## CSE 490 G

Assignment 3
Due Friday, January 27, 2006

1. Consider the probability distribution $a: 1 / 4, b: 1 / 2, c: 1 / 4$.
(a) Use arithmetic coding with scaling to code the string bbbbba. Show the steps in the process and the value of $C$ which keeps track of the number of complementary bits to be output after a 0 or 1 is output. I chose this example because the scaled interval are very easy to calculate.
(b) Use arithmetic decoding with scaling to decode 00000000001 (10 zeros followed by a 1) assuming the string decoded is of length 6 .
2. In some situations a data file has the property of having a relatively small "working set". This means that the current symbol most often comes from a fairly small set of symbols. For example, consider the string $x$ of symbols in he alphabet $\{a, b, c, d, e, f\}$ :

$$
x=\text { abccaabbbcabddcbcbddceddeddeccdeefeffddefdfdeeff }
$$

which tends to have a working set of about size 3 .
In the move-to-front algorithms we first give an initial index to each symbol as follows:

| 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a | b | c | d | e | f |

Suppose symbol $x$ with index $i$ is encountered in the input stream. The index $i$ is output. Then the index of $x$ becomes 0 and all the symbols indexed $<i$ have their index increased by 1 . For example the input $y$

$$
y=\mathrm{bbbfbb}
$$

has output 100510 because after the first b is input the indexing becomes

| 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| b | a | c | d | e | f |

and after the f is input then the indexing becomes

| 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| f | b | a | c | d | e |

and after the fourth b is input the indexing becomes

| 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| b | f | a | c | d | e |

(a) Compute the empirical entropy of the string $x$. (The empirical entropy is done using the frequencies of the symbols found in the string.)
(b) Compute the empirical entropy of the string output in the move-to-front algorithm executed on $x$.
(c) In move-to-front compression both the encoder and decoder know the initial indexing and the output of the move-to-front algorithm is losslessly encoded, say with arithmetic coding. Give one example of a data set that might be amenable to move-to-front compression, and explain why it is so. English text is not an example.

