is closed book and closed notes.
- Please silence and put away all cell phones and other mobile or noise-making devices.
- Remove all hats, headphones, and watches.
- You have 50 minutes to complete this quiz.

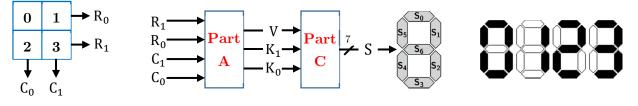
### Advice

- Read questions carefully before starting. Read *all* questions first and start where you feel the most confident to maximize the use of your time.
- There may be partial credit for incomplete answers; please show your work.
- Relax. You are here to learn.

Question	Points	Score
(1) Decoders	12	
(2) Shift Registers	10	
(3) Luhn Algorithm	12	
Total:	34	

#### Question 1: Decoders [12 pts]

We are building a small **keypad-to-7seg decoder circuit** in two parts. The keypad is 4 buttons arranged in a square. The signals  $C_1$ ,  $C_0$ ,  $R_1$ , and  $R_0$  are high (1) only if a button in the corresponding column or row is being pushed. The 2-bit bus K ( $K_1$ ,  $K_0$ ) represents which key is recognized (in binary). The Valid signal (V) is high when at least one key is being pressed.

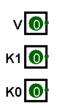


(A) Complete the truth table. We give priority to  $C_1 \mbox{ and } R_1. \ \begin{tabular}{c} 4 \mbox{ pt} \end{bmatrix}$ 

$\mathbf{R}_1$	$\mathbf{R}_{0}$	$\mathbf{C}_1$	$\mathbf{C}_0$	V	$\mathbf{K}_1$	$\mathbf{K}_{0}$
0	0	0	0	0		
0	0	0	1	0		
0	0	1	0	0		
0	0	1	1	0		
0	1	0	0	0		
0	1	0	1	1		
0	1	1	0	1		
0	1	1	1	1		
1	0	0	0	0		
1	0	0	1	1		
1	0	1	0	1		
1	0	1	1	1		
1	1	0	0	0		
1	1	0	1	1		
1	1	1	0	1		
1	1	1	1	1		

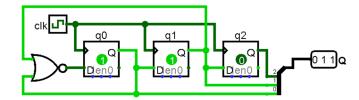
(B) In the space below, solve for the minimal logical expression (using 2-input gates) for  $\mathbf{V}$ . [4 pt]

(C) For the 7seg signal numbering and outputs shown above (lit/black = 1), draw the minimal logic for  $S_4$  in terms of V,  $K_1$ , and  $K_0$ . All signals should be off when V = 0. [4 pt]



#### Question 2: Shift Registers [10 pts]

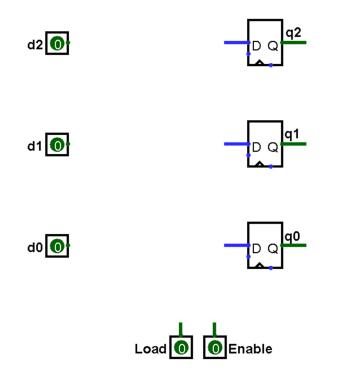
We are using a 3-bit LFSR as a pseudo-random number generator by connecting q1 and q0 to a 2-input NOR gate. Assume we start in state 011.



- (A) <u>Circle one</u>: This LFSR (look at Q) is shifting bits to the LEFT / RIGHT. [1 pt]
- (B) How many distinct states do we end up visiting? What is the length of the cycle we end up in? [4 pt]

States visited:	
Cycle length:	

- (C) Modify the LFSR to add **Enable** and **Load** signals and draw the circuit diagram below using *logic gates* and *routing elements* discussed in class. Give priority to Load. [5 pt]
  - Make sure you label the corresponding selector bits for ports of routing elements.
  - Assume the clock inputs are connected properly for you.



### Question 3: Luhn Algorithm [12 pts]

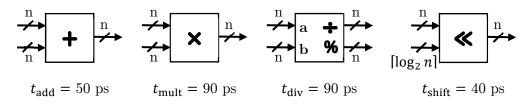
The Luhn algorithm is a "mod 10" formula used for validation of decimal numbers (like credit cards) that uses a "check digit" in the rightmost position. The validation algorithm is as follows:

- 1. From right to left, double every *second* digit.
- 2. Sum all of the *digits*.
- 3. The number is valid if the sum % 10 = 0, invalid otherwise.

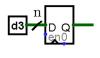
Example:
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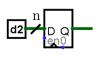
Digits:	$d_3 = 9$	$d_2 = 7$	$d_1 = 6$	check = 1	
Double every other:	18	7	12	1	
Sum digits:	9	7	3	1	= 20, so 9761 is valid $\blacksquare$

Here we will implement a Luhn verifier for 4-digit numbers. You can freely add **registers**, **constants**, and the following **logic blocks** (with specified combinational delays):

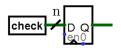


(A) Using these parts, complete the *fastest* 4-digit Luhn verification circuit below. Assume the clock inputs are connected properly for you. [7 pts]









- (B) What is the minimum value of the bus width n for this circuit? [2 pt]
- (C) Assume  $t_{hold} = 10$  ps,  $t_{setup} = 40$  ps,  $t_{C2Q} = 70$  ps and  $t_{comparator} = 55$  ps. How long after a clock trigger does your circuit from Part A take to compute the final value of Valid? [3 pt]

