

## Objectives: parsing lectures

Understand:

- Theory and practice of parsing
- Underlying language theory (CFGs, ...)
- Top-down parsing (and be able to do it)
- Bottom-up parsing (time permitting)
- Today's focus: grammars and ambiguity


## Parsing: two jobs

- Is the program syntactically correct?
a $:=3 *(5+4) ; \quad$ if $x>y$ then $m:=x$;
a $:=3$ * / 4; if $x<y$ else $m:=x ;$
- If so, build the corresponding AST



## CFG terminology

- Terminals: alphabet, or set of legal tokens
- Nonterminals: represent abstract syntax units
- Productions: rules defining nonterminals in terms of a finite sequence of terminals and nonterminals
- Start symbol: root symbol defining the language

Program ::= Stmt
Stmt ::= if Expr then Stmt else Stmt end Stmt ::= while Expr do Stmt end

| Program | $::=$ module Id ; Block Id . |
| :--- | :--- |
| Block | $::=$ DeclList begin StmtList end |
| DeclList | $::=$ \{ Decl ; \} |
| Decl | $::=$ ConstDecl \| ProcDecl | VarDecl |
| ConstDecl | $::=$ const ConstDeclItem $\{$, ConstDeclItem \} |
| ConstDeclItem $::=$ Id $:$ Type $=$ ConstExpr |  |
| ConstExpr | $::=$ Id \| Integer |
| VarDecl | $::=$ var VarDeclItem $\{$, VarDeclItem \} |
| VarDeclItem $::=$ Id $:$ Type |  |

## EBNF description of PL/0

```
OutStmt ::= output := Expr
IfStmt ::= if Test then StmtList end
WhileStmt ::= while Test do StmtList end
Test ::= odd Sum | Sum Relop Sum
Relop ::= <= | <> | < | >= | > | =
Exprs ::= Expr {, Expr }
Expr ::= Sum
Sum ::= Term { (+ | -) Term }
Term ::= Factor { (* | /) Factor }
Factor ::= - Factor | LValue | Integer |
        input ( Expr )
```


## Derivations and parsing

- Derivation
- A sequence of expansion steps,
- Beginning with the start symbol,
- Leading to a string of terminals
- Parsing: inverse of derivation
- Given a target string of terminals,
- Recover nonterminals/productions representing structure


## EBNF description of PL/0

```
ProcDecl ::=
    procedure Id ( [ FormalDecl {, FormalDecl} ] ) ;
                                    Block Id
FormalDecl ::= Id : Type
Type ::= int
StmtList ::= { Stmt ; }
Stmt ::= CallStmt | AssignStmt | OutStmt |
    IfStmt | WhileStmt
CallStmt ::= Id ( [ Exprs ] )
AssignStmt ::= Lvalue := Expr
Lvalue ::= Id
```

Exercise: produce a syntax tree for squares. 0

```
module main;
    var x:int, squareret:int;
    procedure square(n:int);
    begin
        squareret := n * n;
    end square;
begin
    x := input;
    while x <> 0 do
        square(x);
        output := squareret;
        x := input;
    end;
end main.
```


## Parse trees

- We represent derivations and parses as parse trees
- Concrete syntax tree
- Exact reflection of the grammar
- Abstract syntax tree
- Simplified version, reflecting key structural information
- E.g., omit superfluous punctuation \& keywords
meanings for the same program
- Consider the example on the previous slide



## Ambiguity

- Some grammars are ambiguous
- Different parse trees with the same final string
- (Some languages are ambiguous, with no possible non-ambiguous grammar; but we avoid them)
- The structure of the parse tree captures some of the meaning of a program
- Ambiguity is bad since it implies multiple possible


## Ex: An expression grammar

- E : : = E Op E| - E| ( E ) | int Op : : = + | - * * /
- Using this grammar, find parse trees for:
- 3 * 5
- $3+4$ * 5

Resolving ambiguity: \#1

- Add a meta-rule
- For instance, "else associates with the closest previous unmatched if"
$\uparrow$ This works and keeps the original grammar intact
$\downarrow$ But it's ad hoc and informal
- To which then does the else belong?
- The compiler isn't going to be confused
- However, if the compiler chooses a meaning different from what the programmer intended, it could get ugly
- Any ideas for overcoming this problem?



