

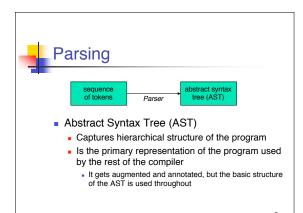


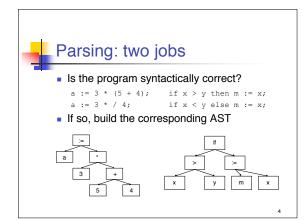
#### Objectives: parsing lectures

#### Understand:

- Theory and practice of parsing
- Underlying language theory (CFGs, ...)
- Top-down parsing (and be able to do it)
- Bottom-up parsing (time permitting)
- Today's focus: grammars and ambiguity

2







## Context-free grammars (CFGs)

- For lexing, we used regular expressions as the underlying notation
- For parsing, we use context-free grammars in much the same way
  - Regular expressions are not powerful enough
     Intuitively, can't express balance/nesting (a<sup>n</sup>b<sup>n</sup>, parens)
  - More general grammars are more powerful than we need



 Well, we could use more power, but instead we delay some checking to semantic analysis instead of doing all the analysis based on the (general, but slow) grammar -

## CFG terminology

- Terminals: alphabet, or set of legal tokens
- Nonterminals: represent abstract syntax units
- Productions: rules defining nonterminals in terms of a finite sequence of terminals and nonterminals
- Start symbol: root symbol defining the language

```
Program ::= Stmt
Stmt ::= if Expr then Stmt else Stmt end
Stmt ::= while Expr do Stmt end
```

```
Program ::= module Id; Block Id.

Block ::= DeclList begin StmtList end

DeclList ::= { Decl; }

Decl ::= ConstDecl | ProcDecl | VarDecl

ConstDecl ::= const ConstDeclItem {, ConstDeclItem }

ConstDeclItem ::= Id : Type = ConstExpr

ConstExpr ::= Id | Integer

VarDecl ::= var VarDeclItem {, VarDeclItem }

VarDeclItem ::= Id : Type
```

```
EBNF description of PL/O

ProcDecl ::=
procedure Id ( [ FormalDecl {, FormalDecl} ] );
Block Id

FormalDecl ::= Id : Type
Type ::= int
StmtList ::= { Stmt ; }
Stmt ::= CallStmt | AssignStmt | OutStmt |
IfStmt | WhileStmt
CallStmt ::= Id ( [ Exprs ] )
AssignStmt ::= Lvalue := Expr
Lvalue ::= Id
```

-

## EBNF description of PL/0

```
OutStmt
         ::= output := Expr
IfStmt
         ::= if Test then StmtList end
WhileStmt ::= while Test do StmtList end
       ::= odd Sum | Sum Relop Sum
         ::= <= | <> | < | >= | > | =
       ::= Expr { , Expr }
Exprs
        ::= Sum
Expr
Sum
        ::= Term { (+ | -) Term }
Term
         ::= Factor { (* | /) Factor }
       ::= - Factor | LValue | Integer |
Factor
             input | ( Expr )
```

9



# Exercise: produce a syntax tree for squares.0

```
module main;
var x:int, squareret:int;
procedure square(n:int);
begin
    squareret := n * n;
end square;
begin
    x := input;
while x <> 0 do
    square(x);
output := squareret;
    x := input;
end;
end main.
```

10



## Derivations and parsing

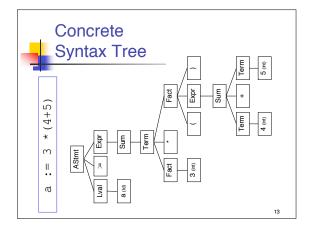
- Derivation
  - A sequence of expansion steps,
  - Beginning with the start symbol,
  - Leading to a string of terminals
- Parsing: inverse of derivation
  - Given a target string of terminals,
  - Recover nonterminals/productions representing structure

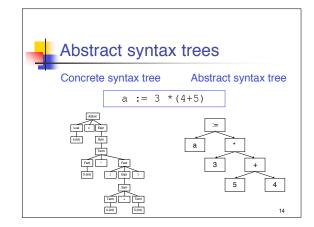
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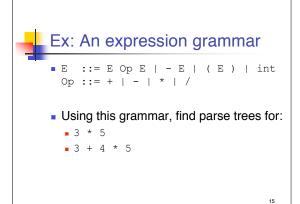
#### Parse trees

