


## CFG terminology

- Terminals: alphabet, or set of legal tokens
- Nonterminals: represent abstract syntax units
- Productions: rules defining nonterminals in terms of a finite sequence of terminals and nonterminals
- Start symbol: root symbol defining the language

| Program $::=$ Stmt |  |
| :--- | :--- |
| Stmt | $::=$ if Expr then Stmt else Stmt end |
| Stmt | $::=$ while Expr do Stmt end |



## EBNF description of PL/O

```
OutStmt ::= output := Expr
    IfStmt ::= if Test then StmtList end
    WhileStmt ::= while Test do StmtList end
    Test ::= odd Sum | Sum Relop Sum
    Relop ::= <= | <> | < | >= | > | =
    Exprs ::= Expr {, Expr }
    Expr ::= Sum
    Sum ::= Term { (+ | -) Term }
    Term ::= Factor { (* | /) Factor }
    Factor ::= - Factor | LValue | Integer |
        input | ( Expr )
```


## Exercise: produce a syntax

## tree for squares. 0

## EBNF description of PL/0

ProcDecl ::=
procedure Id ( [ FormalDecl \{, FormalDecl\} ] ) ; Block Id
FormalDecl ::= Id : Type
Type $::=$ int
StmtList $::=$ \{ Stmt ; \}
Stmt : : = CallStmt | AssignStmt | OutStmt |
IfStmt | WhileStmt
CallStmt : := Id ( Exprs ] )
AssignStmt ::= Lvalue := Expr
Lvalue $::=$ Id

```
module main;
```

module main;
var x:int, squareret:int;
var x:int, squareret:int;
procedure square(n:int);
procedure square(n:int);
begin
begin
squareret := n * n;
squareret := n * n;
end square;
end square;
begin
begin
x := input;
x := input;
while x <> 0 do
while x <> 0 do
square(x);
square(x);
output := squareret;
output := squareret;
x := input;
x := input;
end;
end;
end main.

```
end main.
```

Parse trees

- We represent derivations and parses as parse trees
- Concrete syntax tree
- Exact reflection of the grammar
- Abstract syntax tree
- Simplified version, reflecting key structural information
- E.g., omit superfluous punctuation \& keywords



## Resolving ambiguity: \#1

- Add a meta-rule
- For instance, "else associates with the closest previous unmatched if"
$\uparrow$ This works and keeps the original grammar intact
$\downarrow$ But it's ad hoc and informal


Resolving ambiguity: \#3

- Redesign the programming language to remove the ambiguity

$\uparrow$ Formal, clear, elegant
$\uparrow$ Allows StmtList in then and else branch, without adding begin/end
* Extra end required for every if statement


## Option \#1: add meta-rules

- Add meta-rules for precedence and associativity

E ::= E+E I E-E I E*E I E/E I E^E I (E) I -E I ...

- +,- < *,l < unary - < ^ etc.
- +,-,,, , left-associative; ^ right associative
$\uparrow$ Simple, intuitive
$\downarrow$ But not all parsers can support this $\uparrow$ yacc does



## What about that

expression grammar?
How to resolve its ambiguity?

- Option \#1: add meta-rules for precedence and associativity
- Option \#2: modify the grammar to explicitly resolve the ambiguity
- Option \#3: redefine the language


- Top-down parser => LL(k) grammar
- Bottom-up parser => LR(k) grammar



## Predictive parser

- A top-down parser that can select the correct rhs looking at the next $k$ tokens (lookahead)
- Efficient
- No backtracking is needed
- Linear time to parse
- Implementation
- Table-driven: pushdown automaton (PDA) - like table-driven FSA plus stack for recursive FSA calls
- Recursive-descent parser [used in PL/O]
- Each non-terminal parsed by a procedure
- Call other procedures to parse sub-non-terminals, recursively



## Eliminating common prefixes

- Left factor them, creating a new non-terminal for the common prefix and/or different suffixes
- Before
- If ::= if Test then Stmts end $\begin{aligned} \text { if } & \text { Test then Stmts else Stmts end }\end{aligned}$
- After