## Resolving ambiguity: \#3

- Redesign the programming language to remove the ambiguity

```
Stmt ::= if Expr then Stmt end |
    if Expr then Stmt else Stmt end
```

$\uparrow$ Formal, clear, elegant
$\uparrow$ Allows StmtList in then and else branch, without adding begin/end
$\downarrow$ Extra end required for every if statement $\uparrow$ yacc does

- Add meta-rules for precedence and associativity

$$
\mathrm{E}::=\mathrm{E}+\mathrm{E}|\mathrm{E}-\mathrm{E}| \mathrm{E}^{\star} \mathrm{E}|\mathrm{E} / \mathrm{E}| \mathrm{E}^{\wedge} \mathrm{E}|(\mathrm{E})|-\mathrm{E} \mid .
$$

. +,- < *,/ < unary - < ^ etc.

- +,-,,, l left-associative; ^ right associative
$\uparrow$ Simple, intuitive
$\downarrow$ But not all parsers can support this



## What about that

## expression grammar?

How to resolve its ambiguity?

- Option \#1: add meta-rules for precedence and associativity
- Option \#2: modify the grammar to explicitly resolve the ambiguity
- Option \#3: redefine the language

Resolving ambiguity: \#2 (cont.)
Stmt : : = MatchedStmt | UnmatchedStmt MatchedStmt ::= ... |
if Expr then MatchedStmt else MatchedStmt
UnmatchedStmt : := if Expr then Stmt |
if Expr then MatchStmt else UnmatchedStmt

```
if e1 then if e2 then s1 else s2
```




## Designing a grammar:

on what basis?

- Accuracy
- Readability, clarity
- Unambiguity
- Limitations of CFGs
- Similarity to desired AST structure
- Ability to be parsed by a particular parsing algorithm
- Top-down parser => LL(k) grammar
- Bottom-up parser $=>$ LR(k) grammar


## Option \#3: New language

- Require parens
- E.g., in APL all exprs evaluated left-to-right unless parenthesized
- Forbid parens
- E.g.: RPN calculators


## Parsing algorithms

- Given input (sequence of tokens) and grammar, how do we find an AST that represents the structure of the input with respect to that grammar?
- Two basic kinds of algorithms
- Top-down: expand from grammar's start symbol until a legal program is produced
- Bottom-up: create sub-trees that are merged into larger sub-trees, finally leading to the start symbol


## Top-down parsing

- Build AST from top (start symbol) to leaves (terminals)
- Represents a leftmost derivation (e.g., always expand leftmost nonterminal)
- Basic issue: when replacing a non-terminal
 with a right-hand side
(rhs), which rhs should you use?
- Basic solution: Look at next input tokens


## Predictive parser

- A top-down parser that can select the correct rhs looking at the next $k$ tokens (lookahead)
- Efficient
- No backtracking is needed
- Linear time to parse
- Implementation
- Table-driven: pushdown automaton (PDA) - like table-driven FSA plus stack for recursive FSA calls
- Recursive-descent parser [used in PL/0]
- Each non-terminal parsed by a procedure
- Call other procedures to parse sub-non-terminals, recursively


## LL(k), LR(k), ...?

- These parsers have generally snazzy names
- The simpler ones look like the ones in the title of this slide
- The first $L$ means "process tokens left to right"
- The second letter means "produce a (Right / Left)most derivation" - Leftmost => top-down
- Rightmost => bottom-up
- The $k$ means " $k$ tokens of lookahead"
- We won't discuss LALR(k), SLR, and lots more parsing algorithms


## LL(k) grammars

- It's easy to construct a predictive parser if a grammar is $\operatorname{LL}(\mathrm{k})$
- Leff-to-right scan on input, lookahead lookahead
- Restrictions include
- Unambiguous
- No common prefixes of length $\geq k$ - No left recursion
- ... (more details later).
- Collectively, the restrictions guarantee that, given kinput okens, one can always select the correct rhs to expand

Common prefix
S : : = if Test then Stmts end
if Test then f Test then Stmts else
Stmts end Stmts end

Left recursion
E : : = E op E | ...
correct rhs to expand

## Eliminating common prefixes

- Left factor them, creating a new non-terminal for the common prefix and/or different suffixes
- Before
- If ::= if Test then Stmts end
- After

If $::=$ if Test then Stmts IfCont
IfCont $::=$ end | else Stmts end
$\downarrow$ Grammar is a bit uglier
$\uparrow$ Easy to do manually in a recursive-descent parser

## Eliminating left recursion:

- Before

| $\mathrm{E}::=\mathrm{E}+\mathrm{T}$ | T |  |
| :--- | :--- | :--- |
| $\mathrm{T}:$ | $=\mathrm{T} * \mathrm{~F}$ | F |

$T::=T * F \mid F$
F : : = id | (E) | ...