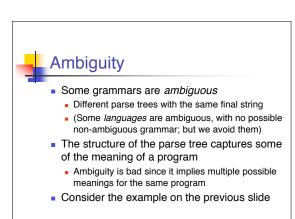
- We represent derivations and parses as parse trees
- Concrete syntax tree
  - Exact reflection of the grammar
- Abstract syntax tree
  - Simplified version, reflecting key structural information
  - E.g., omit superfluous punctuation & keywords

12









Another famous ambiguity:

dangling else

Stmt ::= ... |

if Expr then Stmt |

if Expr then Stmt else Stmt

if el then if else then sl else sl

To which then does the else belong?

The compiler isn't going to be confused

However, if the compiler chooses a meaning different from what the programmer intended, it could get ugly

Any ideas for overcoming this problem?



## Resolving ambiguity: #1

- Add a meta-rule
  - For instance, "else associates with the closest previous unmatched if"
- This works and keeps the original grammar intact
- But it's ad hoc and informal



## Resolving ambiguity: #2

Rewrite the grammar to resolve it explicitly

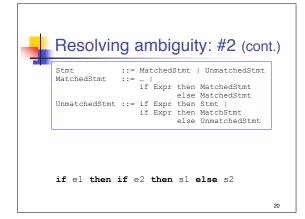
```
Stmt ::= MatchedStmt | UnmatchedStmt
MatchedStmt ::= ... |

if Expr then MatchedStmt
else MatchedStmt
UnmatchedStmt ::= if Expr then Stmt |

if Expr then MatchStmt
else UnmatchedStmt
```

- ♠ Formal, no additional meta-rules
- Somewhat more obscure grammar

19





## Resolving ambiguity: #3

 Redesign the programming language to remove the ambiguity

```
Stmt ::= if Expr then Stmt end |
    if Expr then Stmt else Stmt end
```

- ♠ Formal, clear, elegant
- ↑ Allows StmtList in then and else branch, without adding begin/end
- ◆ Extra end required for every if statement

21



## What about that expression grammar?

How to resolve its ambiguity?

- Option #1: add meta-rules for precedence and associativity
- Option #2: modify the grammar to explicitly resolve the ambiguity
- Option #3: redefine the language

22



#### Option #1: add meta-rules

 Add meta-rules for precedence and associativity

```
E ::= E+E | E-E | E*E | E/E | E^E | (E) | -E | ...
```

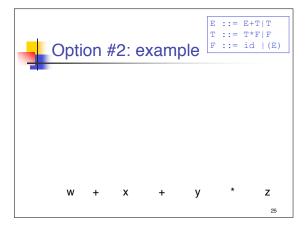
- +,- < \*,/ < unary < ^ etc.
- +,-,\*,/ left-associative; ^ right associative
- ↑ Simple, intuitive
- ◆ But not all parsers can support this
  - ◆ yacc does

4

#### Option #2: new BNF

E ::= E+T|T T ::= T\*F|F F ::= id | (E)

- Create a nonterminal for each precedence level
- Expr is the lowest precedence nonterminal
  - Each nonterminal can be rewritten with higher precedence operator
  - Highest precedence operator includes atomic expressions
- At each precedence level use
  - Left recursion for left-associative operators
  - Right recursion for right-associative operators
  - No recursion for non-associative operators





## Option #3: New language

- Require parens
  - E.g., in APL all exprs evaluated left-to-right unless parenthesized
- Forbid parens
  - E.g.: RPN calculators

26



#### Designing a grammar:

on what basis?

- Accuracy
- Readability, clarity
- Unambiguity
- Limitations of CFGs
- Similarity to desired AST structure
- Ability to be parsed by a particular parsing algorithm
  - Top-down parser => LL(k) grammar
  - Bottom-up parser => LR(k) grammar

27



## Parsing algorithms

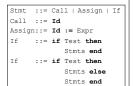
- Given input (sequence of tokens) and grammar, how do we find an AST that represents the structure of the input with respect to that grammar?
- Two basic kinds of algorithms
  - Top-down: expand from grammar's start symbol until a legal program is produced
  - Bottom-up: create sub-trees that are merged into larger sub-trees, finally leading to the start symbol

