CSE 401 – Compilers

Lecture 3: Regular Expressions & Scanning, continued…

Michael Ringenburg

Winter 2013

Today's Agenda

- Last time we reviewed languages and grammars, and briefly started discussing regular expressions.
- Today I'll restart the regular expression discussion, since it felt a bit rushed.
- I'll then describe how to build finite automata that recognize regular expressions.
- On Monday, I'll discuss how scanners are implemented.

Announcements

- Homework 1 will be out later today.
  - I'll post on course website and send email.
  - Due next Friday (January 18).
- First part of the project (the scanner) will be assigned early next week.
- Office hours selected, starting next week:
  - Laure: Mondays (except 1/21 & 2/18), 4-5, CSE 218
  - Mike: Wednesdays, 2:30-3:30, CSE 212
    - Or by appointment on Tuesdays
  - Zach: Fridays, 1:30-2:30, CSE 218

Regular Expressions

- Defined over some alphabet Σ
  - For programming languages, alphabet is usually ASCII or Unicode
- If *re* is a regular expression, *L*(*re*) is the language (set of strings) generated by *re*

Fundamental REs

re	*L*(*re*)	Notes
a	{ a }	Singleton set, for each symbol a in the alphabet Σ
ε	{ ε }	Empty string
∅	{ }	Empty language

These are the basic building blocks that other regular expressions are built from.

Operations on REs

re	*L*(*re*)	Notes
rs	L(r)L(s)	Concatenation: a string from r followed by a string from s
r|s	L(r) ∪ L(s)	Combination (union): a string from either r or s
r*	L(r)*	Kleene closure: sequence of 0 or more strings from r

Precedence: * (highest), concatenation, | (lowest)

Parentheses can be used to group REs as needed

Examples

re	Meaning
+	single + character
!	single ! character
!=	2 character sequence "!="
xyzzy	5 character sequence "xyzzy"
(1|0)*	Zero or more binary digits
(1|0)(1|0)*	Binary constant (possible leading 0s)
0|1(1|0)*	Binary constant without extra leading 0s, i.e, 0 or starts with 1 (| has lowest precedence)

Abbreviations

The basic operations generate all possible regular expressions, but there are common abbreviations used for convenience. Some examples:

Abbr.	Meaning	Notes
r+	(rr*)	1 or more occurrences
r?	(r | ε)	0 or 1 occurrence
[a-z]	(a|b|…|z)	1 character in given range
[abxyz]	(a|b|x|y|z)	1 of the given characters

Exercise: What do these represent?

re	Meaning
[abc]+	
[abc]*	
[0-9]+	
[1-9][0-9]*	
[a-zA-Z][a-zA-Z0-9_]*	

Exercise: What do these represent?

re	Meaning
[abc]+	Sequence of one or more a's, b's and c's
[abc]*	Zero or more a's, b's, and c's
[0-9]+	Non-negative integer (possibly with leading 0s)
[1-9][0-9]*	Positive integer (no leading 0s)
[a-zA-Z][a-zA-Z0-9_]*	One or more letters or digits, must start with a letter.

Abbreviations

- Many systems allow abbreviations to make writing and reading definitions or specifications easier

name ::= *re*

  – Restriction: abbreviations may not be circular (recursive) either directly or indirectly (else would be not be a regular language)
    - **digit ::= [0-9]** is okay
    - **number ::= digit number** is not

Example

- Possible syntax for numeric constants

digit ::= [0-9]
digits ::= *digit*+
number ::= *digits* (. *digits*)?
([eE] (+ | -)? *digits*) ?

- Notice that this allows (unnecessary) leading 0s, e.g., 00045.6. (0, or 0.14 would be necessary 0s.)
- How would you prevent that?

Example

- Possible syntax for numeric constants

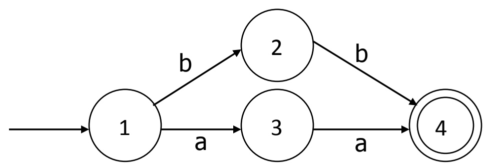
digit ::= [0-9]
nonzero_digit ::= [1-9]
digits ::= *digit*+
number ::= (0 | *nonzero_digit digits?)*
  (. *digits*)?
  ([eE] (+ | -)? *digits*) ?

Recognizing REs

- Finite automata can be used to recognize languages generated by regular expressions
- Can build by hand or automatically
  - Reasonably straightforward, and can be done systematically
  - Tools like Lex, Flex (for compilers written in C++), and JFlex (for compilers written in Java) do this automatically, given a set of REs.

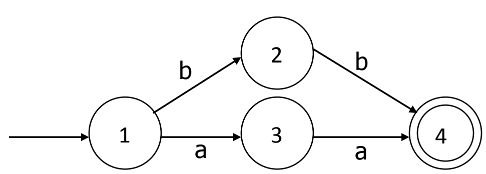
Finite State Automaton

- Review from your CS theory class …
- A finite set of states
  - One marked as initial state
  - One or more marked as final states
  - States sometimes labeled or numbered
- A set of transitions from state to state
  - Each labeled with symbol from Σ (the alphabet), or ε
  - The symbols correspond to characters in the input stream.