$\downarrow$ Grammar is a bit uglier
$\uparrow$ Easy to do manually in a recursive-descent parser



## LL(1) Parsing Theory

Goal: Formal, rigorous description of those grammars for which "I can figure out how to do a top-down parse by looking ahead just one token", plus corresponding algorithms.
Notation:
$\mathrm{T}=$ Set of Terminals (Tokens)
$\mathrm{N}=$ Set of Nonterminals
$\$=$ End-of-file character ( T -like, but not in $\mathrm{N} \cup \mathrm{T}$ )



## Computing FIRST (Cont.)

4. For all any string $Y_{1} \ldots Y_{k} \in(N \cup T)^{*}$, similar: $\operatorname{add}\left(\operatorname{FIRST}_{1}\left(\mathrm{Y}_{1}\right)-\{\varepsilon\}\right)$
if $\varepsilon \in \operatorname{FIRST}\left(Y_{1}\right)$ then $\operatorname{add}\left(\operatorname{FIRST}\left(Y_{2}\right)-\{\varepsilon\}\right)$
if $\varepsilon \in \operatorname{FIRST}\left(Y_{1} Y_{2}\right)$ then $\operatorname{add}\left(\operatorname{FIRST}\left(Y_{3}\right)-\{\varepsilon\}\right)$
...
if $\varepsilon \in \operatorname{FIRST}\left(Y_{1} Y_{2} \ldots Y_{k}\right)$ then $\operatorname{add}(\varepsilon)$
[Note: defined for all strings; really only care about FIRST(right hand sides).]

## Computing FIRST- 4 cases

$\operatorname{FIRST}(\varepsilon)=\{\varepsilon\}$
For all $a \in T, \operatorname{FIRST}(a)=\{a\}$
3. For all $A \in N$, repeat until no change If there is a rule $\mathrm{A} \rightarrow \varepsilon, \operatorname{add}(\varepsilon)$ to $\operatorname{FIRST}(\mathrm{A})$
For all rules $A \rightarrow Y_{1} \ldots Y_{k} \quad \operatorname{add}\left(\operatorname{FIRST}\left(Y_{1}\right)-\{\varepsilon\}\right)$ if $\varepsilon \in \operatorname{FIRST}\left(Y_{1}\right) \quad$ then $\operatorname{add}\left(\operatorname{FIRST}\left(Y_{2}\right)-\{\varepsilon\}\right)$ if $\varepsilon \in \operatorname{FIRST}\left(Y_{1} Y_{2}\right)$ then $\operatorname{add}\left(\operatorname{FIRST}\left(Y_{3}\right)-\{\varepsilon\}\right)$ if $\varepsilon \in \operatorname{FIRST}\left(Y_{1} Y_{2} \ldots Y_{k}\right)$ then $\operatorname{add}(\varepsilon)$
$\left.\begin{array}{l}\text { Fefinition: for any non-terminal B, FOLLOW(B) } \\ \text { is the set of terminals that can appear } \\ \text { immediately after B in some derivation from } \\ \text { the start symbol, together with \$, if B can be } \\ \text { the end of such a derivation. (\$ represents } \\ \text { "end of input".) More precisely: For all } \mathrm{B} \in \mathrm{N}, \\ \text { FOLLOW }(\mathrm{B})=\left\{\mathrm{a} \in(\mathrm{T} \cup\{\$\}) \mid \mathrm{S} \$ \Rightarrow^{*} \alpha \mathrm{~B} \mathrm{a} \beta\right. \\ \left.\text { for some } \alpha, \beta \in(\mathrm{N} \cup \mathrm{T} \cup\{\$\})^{*}\right\} \\ \text { never } \varepsilon\end{array}\right]$