28



#### Top-down parsing

- Build AST from top (start symbol) to leaves (terminals)
  - Represents a leftmost derivation (e.g., always expand leftmost nonterminal)
- Basic issue: when replacing a non-terminal with a right-hand side (rhs), which rhs should you use?
- Basic solution: Look at next input tokens



29



## Predictive parser

- A top-down parser that can select the correct rhs looking at the next k tokens (lookahead)
- Efficient
  - No backtracking is needed
  - Linear time to parse
- Implementation
  - Table-driven: pushdown automaton (PDA) like table-driven FSA plus stack for recursive FSA calls
  - Recursive-descent parser [used in PL/0]
    - Each non-terminal parsed by a procedure
    - Call other procedures to parse sub-non-terminals.



## LL(k), LR(k), ...?

- These parsers have generally snazzy names
- The simpler ones look like the ones in the title of this slide
  - The first L means "process tokens left to right"
  - The second letter means "produce a (Right / Left)most derivation"

  - Leftmost => top-down Rightmost => bottom-up

  - The k means "k tokens of lookahead"
- We won't discuss LALR(k), SLR, and lots more parsing algorithms



#### LL(k) grammars

- It's easy to construct a predictive parser if a grammar is LL(k)
  - Left-to-right scan on input, Leftmost derivation, k tokens of
- Restrictions include

  - Unambiguous
     No common prefixes of length ≥ k
- No left recursion
- ... (more details later)...
- Collectively, the restrictions guarantee that, given k input tokens, one can always select the correct rhs to expand

Common prefix S ::= if Test then Stmts end if Test then Stmts end |

Left recursion

E ::= E op E | ...



## Eliminating common prefixes

- Left factor them, creating a new non-terminal for the common prefix and/or different suffixes
- Before
  - If ::= if Test then Stmts end | if Test then Stmts else Stmts end
- After
  - If ::= if Test then Stmts IfCont
    IfCont ::= end | else Stmts end
- Grammar is a bit uglier
- ↑ Easy to do manually in a recursive-descent

33



## Eliminating left recursion:

Before

```
E ::= E + T | T
T ::= T * F | F
F ::= id | (E) | ...
```

After

```
::= T ECont
ECont ::= + T ECont | \epsilon
T ::= F TCont TCont ::= * F TCont | \epsilon
       ::= id | ( E ) | ...
```



#### Just add sugar

```
T ::= F { * F }
F ::= id | (E) | ...
```

- ↑ Sugared form is still pretty readable
- ↑ Easy to implement in hand-written recursive descent parser
- Concrete syntax tree is not as close to abstract syntax tree

## LL(1) Parsing Theory

Goal: Formal, rigorous description of those grammars for which "I can figure out how to do a top-down parse by looking ahead just one token", plus corresponding algorithms.

T = Set of Terminals (Tokens)

N = Set of Nonterminals

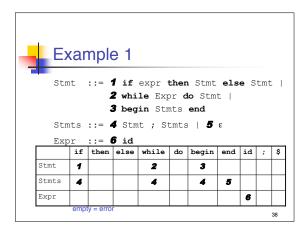
= End-of-file character (T-like, but not in N  $\cup$  T)

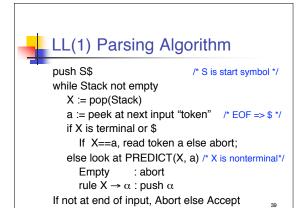


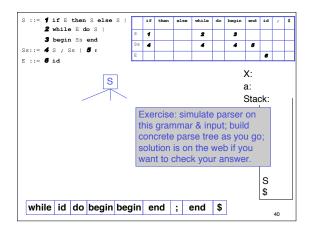
#### Table-driven predictive parser

- Automatically compute PREDICT table from grammar
- PREDICT(nonterminal,input-symbol)
  - → action, e.g. which rhs or error

37









## Constructing PREDICT: overview

- Compute FIRST set for each rhs
  - All tokens that can appear first in a derivation from that rhs
- In case rhs can be empty, compute FOLLOW set for each non-terminal
  - All tokens that can appear right after that nonterminal in a derivation
- Constructions of FIRST and FOLLOW sets are interdependent
- PREDICT depends on both

•

#### Example 1 (cont.)

	FIRST	FOLLOW
1 S ::= if E then S else S		
2   while E do S		
3   begin Ss end		
4 Ss ::= S ; Ss		
<b>5</b>   ε		
6 E ::= id		