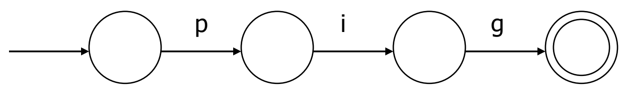


Finite State Automaton

- Operate by reading input symbols (usually characters)
  - Transition can be taken if labeled with current symbol
  - ε-transition can be taken at any time
- Accept when final state reached and no more input
  - Slightly different in a scanner, where the FSA is used as a subroutine to find the longest input string that matches a token RE.
- Reject if no transition possible, or no more input and not in final state (DFA)

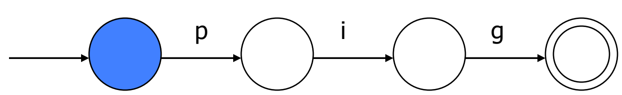


Example: FSA for "pig"



Example: FSA for "pig"

Input 1: pig



Status: Executing…

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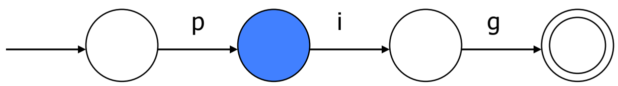

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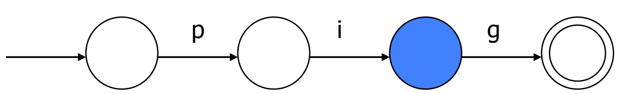
Input 1: pig



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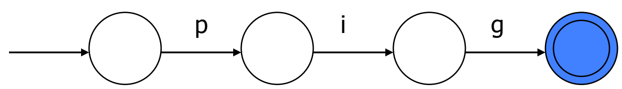
Example: FSA for "pig"

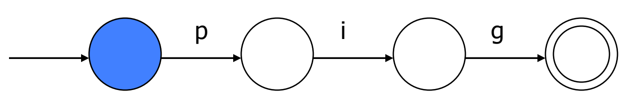
Input 1: pig



Status: Accept! (In a final state, and no more input.)

Example: FSA for "pig"

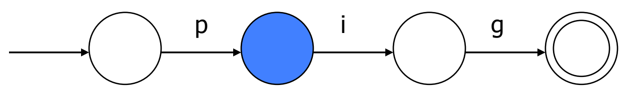
Input 2: pit



Status: Executing…

Example: FSA for "pig"

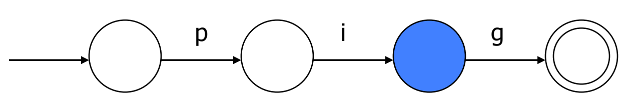
Input 1: pit



Status: Executing…

Example: FSA for "pig"

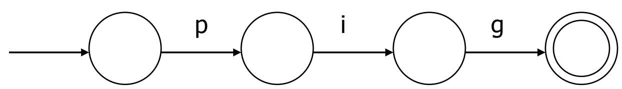
Input 1: pit



Status: Executing…

Example: FSA for "pig"

Input 1: pit



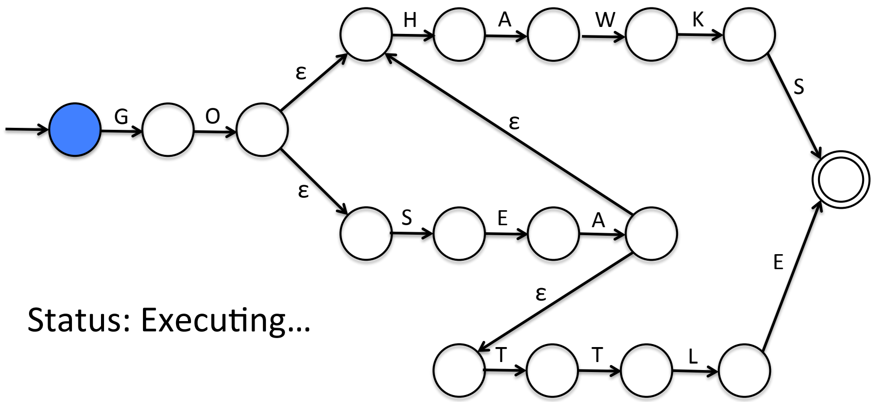
Status: Reject! (No legal transitions on 't'.)