PREDICT - Given Ins, which rhs?
For all rules $\mathrm{A} \rightarrow \alpha$
For all $a \in \operatorname{FIRST}(\alpha)-\{\varepsilon\}$
$\operatorname{Add}(\mathrm{A} \rightarrow \alpha)$ to $\operatorname{PREDICT}(\mathrm{A}, \mathrm{a})$
If $\varepsilon \in \operatorname{FIRST}(\alpha)$ then
For all $b \in \operatorname{FOLLOW}(A)$
$\operatorname{Add}(A \rightarrow \alpha)$ to $\operatorname{PREDICT}(A, b)$
Defn: $G$ is $L L(1)$ iff every cell has $\leq 1$ entry


## Example 2: PREDICT

|  | id | + | - | $*$ | $/$ | ( | ) | \$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| E |  |  |  |  |  |  |  |  |
| $\mathrm{E}^{\prime}$ |  |  |  |  |  |  |  |  |
| T |  |  |  |  |  |  |  |  |
| $\mathrm{T}^{\prime}$ |  |  |  |  |  |  |  |  |
| F |  |  |  |  |  |  |  |  |




Example 2 (cont.)

## PREDICT and LL(1)

- The PREDICT table has at most one entry in each cell if and only if the grammar is LL(1)
- $\therefore$ there is only one choice (it's predictive), making it fast to parse and easy to implement
- Multiple entries in a cell
- Arise with left recursion, ambiguity, common prefixes, etc.
- Can patch by hand, if you know what to do
- Or use more powerful parser (LL(2), or LR(k), or ...?)
- Or change the grammar



## LL(1) and Recursive Descent

- If the grammar is LL(1), it's easy to build a recursive descent parser
- One nonterminal/row $\rightarrow$ one procedure
- Use 1 token lookahead to decide which rhs
- Table-driven parser's stack $\rightarrow$ recursive call stack
- Recursive descent can handle some non-LL(1) features, too.



## It's demo time...

- Let's look at some of the PL/0 code to see how the recursive descent parsing works in practice





<term> $::=<$ factor> $\{(* \mid /)$ <factor> $\}$


## Parser::ParseTerm()

```
Expr* Parser::ParseTerm()
Expr* expr = ParseFactor();
    for (;;) {
        Token* t = scanner->Peek();
        if (t->kind() == MUL | t->kind() == DIVIDE) {
            scanner->Get(); // eat the token
            Expr* expr2 = ParseFactor();
            expr = new BinOp(t->kind(), expr, expr2)
            } else {
            return expr;
        }
    }
    }
```



| Yacc input grammar Example |
| :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Yacc with actions

assignstmt: IDENT GETS expr $\{\$ \$=$ new AssignStmt (\$1, \$3); ifstmt: ;
ifstmt: IF be THEN stmts END $\left\{\begin{array}{l}\text { \$ } \\ = \\ \text { new } \\ \operatorname{IfStmt}(\$ 2, \$ 4, N U L L) ;\}\end{array}\right.$
| IF be THEN stmts
ELSE stmts END $\{\$ \$=$ new $\operatorname{IfStmt}(\$ 2, \$ 4, \$ 6) ;\}$
expr: 't
term $\{\$ \$=\$ 1$;
expr '+' term $\{\$ \$=$ new BinOp(PLUS, \$1, \$3);
| expr '-' term $\{\$ \$=$ new BinOp(MINUS, $\$ 1, \$ 3$ );
factor: ;
'-' factor $\{\$ \$=$ new UnOp(MINUS, \$2); \}
$\mid$ IDENT $\{\$ \$=$ new $\operatorname{VarRef}(\$ 1) ;\}$
I INTEGER $\{\$ \$=$ new IntLiteral (\$1); \}
INPUT \{ $\$ \$=$ new InputExpr; \}
' (' expr ')' $\{\$ \$=\$ 2 ;\}$

## Parsing summary

- Discover/impose a useful (hierarchical) structure on flat token sequence
- Represented by Abstract Syntax Tree
- Validity check syntax of input
- Could build concrete syntax tree (but don't)
- Many methods available
- Top-down: LL(1)/recursive descent common for simple, by-hand projects
- Bottom-up: $\operatorname{LR}(1) / \operatorname{LALR}(1) / \operatorname{SLR}(1)$ common for more complex projects
- parser generator (e.g., yacc) almost necessary