- After
$\mathrm{E} \quad::=\mathrm{T}$ ECont
ECont ::= + T ECont $\mid \varepsilon$
$\mathrm{T} \quad::=\mathrm{F}$ TCont
TCont $::=$ * $F$ TCont $\mid \varepsilon$
F : := id| (E) |.


## Just add sugar

## LL(1) Parsing Theory

$\mathrm{E}::=\mathrm{T}\{+\mathrm{T}\}$
$\mathrm{T}::=\mathrm{F}\{* \mathrm{~F}\}$
F : : = id | ( E ) | ..
$\uparrow$ Sugared form is still pretty readable
$\uparrow$ Easy to implement in hand-written recursive descent parser
$\downarrow$ Concrete syntax tree is not as close to abstract syntax tree

Goal: Formal, rigorous description of those grammars for which "I can figure out how to do a top-down parse by looking ahead just one token", plus corresponding algorithms.

## Notation:

T = Set of Terminals (Tokens)
$\mathrm{N}=$ Set of Nonterminals
\$ = End-of-file character ( T -like, but not in $\mathrm{N} \cup \mathrm{T}$ )

## Table-driven predictive parser

- Automatically compute PREDICT table from grammar
- PREDICT(nonterminal,input-symbol)
$\rightarrow$ action, e.g. which rhs or error


## Example 1

| Stmt : : = 1 if expr then Stmt else Stmt <br> 2 while Expr do Stmt <br> 3 begin Stmts end |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Stmts ::=4 Stmt ; Stmts \| $\mathbf{5}$ ع Expr ::= 6 id |  |  |  |  |  |  |  |  |  |  |
|  | if | then | else | while | do | begin | end | id | ; | \$ |
| Stmt | 1 |  |  | 2 |  | 3 |  |  |  |  |
| Stmts | 4 |  |  | 4 |  | 4 | 5 |  |  |  |
| Expr |  |  |  |  |  |  |  | 6 |  |  |
| empty $=$ error |  |  |  |  |  |  |  |  |  |  |

## Constructing PREDICT: overview

- Compute FIRST set for each rhs
- All tokens that can appear first in a derivation from that rhs
- In case rhs can be empty, compute FOLLOW set for each non-terminal
- All tokens that can appear right after that nonterminal in a derivation
- Constructions of FIRST and FOLLOW sets are interdependent
- PREDICT depends on both


## FIRST $(\alpha)-1^{\text {st }}$ "token" from $\alpha$

Definition: For any string $\alpha$ of terminals and nonterminals, $\operatorname{FIRST}(\alpha)$ is the set of terminals that begin strings derived from $\alpha$, together with $\varepsilon$, if $\alpha$ can derive $\varepsilon$. More precisely:

For any $\alpha \in(N \cup T)^{*}$,
$\operatorname{FIRST}(\alpha)=$
$\left\{a \in T \mid \alpha \Rightarrow^{*} a \beta\right.$ for some $\left.\beta \in(N \cup T)^{*}\right\} \cup$ $\left\{\varepsilon\right.$, if $\left.\alpha \Rightarrow^{*} \varepsilon\right\}$

## Computing FIRST- 4 cases

$\operatorname{FIRST}(\varepsilon)=\{\varepsilon\}$
For all $a \in T, \operatorname{FIRST}(a)=\{a\}$
For all $A \in N$, repeat until no change If there is a rule $A \rightarrow \varepsilon$, add( $\varepsilon$ ) to $\operatorname{FIRST}(\mathrm{A})$ For all rules $A \rightarrow Y_{1} \ldots Y_{k} \quad \operatorname{add}\left(\operatorname{FIRST}\left(Y_{1}\right)-\{\varepsilon\}\right)$ if $\varepsilon \in \operatorname{FIRST}\left(\mathrm{Y}_{1}\right) \quad$ then $\operatorname{add}\left(\operatorname{FIRST}\left(\mathrm{Y}_{2}\right)-\{\varepsilon\}\right)$ if $\varepsilon \in \operatorname{FIRST}\left(Y_{1} Y_{2}\right)$ then $\operatorname{add}\left(\operatorname{FIRST}\left(Y_{3}\right)-\{\varepsilon\}\right)$
if $\varepsilon \in \operatorname{FIRST}\left(Y_{1} Y_{2} \ldots Y_{k}\right)$ then $\operatorname{add}(\varepsilon)$

FOLLOW(B) - Next "token" after B
Definition: for any non-terminal B, FOLLOW(B) is the set of terminals that can appear immediately after B in some derivation from the start symbol, together with $\$$, if B can be the end of such a derivation. (\$ represents "end of input".) More precisely: For all $B \in N$,
$\operatorname{FOLLOW}(B)=\left\{a \in(T \cup\{\$\}) \mid S \$ \Rightarrow^{*} \alpha B a \beta\right.$ for some $\left.\alpha, \beta \in(N \cup T \cup\{\$\})^{*}\right\}$
( S is the Start symbol of the grammar.)