DFA vs NFA

- Deterministic Finite Automata (DFA)
  - No choice of which transition to take
- Non-deterministic Finite Automata (NFA)
  - Choice of transition in at least one case
  - ε transitions (arcs): If the current state has any outgoing ε arcs, we can follow any of them *without* consuming any input
  - Accept if some way to reach a final state on given input
  - Reject if no possible way to final state
  - Modeling choice option 1: guess path, backtrack if rejects
  - Option 2: "clone" at choice point, accept if any clone accepts

Example NFA

Input 1: GOSEAHAWKS



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Example NFA

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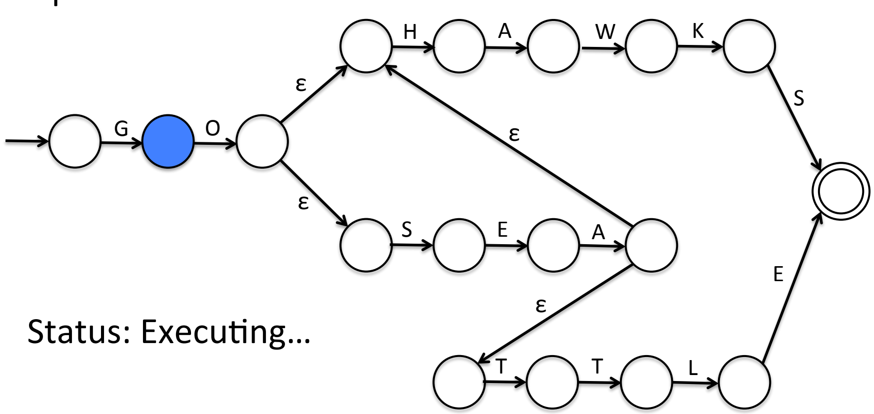

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Example NFA

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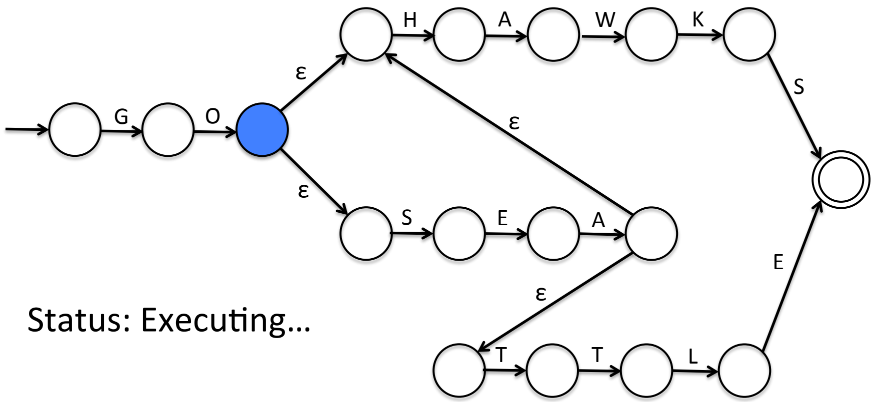

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Example NFA

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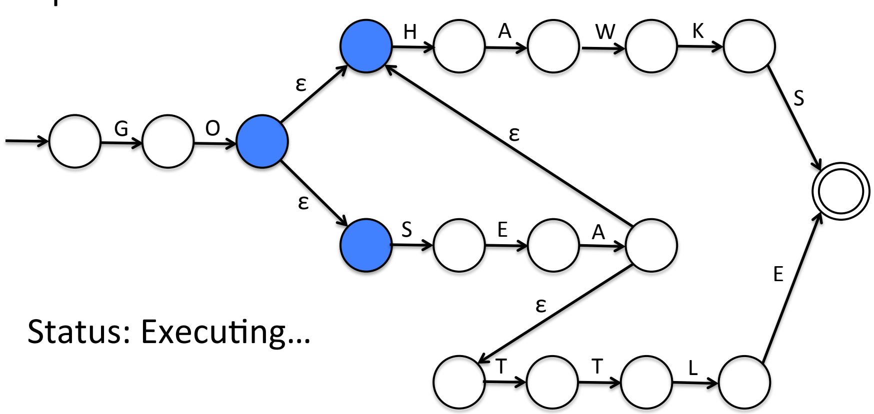

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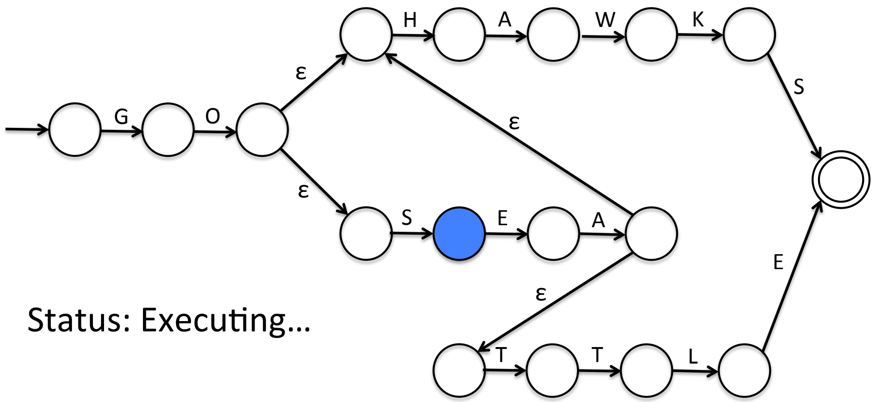

