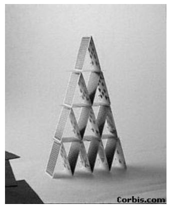
Introduction to Bottom-Up Parsing



Outline

- The strategy: *shift-reduce* parsing
- LR(0) example

Predictive Parsing Summary

- First and Follow sets are used to construct predictive tables
  - For non-terminal *A* and input t, use a production $A \rightarrow \alpha$ where $t \in \text{First}(\alpha)$
  - For non-terminal *A* and input t, if $\varepsilon \in \text{First}(A)$ and $t \in \text{Follow}(\alpha)$, then use a production $A \rightarrow \alpha$ where $\varepsilon \in \text{First}(\alpha)$

Bottom-Up Parsing

- Bottom-up parsing is more general than top-down parsing
  - And just as efficient
  - Builds on ideas in top-down parsing
- Bottom-up is the preferred method in practice

An Introductory Example

- Bottom-up parsers don't need left-factored grammars
- Hence we can revert to a "natural" grammar for our example:

  $E \rightarrow T + E \mid T$

  $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$
- Consider the string: int * int + int

The Idea

Bottom-up parsing *reduces* a string to the start symbol by inverting productions:

int * int + int	$T \rightarrow \text{int}$
int * T + int	$T \rightarrow \text{int} * T$
T + int	$T \rightarrow \text{int}$
T + T	$E \rightarrow T$
T + E	$E \rightarrow T + E$
E	

Observation

- Read the productions found by bottom-up parse in reverse (i.e., from bottom to top)
- This is a rightmost derivation!

int * int + int	T → int
int * T + int	T → int * T
T + int	T → int
T + T	E → T
T + E	E → T + E
E	

Important Fact #1

Important Fact #1 about bottom-up parsing:

A bottom-up parser traces a rightmost derivation in reverse

A Bottom-up Parse

int * int + int

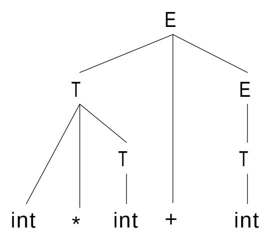
int * T + int

T + int

T + T

T + E

E



A Bottom-up Parse in Detail (1)

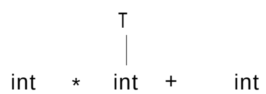
int * int + int

int * int + int

A Bottom-up Parse in Detail (2)

int * int + int

int * T + int

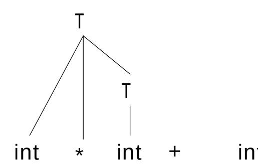


A Bottom-up Parse in Detail (3)

int * int + int

int * T + int

T + int



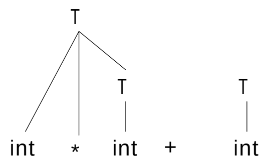
A Bottom-up Parse in Detail (4)

int * int + int

int * T + int

T + int

T + T



A Bottom-up Parse in Detail (5)

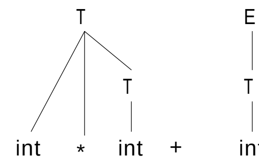
int * int + int

int * T + int

T + int

T + T

T + E



A Bottom-up Parse in Detail (6)

int * int + int

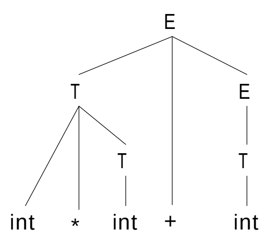
int * T + int

T + int

T + T

T + E

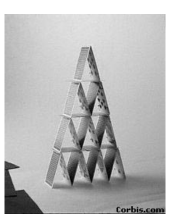
E



Question

How do we choose the substring to reduce at each step?

How to build the house of cards?



Where Do Reductions Happen

Important Fact #1 has an interesting consequence:

- Let $\alpha\beta\omega$ be a step of a bottom-up parse
- Assume the next reduction is by $X \rightarrow \beta$
- Then ω is a string of terminals

Why? Because $\alpha X\omega \rightarrow \alpha\beta\omega$ is a step in a right-most derivation

Notation

- Idea: Split string into two substrings
  - Right substring is as yet unexamined by parsing (a string of terminals)
  - Left substring has terminals and non-terminals
- The dividing point is marked by a |
  - The | is not part of the string
  - Some texts use •
- Initially, all input is unexamined $|x_1x_2 \ldots x_n$

Shift-Reduce Parsing

Bottom-up parsing uses only two kinds of actions:

Shift

Reduce

Shift

- *Shift:* Move | one place to the right
  - Shifts a terminal to the left string

ABC|xyz ⇒ ABCx|yz

Reduce

- Apply an *inverse production* at the right end of the left string
  - If A → xy is a production, then

Cbxy|ijk ⇒ CbA|ijk

The Example with Shift-Reduce Parsing

|int * int + int	shift
int | * int + int	shift
int * | int + int	shift
int * int | + int	reduce T → int
int * T | + int	reduce T → int * T
T | + int	shift
T + | int	shift
T + int |	reduce T → int
T + T |	reduce E → T
T + E |	reduce E → T + E
E |	