PREDICT - Given Ins, which rhs?
For all rules $\mathrm{A} \rightarrow \alpha$
For all $\mathrm{a} \in \operatorname{FIRST}(\alpha)-\{\varepsilon\}$ $\operatorname{Add}(\mathrm{A} \rightarrow \alpha)$ to $\operatorname{PREDICT}(\mathrm{A}, \mathrm{a})$
If $\varepsilon \in \operatorname{FIRST}(\alpha)$ then
For all $b \in \operatorname{FOLLOW}(A)$

$$
\operatorname{Add}(\mathrm{A} \rightarrow \alpha) \text { to } \operatorname{PREDICT}(\mathrm{A}, \mathrm{~b})
$$

Defn: $G$ is $\operatorname{LL}(1)$ iff every cell has $\leq 1$ entry

## Computing FIRST (Cont.)

4. For all any string $Y_{1} \ldots Y_{k} \in(N \cup T)^{*}$, similar: $\operatorname{add}\left(\operatorname{FIRST}\left(\mathrm{Y}_{1}\right)-\{\varepsilon\}\right)$
if $\varepsilon \in \operatorname{FIRST}\left(Y_{1}\right)$ then $\operatorname{add}\left(\operatorname{FIRST}\left(Y_{2}\right)-\{\varepsilon\}\right)$
if $\varepsilon \in \operatorname{FIRST}\left(Y_{1} Y_{2}\right)$ then $\operatorname{add}\left(\operatorname{FIRST}\left(Y_{3}\right)-\{\varepsilon\}\right)$
if $\varepsilon \in \operatorname{FIRST}\left(Y_{1} Y_{2} \ldots Y_{k}\right)$ then $\operatorname{add}(\varepsilon)$
[Note: defined for all strings; really only care about FIRST(right hand sides).]
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## Computing FOLLOW(B)

```
Add \$ to FOLLOW(S)
Repeat until no change
For all rules \(A \rightarrow \alpha B \beta\) [i.e. all rules with a \(B\) in r.h.s],
Add (FIRST( \(\beta\) ) - \(\{\varepsilon\}\) ) to FOLLOW(B)
If \(\varepsilon \in \operatorname{FIRST}(\beta)\) [in particular, if \(\beta\) is empty] then Add \(\operatorname{FOLLOW}(\mathrm{A})\) to \(\operatorname{FOLLOW}(\mathrm{B})\)
Assume for all \(A\) that \(S \Rightarrow^{*} \alpha A \beta\) for some \(\alpha, \beta \in(N \cup T)^{*}\), else \(A\) irrelevant

\section*{Properties of LL(1) Grammars}
- Clearly, given a conflict-free PREDICT table ( \(\leq 1\) entry/cell), the parser will do something unique with every input
- Key fact is, if the table is built as above, that something is the correct thing
- I.e., the PREDICT table will reliably guide the LL(1) parsing algorithm so that it will
- Find a derivation for every string in the language
- Declare an error on every string not in the language

\section*{Exercises (1st especially recommended)}
- Easy: Pick some grammar with common prefixes, left recursion, and/or ambiguity. - Build PREDICT; it will have conflicts
- Harder: prove that every grammar with \(\geq 1\) of those properties will have PREDICT conflicts
- Harder: Find a grammar with none of those features that nevertheless gives conflicts.
- I.e., absence of those features is necessary but not sufficient for a grammar to be LL(1).
- Harder, for theoryheads: if the table has conflicts, and the parser chooses among them nondeterministically, it will work correctly

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\section*{Example 2}


\section*{Example 2: PREDICT}
\begin{tabular}{|l|l|l|l|l|l|l|l|l|}
\hline & id & \(\mathbf{+}\) & - & \(*\) & \(/\) & \(\mathbf{(}\) & \(\mathbf{)}\) & \(\mathbf{\$}\) \\
\hline E & & & & & & & & \\
\hline \(\mathrm{E}^{\prime}\) & & & & & & & & \\
\hline T & & & & & & & & \\
\hline \(\mathrm{T}^{\prime}\) & & & & & & & & \\
\hline F & & & & & & & & \\
\hline
\end{tabular}

\section*{PREDICT and LL(1)}
- The PREDICT table has at most one entry in each cell if and only if the grammar is LL(1)
- \(\therefore\) there is only one choice (it's predictive), making it fast to parse and easy to implement
- Multiple entries in a cell
- Arise with left recursion, ambiguity, common prefixes, etc.
- Can patch by hand, if you know what to do
- Or use more powerful parser (LL(2), or LR(k), or ...?)
- Or change the grammar

\section*{Recursive descent parsers}
- Write procedure for each non-terminal
- Each procedure selects the correct right-hand side by peeking at the input tokens
- Then the r.h.s. is consumed
- If it's a terminal symbol, verify it is next and then advance through the token stream
- If it's a non-terminal, call corresponding procedure
- Build and return AST representing the r.h.s.


\section*{\(\mathrm{LL}(1)\) and Recursive Descent}
- If the grammar is LL(1), it's easy to build a recursive descent parser
- One nonterminal/row \(\rightarrow\) one procedure
- Use 1 token lookahead to decide which rhs
- Table-driven parser's stack \(\rightarrow\) recursive call stack
- Recursive descent can handle some non-LL(1) features, too.


It's demo time...
- Let's look at some of the PL/0 code to see how the recursive descent parsing works in practice



\section*{Parser::ParseWhileStmt()}
```