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Example NFA

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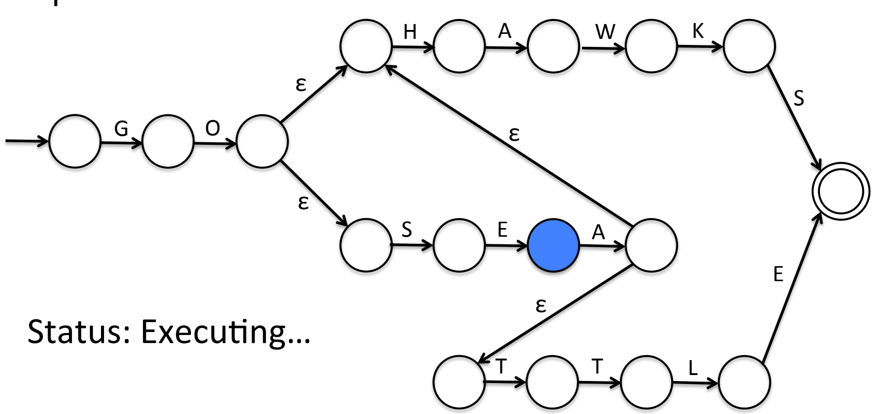

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Example NFA

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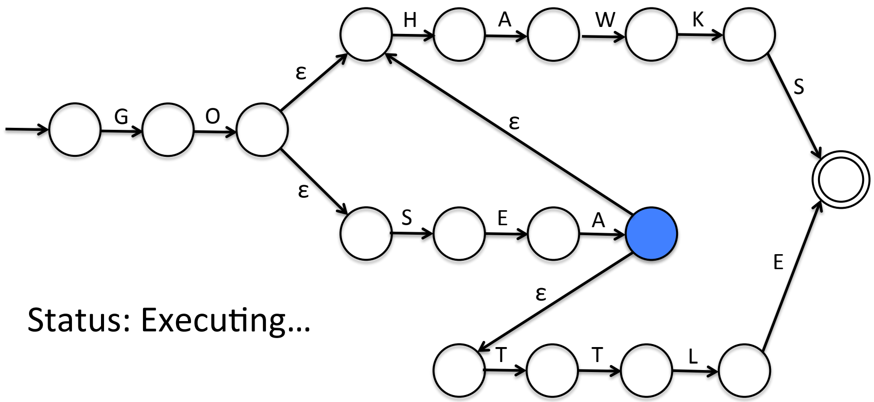

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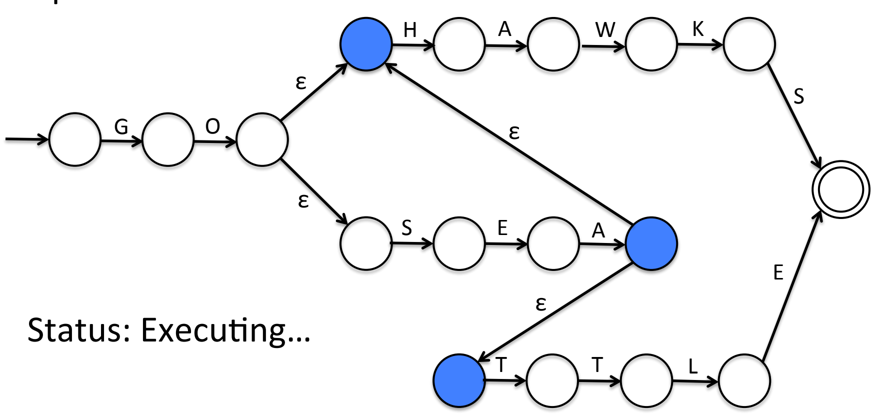

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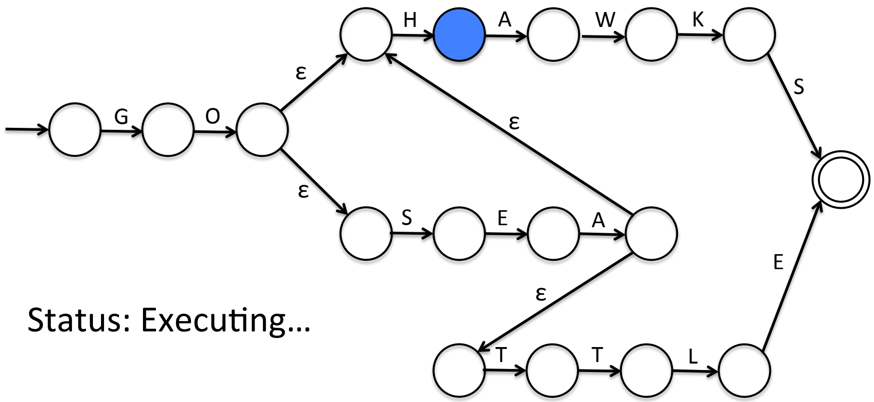

