

## Solution A

- If $x_{n+1} x_{n+2} \ldots x_{n+k}$ is a substring of $x_{1} x_{2} \ldots x_{n}$ then $x_{n+1} x_{n+2} \ldots x_{n+k}$ can be coded by <j,k> where $j$ is the beginning of the match.
- Example
$\frac{\text { ababababa }}{\text { coded }}$ babababababababab....
$\frac{\text { ababababa }}{<2,8>}$


## The Dictionary is Implicit

- Ziv and Lempel, 1977
- Use the string coded so far as a dictionary.
- Given that $x_{1} x_{2} \ldots x_{n}$ has been coded we want to code $x_{n+1} x_{n+2} \ldots x_{n+k}$ for the largest $k$ possible.

| Solution A <br> - If $x_{n+1} x_{n+2} \ldots x_{n+k}$ is a substring of $x_{1} x_{2} \ldots x_{n}$ then $x_{n+1} x_{n+2} \ldots x_{n+k}$ can be coded by <j,k> where $j$ is the beginning of the match. <br> - Example $\begin{aligned} & \frac{\text { ababababa }}{\text { coded }} \text { babababababababab.... } \\ & \frac{\text { ababababa }}{\frac{\text { babababa babababab.... }}{<2,8>}} \end{aligned}$ |
| :---: |
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## Solution A Problem

- What if there is no match at all in the dictionary?
ababababa cabababababababab.... coded
- Solution B. Send tuples $<j, k, x>$ where - If $k=0$ then $x$ is the unmatched symbol
- If $k>0$ then the match starts at j and is k long and the unmatched symbol is $x$.


## Solution B

- If $x_{n+1} x_{n+2} \ldots x_{n+k}$ is a substring of $x_{1} x_{2} \ldots x_{n}$ and $x_{n+1} x_{n+2} \cdots x_{n+k} x_{n+k+1}$ is not then $x_{n+1} x_{n+2} \cdots x_{n+k}$ $x_{n+k+1}$ can be coded by
$<j, k, x_{n+k+1}>$
where j is the beginning of the match.
- Examples

$$
\begin{aligned}
& \text { ababababa cabababababababab.... } \\
& \frac{\text { ababababa }}{<0} \frac{\text { ababababab }}{} \text { ababab.... } \\
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\end{aligned}
$$

## Solution B Example

$$
\begin{aligned}
& \underline{a} \text { bababababababababababab..... } \\
& <0,0, a> \\
& \underline{a} \underline{b} \text { ababababababababababab..... } \\
& <0,0, b> \\
& \underline{a} \underline{b} \frac{a b a}{<1,2, a>} \\
& \underline{a} \underline{b} \frac{a b a}{} \frac{b a b a b}{<2,4, b>} \text { ababababababab.... } \\
& \underline{a} \underline{b} \frac{a b a}{} \frac{b a b a b}{} \frac{a b a b a b a b a b a}{<1,10, a>}
\end{aligned}
$$



## Surprise Decoding

$$
\begin{array}{ll}
<0,0, a><0,0, b><1,22, \$> \\
& \\
<0,0, a> & a \\
<0,0, b> & b \\
<1,22, \$> & a \\
<2,21, \$> & b \\
<3,20, \$> & a \\
<4,19, \$> & b \\
\ldots & \\
<22,1, \$> & b \\
<23,0, \$> & \$
\end{array}
$$

| Surprise Decoding |  |  |
| :---: | :---: | :---: |
| $<0,0, a\rangle<0,0, b><1,22, \$>$ |  |  |
| $\begin{aligned} & <0,0, \mathrm{a}> \\ & <0,0, \mathrm{~b}> \end{aligned}$ | $a-$ |  |
| <1,22,\$> |  |  |
| <2,21,\$> |  |  |
| <3,20,\$> |  |  |
| <4,19,\$> |  |  |
|  |  |  |
| <22,1,\$> | b |  |
| <23,0,\$> | \$ |  |
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## Solution C

- The matching string can include part of itself!
- If $x_{n+1} x_{n+2} \ldots x_{n+k}$ is a substring of

$$
x_{1} x_{2} \ldots x_{n} x_{n+1} x_{n+2} \ldots x_{n+k}
$$

that begins at $j \leq n$ and $x_{n+1} x_{n+2} \cdots x_{n+k} x_{n+k+1}$ is not then $x_{n+1} x_{n+2} \ldots x_{n+k} x_{n+k+1}$ can be coded by $<j, k, x_{n+k+1}>$

## In Class Exercise

- Use Solution C to code the string - abaabaaabaaaab\$
- aaaabaaabaabab\$


## Bounded Buffer - Sliding Window

- We want the triples $<j, k, x>$ to be of bounded size. To achieve this we use bounded buffers.
- Search buffer of size $s$ is the symbols $x_{n-s+1} \ldots x_{n}$ $j$ is then the offset into the buffer.
- Look-ahead buffer of size $t$ is the symbols $x_{n+1} \cdots x_{n+t}$
- Match pointer can start in search buffer and go into the look-ahead buffer but no farther.

Sliding window \begin{tabular}{ccc}
match pointer \& uncoded text pointer <br>

| search buffer |
| :---: |
| coded | \& | look-ahead buffer |
| :---: |
| uncoded | <br>

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\end{tabular}

| Search in the Sliding Window |  |  |  |
| :---: | :---: | :---: | :---: |
| a fl |  |  |  |
| a ${ }_{\text {aaablababaaab }}$ | 2 | 1 |  |
| a aaabababaaab | 2 | 2 |  |
| a aaabababaab | 2 | 3 |  |
| a aaabababaaab | 2 | 4 |  |
| a aaabababáaabs | 2 | 5 | ${ }_{\ll 2,5, a>}$ |
|  |  |  |  |


| Coding Example$s=4, t=4, a=3$ |  |  |
| :---: | :---: | :---: |
|  | tuple |  |
| -aaaabababaaab | <0, 0, a> |  |
| alaabababaaab\$ | <1, $3, \mathrm{~b}$ > |  |
| a aaabababaaab\$ | <2, 5 , a> |  |
| aaaabababaalab\$ | <4,2, \$> |  |
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## Coding the Tuples

- Simple fixed length code

$$
\begin{gathered}
\left\lceil\log _{2}(\mathrm{~s}+1)\right\rceil+\left\lceil\log _{2}(\mathrm{~s}+\mathrm{t}+1)\right\rceil+\left\lceil\log _{2} \mathrm{a}\right\rceil \\
\mathrm{s}=4, \mathrm{t}=4, \mathrm{a}=3 \quad \text { tuple } \quad \text { fixed code } \\
<2,5, \mathrm{a}> \\
010010100
\end{gathered}
$$

- Variable length code using adaptive Huffman or arithmetic code on Tuples
- Two passes, first to create the tuples, second to code the tuples
- One pass, by pipelining tuples into a variable length coder



## Notes on LZ77

- Very popular especially in unix world
- Many variants and implementations - Zip, Gzip, PNG, PKZip,Lharc, ARJ
- Tends to work better than LZW
- LZW has dictionary entries that are never used
- LZW has past strings that are not in the dictionary
- LZ77 has an implicit dictionary. Common tuples are coded with few bits.