Stmt* Parser::ParseWhileStmt() {
scanner->Read(WHILE);
Expr* test = ParseTest();
scanner->Read (DO);
StmtArray* stmts = ParseStmts();
scanner->Read(END);
return new WhileStmt(test, stmts);
}


## <sum> $\quad::=$ <term> $\{(+\mid-)$ <term> $\}$

Parser::ParseSum()

```
Expr* Parser::ParseSum() {
```

    Expr* expr \(=\) ParseTerm();
    for (; ; )
        Token* t = scanner->Peek ()
        if (t->kind() == PLUS || t->kind() == MINUS) \{
            scanner->Get(); // eat the token
            Expr* expr2 = ParseTerm ();
            expr = new BinOp(t->kind(), expr, expr2)
        \} else \{
            return expr;
        ,
    \(\}^{3}\)
    Yacc - A bottom-up-parser generator

- "yet another compiler-compiler"
- Input:
- grammar, possibly augmented with action code
- Output:
- C code to parse it and perform actions
- LALR(1) parser generator
- practical bottom-up parser
- more powerful than LL(1)
- modern updates of yacc
- yacc++, bison, byacc, .



## Yacc with actions

assignstmt: IDENT GETS expr $\{\$ \$=$ new AssignStmt $(\$ 1, \$ 3) ;\}$
ifstmt: IF be THEN stmts END $\{\$ \$=$ new $\operatorname{IfStmt}(\$ 2, \$ 4$,NULL); \} | IF be THEN stmts

ELSE stmts END $\{\$ \$=$ new $\operatorname{IfStmt}(\$ 2, \$ 4, \$ 6) ;\}$
expr: term
\$ $\$=\$ 1$; \}
| expr ' + ' term $\{\$ \$=$ new BinOp(PLUS, \$1, \$3);
| expr '_' term $\{\$ \$=$ new BinOp(MINUS, \$1, \$3);\}


## Parsing summary

- Discover/impose a useful (hierarchical) structure on flat token sequence
- Represented by Abstract Syntax Tree
- Validity check syntax of input
- Could build concrete syntax tree (but don't)
- Many methods available
- Top-down::LL(1)/recursive descent common for simple, by-hand projects
- Bottom-up::LR(1)/LALR(1)/SLR(1) common for more complex projects - parser generator (e.g., yacc) almost necessary

Parsing summary -
Technical details you should know

- Context-free grammars
- Definitions
- Manipulations
(algorithmic)
Left factor common
prefixes
Eliminating left recursion
- Ambiguity \& (semiheuristic) fixes
- meta-rules (code/ precedence tables)
- rewrite grammar
- change language
- Building a table-driven predictive parser
- LL(1) grammar: definition \& common obstacles
- PREDICT(nonterminal, input symbol)
- FIRST(RHS)
- FOLLOW(nonterminal)
- Building a recursive descent parser