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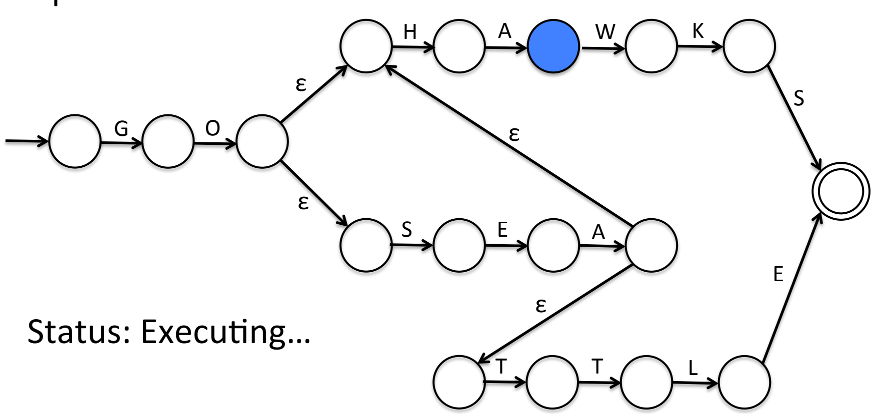

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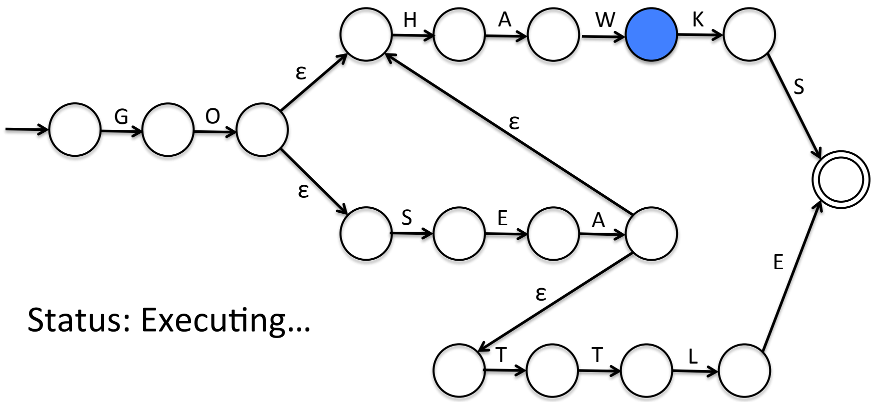

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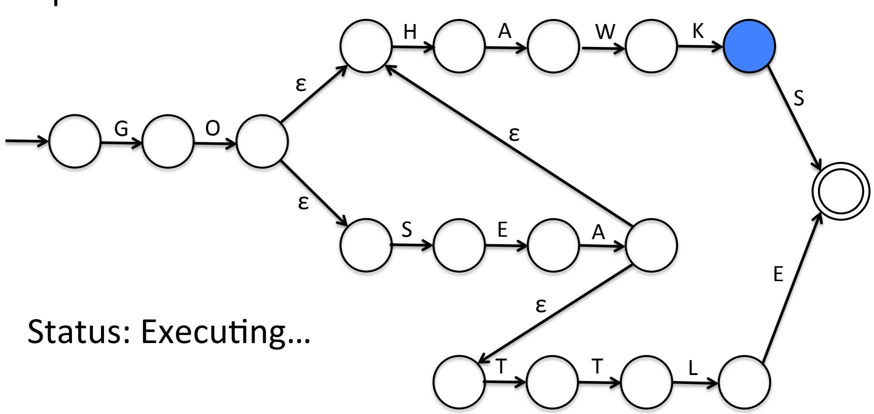

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Example NFA

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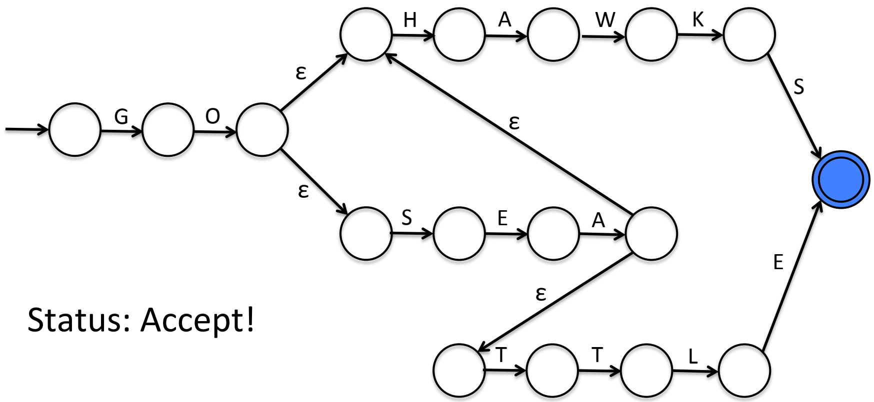


Status: Accept!

FAs in Scanners

- Want DFA for speed (no backtracking or cloning)
- But conversion from regular expressions to NFA is easier
- Luckily, there is a well-defined procedure for converting an NFA to an equivalent DFA

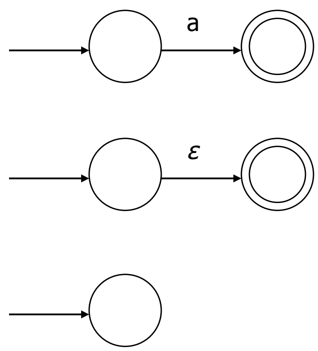
From RE to NFA: base cases

These correspond to the "Fundamental REs" shown earlier.