42

May be ε, never \$



### FIRST( $\alpha$ ) – 1<sup>st</sup> "token" from $\alpha$

Definition: For any string  $\alpha$  of terminals and non-terminals, FIRST( $\alpha$ ) is the set of terminals that begin strings derived from  $\alpha$ , together with  $\epsilon$ , if  $\alpha$  can derive  $\epsilon$ . More precisely:

```
For any \alpha \in (N \cup T)^*,

FIRST(\alpha) = \{ a \in T \mid \alpha \Rightarrow^* a \ \beta \text{ for some } \beta \in (N \cup T)^* \} \cup \{ \epsilon, \text{ if } \alpha \Rightarrow^* \epsilon \}
```

43



#### Computing FIRST– 4 cases

- 1. FIRST( $\varepsilon$ ) = { $\varepsilon$ }
- 2. For all  $a \in T$ , FIRST(a) = {a}
- 3. For all  $A \in N$ , repeat until no change

If there is a rule  $A \rightarrow \varepsilon$ , add( $\varepsilon$ ) to FIRST(A)

For all rules  $A \rightarrow Y_1...Y_k$  add(FIRST( $Y_1$ ) -  $\{\epsilon\}$ )

if  $\varepsilon \in FIRST(Y_1)$  then  $add(FIRST(Y_2) - \{\varepsilon\})$ 

if  $\varepsilon \in FIRST(Y_1Y_2)$  then add(FIRST(Y\_3) -  $\{\varepsilon\}$ )

if  $\varepsilon \in \mathsf{FIRST}(\mathsf{Y}_1\mathsf{Y}_2)$  then  $\mathsf{add}(\mathsf{FIRST}(\mathsf{Y}_3) - \{\dots$ 

if  $\varepsilon \in FIRST(Y_1, Y_2...Y_k)$  then  $add(\varepsilon)$ 

44

May be \$,



#### Computing FIRST (Cont.)

For all any string  $Y_1...Y_k \in (N \cup T)^*$ , similar:  $add(FIRST(Y_1) - \{\epsilon\})$  if  $\epsilon \in FIRST(Y_1)$  then  $add(FIRST(Y_2) - \{\epsilon\})$  if  $\epsilon \in FIRST(Y_1Y_2)$  then  $add(FIRST(Y_3) - \{\epsilon\})$  ... if  $\epsilon \in FIRST(Y_1Y_2...Y_k)$  then  $add(\epsilon)$ 

[Note: defined for all strings; really only care about FIRST(right hand sides).]

45



#### FOLLOW(B) – Next "token" after B

Definition: for any non-terminal B, FOLLOW(B) is the set of terminals that can appear immediately after B in some derivation from the start symbol, together with \$, if B can be the end of such a derivation. (\$ represents "end of input".) More precisely: For all  $B \in \mathbb{N}$ ,

FOLLOW(B) = {  $a \in (T \cup \{\$\}) \mid S\$ \Rightarrow^* \alpha B a \beta$ for some  $\alpha, \beta \in (N \cup T \cup \{\$\})^* \}$ 

(S is the Start symbol of the grammar.)

..



## Computing FOLLOW(B)

Add \$ to FOLLOW(S) Repeat until no change For all rules  $A \to \alpha B\beta$  [i.e. all rules with a B in r.h.s], Add (FIRST( $\beta$ ) - { $\epsilon$ }) to FOLLOW(B) If  $\epsilon \in \mathsf{FIRST}(\beta)$  [in particular, if  $\beta$  is empty] then Add FOLLOW(A) to FOLLOW(B)

Assume for all A that  $S \Rightarrow^* \alpha A \beta$  for some  $\alpha, \beta \in (N \cup T)^*$ , else A irrelevant

-

#### PREDICT - Given lhs, which rhs?

For all rules  $A \rightarrow \alpha$ For all  $a \in FIRST(\alpha)$  -  $\{\epsilon\}$   $Add(A \rightarrow \alpha)$  to PREDICT(A,a) If  $\epsilon \in FIRST(\alpha)$  then For all  $b \in FOLLOW(A)$  $Add(A \rightarrow \alpha)$  to PREDICT(A,b)