A Shift-Reduce Parse in Detail (1)

|int * int + int

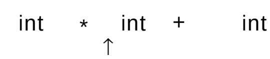
int * int + int
↑

A Shift-Reduce Parse in Detail (2)

|int * int + int
int | * int + int

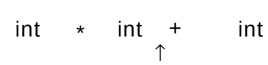
A Shift-Reduce Parse in Detail (3)

|int * int + int
int | * int + int
int * | int + int



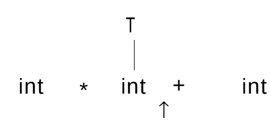
A Shift-Reduce Parse in Detail (4)

|int * int + int
int | * int + int
int * | int + int
int * int | + int



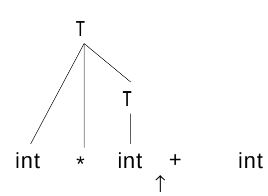
A Shift-Reduce Parse in Detail (5)

|int * int + int
int | * int + int
int * | int + int
int * int | + int
int * T | + int



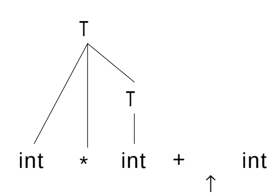
A Shift-Reduce Parse in Detail (6)

|int * int + int
int | * int + int
int * | int + int
int * int | + int
int * T | + int
T | + int



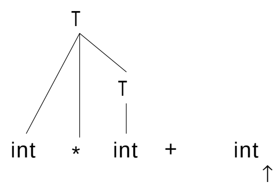
A Shift-Reduce Parse in Detail (7)

|int * int + int
int | * int + int
int * | int + int
int * int | + int
int * T | + int
T | + int
T + | int



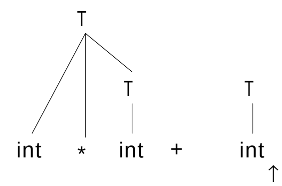
A Shift-Reduce Parse in Detail (8)

|int * int + int
int | * int + int
int * | int + int
int * int | + int
int * T | + int
T | + int
T + | int
T + int |



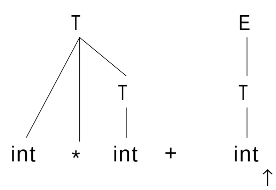
A Shift-Reduce Parse in Detail (9)

|int * int + int
int | * int + int
int * | int + int
int * int | + int
int * T | + int
T | + int
T + | int
T + int |
T + T |



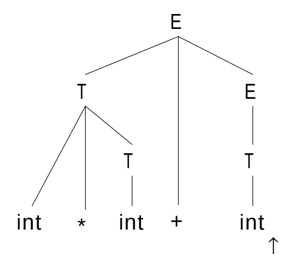
A Shift-Reduce Parse in Detail (10)

|int * int + int
int | * int + int
int * | int + int
int * int | + int
int * T | + int
T | + int
T + | int
T + int |
T + T |
T + E |



A Shift-Reduce Parse in Detail (11)

|int * int + int
int | * int + int
int * | int + int
int * int | + int
int * T | + int
T | + int
T + | int
T + int |
T + T |
T + E |
E |



The Stack

- Left string can be implemented by a stack
  - Top of the stack is the |
- Shift pushes a terminal on the stack
- Reduce pops 0 or more symbols off of the stack (production rhs) and pushes a non-terminal on the stack (production lhs)

Key Issue (will be resolved by algorithms)

- How do we decide when to shift or reduce?
  - Consider step int | * int + int
  - We could reduce by T → int giving T | * int + int
  - A fatal mistake: No way to reduce to the start symbol E

Conflicts

- Generic shift-reduce strategy:
  - If there is a handle on top of the stack, reduce
  - Otherwise, shift
- But what if there is a choice?
  - If it is legal to shift or reduce, there is a *shift-reduce* conflict
  - If it is legal to reduce by two different productions, there is a *reduce-reduce* conflict

Source of Conflicts

- Ambiguous grammars always cause conflicts
- But beware, so do many non-ambiguous grammars

Conflict Example

Consider our favorite ambiguous grammar:

E	→	E + E
	|	E * E
	|	(E)
	|	int

One Shift-Reduce Parse

|int * int + int	shift
. . .	. . .
E * E | + int	reduce E → E * E
E | + int	shift
E + | int	shift
E + int|	reduce E → int
E + E |	reduce E → E + E
E |	

Another Shift-Reduce Parse

|int * int + int	shift
. . .	. . .
E * E | + int	shift
E * E + | int	shift
E * E + int |	reduce E → int
E * E + E|	reduce E → E + E
E * E |	reduce E → E * E
E |	

Example Notes

- In the second step E * E | + int we can either shift or reduce by E → E * E
- Choice determines associativity of + and *
- As noted previously, grammar can be rewritten to enforce precedence
- Precedence declarations are an alternative

Precedence Declarations Revisited

- Precedence declarations cause shift-reduce parsers to resolve conflicts in certain ways
- Declaring "* has greater precedence than +" causes parser to reduce at E * E | + int
- More precisely, precedence declaration is used to resolve conflict between reducing a * and shifting a +

Precedence Declarations Revisited (Cont.)

- The term "precedence declaration" is misleading
- These declarations do not define precedence; they define conflict resolutions
  - Not quite the same thing!

Nitty Gritty Algorithms

- See pages 215-257 in the Dragon Book
  - How to determine handles
  - Algorithms to construct a DFA describing a parse
  - LR(0), LR(1), SLR, LALR
- Next class
  - Yacc does most of it for you