NFA for symbol 'a'

NFA for empty string (*ε)*

NFA for empty set (∅)

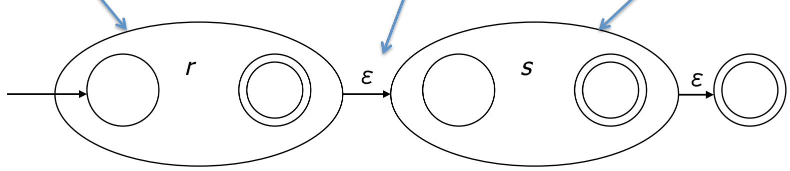


Concatenation: *r s*

This represent an NFA that accepts the regular expression *r*

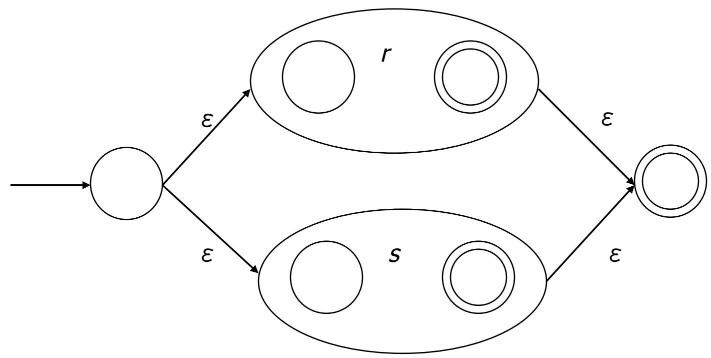
An ε- transition from every final state of the *r* machine to start state of the *s* machine.

An NFA for RE *s*



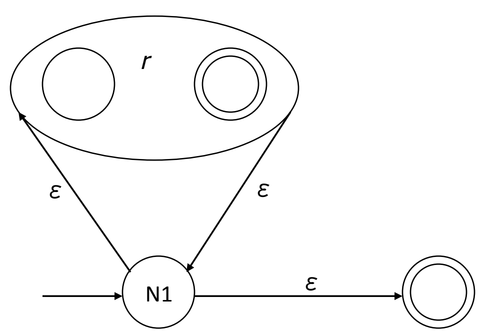
The idea: When we find a string that matches the regular expression *r*, we start trying to match the regular expression *s*. Since this is an NFA, it's okay if we guess wrong – we will make an ε transition from every prefix of the input that matches *r*, and thus check all possible matches.

Union/Combination: *r* | *s*



The idea: Non-deterministically check if the input matches either *r* or *s*. If either sub-machine reaches a final state, jump to the union machine's final state. If the entire input has been consumed at this point (i.e., the entire string matches *r* or *s)*, the union machine will accept.

Kleene star: *r* *



The idea: At the start node (N1), we attempt to match either the empty string (to account for the possibility of zero occurrence of *r*) or a single match of r. Every time the *r* machine find a potential match, it non-deterministically jumps back to N1 and repeats the process. Since this is an NFA, it's okay if we guess the wrong match of r – we'll try all of them.

Example

- Draw the NFA for (ab|c):

Example

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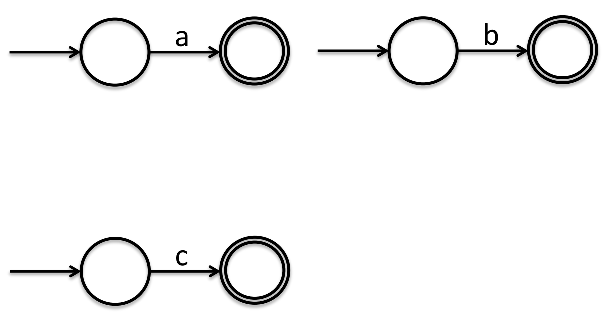

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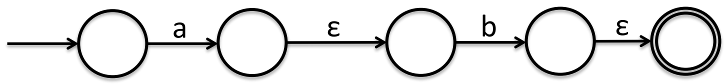

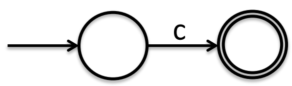

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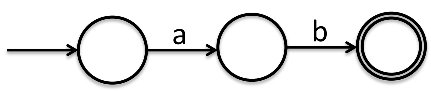
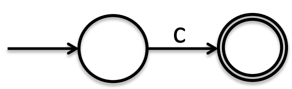


(If a state has a single outgoing ε-transition, and no other outgoing transitions, you can merge it into the ε target.)