Defn: G is LL(1) iff every cell has ≤ 1 entry

41



#### Properties of LL(1) Grammars

- Clearly, given a conflict-free PREDICT table (≤ 1 entry/cell), the parser will do something unique with every input
- Key fact is, if the table is built as above, that something is the correct thing
- I.e., the PREDICT table will reliably guide the LL(1) parsing algorithm so that it will
  - Find a derivation for every string in the language
  - Declare an error on every string not in the language

49



#### Exercises (1st especially recommended)

- Easy: Pick some grammar with common prefixes, left recursion, and/or ambiguity.
  - Build PREDICT; it will have conflicts
- Harder: prove that every grammar with ≥1 of those properties will have PREDICT conflicts
- Harder: Find a grammar with none of those features that nevertheless gives conflicts.
  - I.e., absence of those features is necessary but not sufficient for a grammar to be LL(1).
- Harder, for theoryheads: if the table has conflicts, and the parser chooses among them nondeterministically, it will work correctly

50



#### Example 2

E ::= T { + T }

T ::= F { \* F }

F ::= - F | id | (E)

E ::= 1 T E'

E' ::= 2 + T E' | 3 E

T ::= 4 F T'

T' ::= 5 \* F T' | 6 E

F ::= 7 - F | 8 id | 9 (E)

51

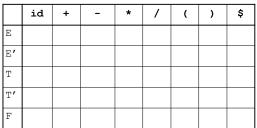


## Example 2 (cont.)

	FIRST	FOLLOW
1 E ::= T E'		
2 E' ::= + T E'		
3   ε		
4 T ::= F T'		
5 T' ::= * F T'		
6   ε		
7 F ::= - F		
8   id		
9   (E)		



#### **Example 2: PREDICT**



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#### PREDICT and LL(1)

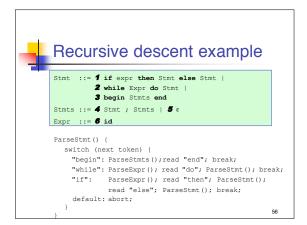
- The PREDICT table has at most one entry in each cell if and only if the grammar is LL(1)
  - there is only one choice (it's predictive) , making it fast to parse and easy to implement
- Multiple entries in a cell
  - Arise with left recursion, ambiguity, common prefixes, etc.
  - Can patch by hand, if you know what to do
  - Or use more powerful parser (LL(2), or LR(k), or...?)
  - Or change the grammar



#### Recursive descent parsers

- Write procedure for each non-terminal
- Each procedure selects the correct right-hand side by peeking at the input tokens
- Then the r.h.s. is consumed
  - If it's a terminal symbol, verify it is next and then advance through the token stream
  - If it's a non-terminal, call corresponding procedure
- Build and return AST representing the r.h.s.

55

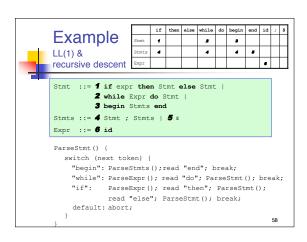


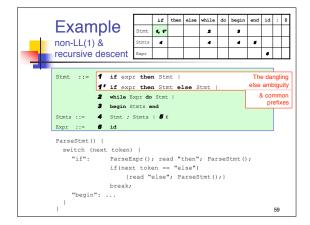


## LL(1) and Recursive Descent

- If the grammar is LL(1), it's easy to build a recursive descent parser
  - One nonterminal/row → one procedure
  - Use 1 token lookahead to decide which rhs
  - Table-driven parser's stack → recursive call stack
- Recursive descent can handle some non-LL(1) features, too.

57







#### It's demo time...

 Let's look at some of the PL/0 code to see how the recursive descent parsing works in practice

```
stmt lisb ::= { <stmt };}
<stmt >::= <dstmb | <out stmb |
| <if stmb | <while stmb |
| <if stmb | <if stmb | </ti>

Parser::ParseStmts() {

StmtArray* Parser::ParseStmts() {

StmtArray* stmts = new StmtArray; Stmt* stmt;

for (;;) {

Token t = scanner->Peek();

switch (t->kind()) {

case IDENT: stmt = ParseIdentStmt(); break;

case OUTPUT: stmt = ParseOutputStmt(); break;

case IF: stmt = ParseIfStmt(); break;

case WHILE: stmt = ParseWhileStmt(); break;

default: return stmts; // no more stmts

}

stmts->add(stmt);

scanner->Read(SEMICOLON);
}

}

61
```

```
Parser::ParseIfStmt()

Stmt* Parser::ParseIfStmt() {
   scanner->Read(IF);
   Expr* test = ParseTest();
   scanner->Read(THEN);
   StmtArray* stmts = ParseStmts();
   scanner->Read(END);
   return new IfStmt(test, stmts);
}
```

```
Parser::ParseWhileStmt()

Stmt* Parser::ParseWhileStmt()

Stmt* Parser::ParseWhileStmt() {
    scanner->Read(WHILE);
    Expr* test = ParseTest();
    scanner->Read(DO);
    StmtArray* stmts = ParseStmts();
    scanner->Read(END);
    return new WhileStmt(test, stmts);
}
```

```
<id stmt>
                                      ::= <call stmt> | <assign stmt>
                           <call stmt> ::= IDENT "(" [ <exprs> ] ")"
                           <assign stmt>::= <lvalue> := <expr>
Parser::
                                      ::= IDENT
                           < lvalue>
ParseldentStmt()
Stmt* Parser:: ParseIdentStmt() {
 Token* id = scanner->Read(IDENT);
if (scanner->CondRead(LPAREN)){
      ExprArray* args;
      if (scanner->CondRead (RPAREN)) {
           args = NULL;
      } else {
          args = ParseExprs();
          scanner->Read(RPAREN);
      return new CallStmt(id->ident(), args);
      LValue* lvalue = new VarRef(id->ident());
      scanner->Read(GETS);
      return new AssignStmt(lvalue, ParseExpr());
```

```
Parser::ParseSum()

Expr* Parser::ParseSum() {

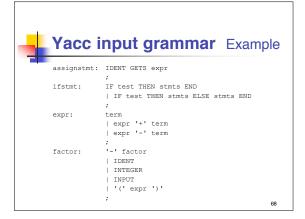
Expr* expr = ParseTerm();
  for (;;) {
    Token* t = scanner->Peek();
    if (t->kind() == PLUS || t->kind() == MINUS) {
        scanner->Get(); // eat the token
        Expr* expr2 = ParseTerm();
        expr = new BinOp(t->kind(), expr, expr2);
    } else {
        return expr;
    }
}
```



#### Yacc - A bottom-up-parser generator

- "yet another compiler-compiler"
- Input:
- grammar, possibly augmented with action code
- Output:
  - C code to parse it and perform actions
- LALR(1) parser generator
  - practical bottom-up parser
  - more powerful than LL(1)
- modern updates of yacc
  - yacc++, bison, byacc, ...

67





#### Yacc with actions

```
assignstmt: IDENT GETS expr { $$ = new AssignStmt($1, $3);}
ifstmt: IF be THEN stmts END { $$ = new IfStmt($2,$4,NULL);}
       | IF be THEN stmts
               ELSE stmts END { $$ = new IfStmt($2,$4,$6);}
                               { $$ = $1; }
        | expr '+' term
                              { $$ = new BinOp(PLUS, $1, $3);}
         expr '-' term
                               { $$ = new BinOp(MINUS, $1, $3);}
factor: '-' factor
                              { $$ = new UnOp(MINUS, $2); }
                               { $$ = new VarRef($1); }
                              { $$ = new IntLiteral($1); }
{ $$ = new InputExpr; }
         INTEGER
         INPUT
         '(' expr ')'
                               { $$ = $2; }
```



## Parsing summary

- Discover/impose a useful (hierarchical) structure on flat token sequence
  - Represented by Abstract Syntax Tree
- Validity check syntax of input
  - Could build concrete syntax tree (but don't)
- Many methods available
  - Top-down: LL(1)/recursive descent common for simple, by-hand projects
  - Bottom-up: LR(1)/LALR(1)/SLR(1) common for more complex projects
    - parser generator (e.g., yacc) almost necessary

70



## Parsing summary – Technical details you should know

- Context-free grammars
- Definitions
- Manipulations (algorithmic)
- Left factor common prefixes
- Eliminating left recursion
- Ambiguity & (semi-
- heuristic) fixes
  - meta-rules (code/ precedence tables)
  - rewrite grammarchange language
- Building a table-driven predictive parser
  - LL(1) grammar: definition & common obstacles
  - PREDICT(nonterminal, input symbol)
  - FIRST(RHS)
  - FOLLOW(nonterminal)
- Building a recursive descent parser
  - Including AST