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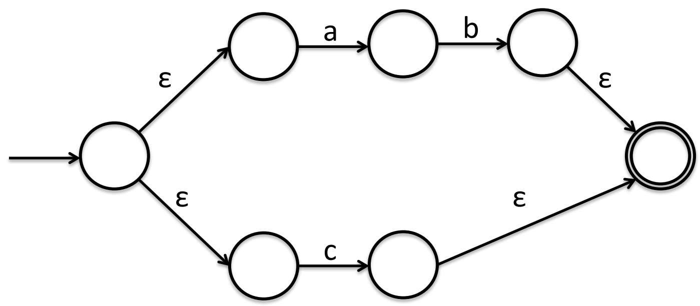

Exercise

- Draw the NFA for: b(at|ag) | bug

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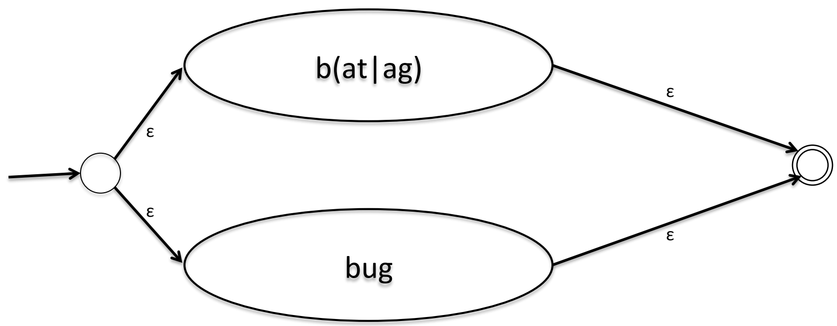

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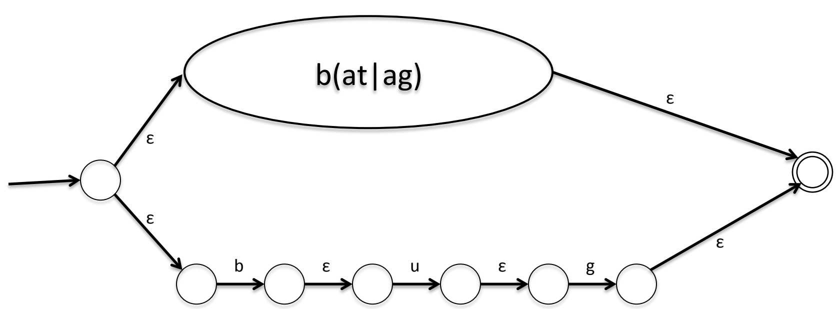

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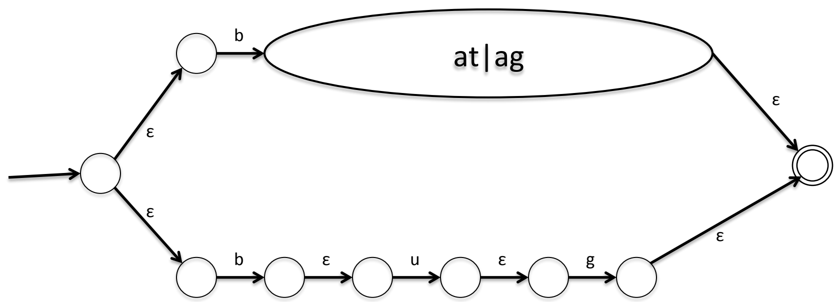

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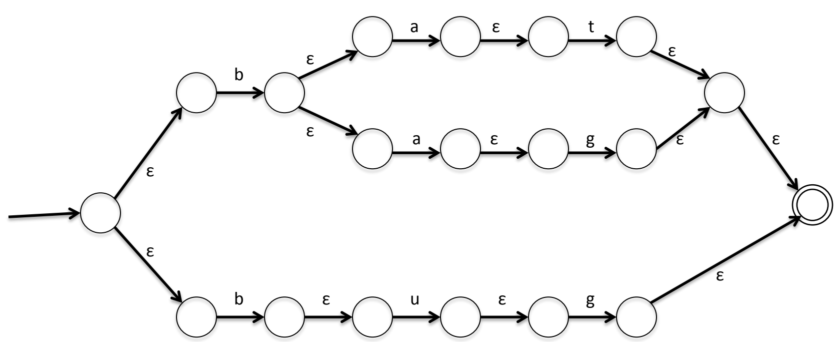
Exercise

- Draw the NFA for: b(at|ag) | bug



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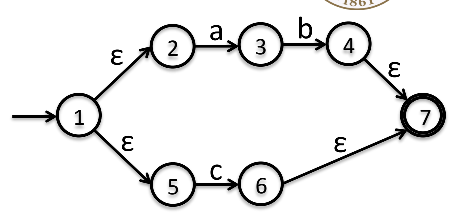
From NFA to DFA

- Subset construction: construct a DFA from an NFA. Each DFA state represents a *set* of NFA states.
- Key idea: State of DFA after reading some input is the set of *all* states that NFA could have reached after reading the same input
- Algorithm (example of a fixed-point computation):
  - Find ε-closure (all states reachable via 0 or more ε-transitions) of start state. Create DFA state corresponding to this set. Add to unvisited list.
  - While there exist unvisited DFA states, select one (call it *d*):
    - For each symbol *s* in the alphabet, determine the NFA states reachable by any NFA state in the set corresponding to *d*.
    - Determine the ε closure of these states. Create a transition from *d* on symbol *s* to a DFA state corresponding to this closure set.
    - If this state is new, add to the unvisited list.

Example

- Convert NFA to a DFA:



Example

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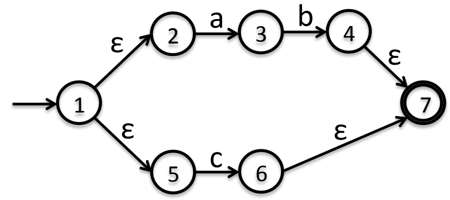
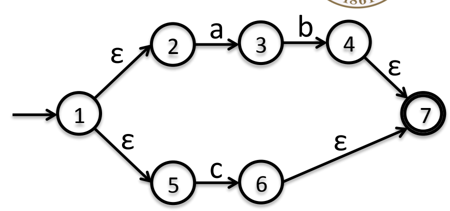
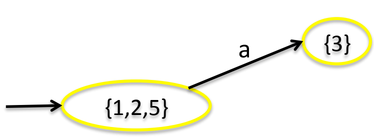


Epsilon closure of start state

Example

- Convert NFA to a DFA:

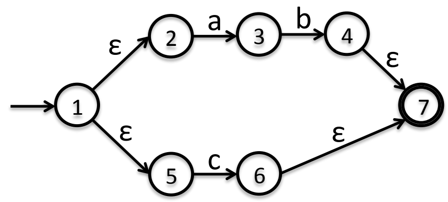


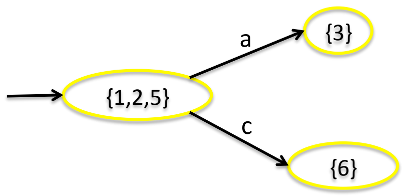


Visit {1,2,5}: Transitions on ‘a’.
No ε transitions from 3.

Example

- Convert NFA to a DFA:

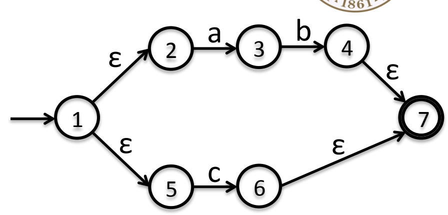


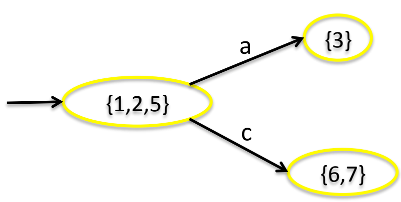


Visit {1,2,5}: Transitions on ‘c’.

Example

- Convert NFA to a DFA:

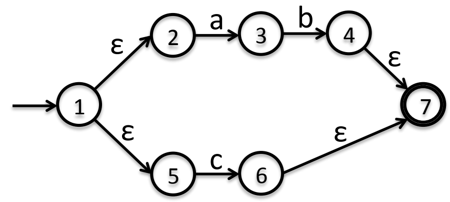


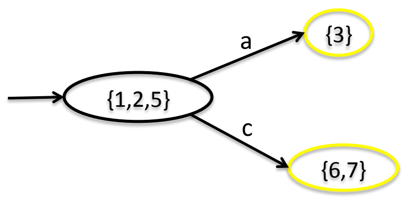


Epsilon closure of {6}

Example

- Convert NFA to a DFA:

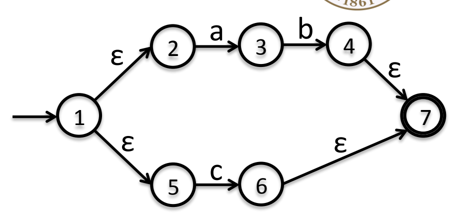


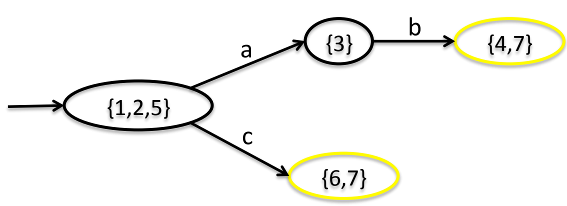


Done with {1,2,5}

Example

- Convert NFA to a DFA:

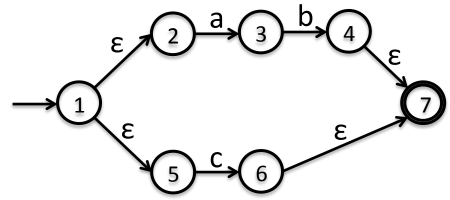


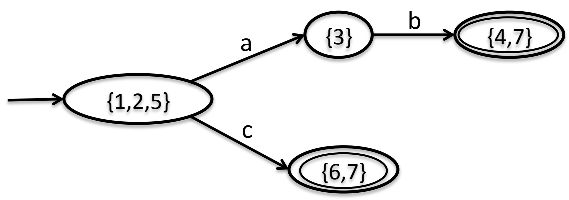


Visit {3}: Just one transition.
Do ε closure of new state. Mark {3} as visited.

Example

- Convert NFA to a DFA:





Last two states have no transitions, but contain a final state, so mark as final.

Next Time

- Implementing a scanner
  - By hand
  - Via automated tools
- Enjoy your weekend
  - Go Hawks!