More refined representations

Problem: control-flow edges in CFG overspecify evaluation order

Solution: introduce more refined notions w/ fewer constraining edges that still capture required orderings

- side-effects occur in proper order
- side-effects occur only under right conditions

Some ideas:

- explicit control dependence edges,
  control-equivalent regions,
  control-dependence graph (PDG)
- operators as nodes (Click, VDG, Whirlwind, etc.)
  - computable ϕ-function operator nodes
- control dependence via data dependence (VDG)

Control dependence graph

Program dependence graph (PDG):
data dependence graph + control dependence graph (CDG)
[Ferrante, Ottenstein, & Warren, TOPLAS 87]

Idea: represent controlling conditions directly

- complements data dependence representation

A node (basic block) *Y* is **control-dependent** on another *X* iff *X* determines whether *Y* executes, i.e.

- there exists a path from *X* to *Y* s.t. every node in the path other than *X* & *Y* is **post-dominated** by *Y*
- *X* is not post-dominated by *Y*

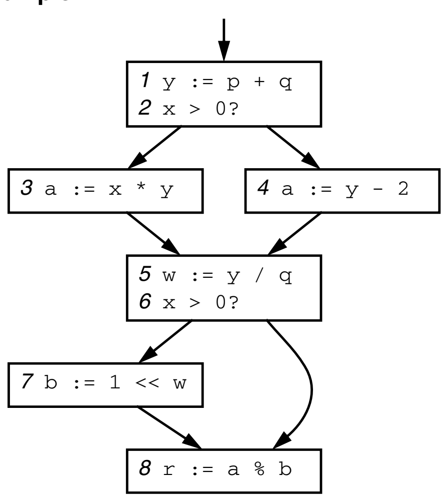
Control dependence graph:
Y proper descendant of *X* iff *Y* control-dependent on *X*

- label each child edge with required branch condition
- group all children with same condition under **region** node

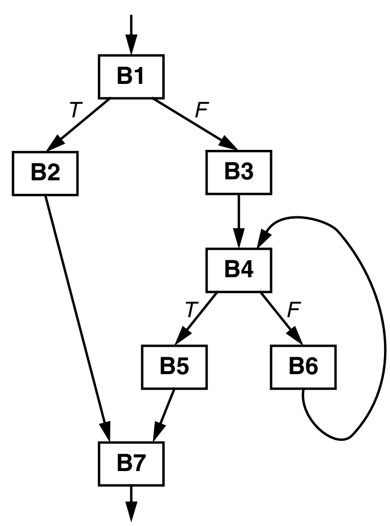
Two sibling nodes execute under same control conditions ⇒ can be reordered or parallelized, as data dependences allow

(Challenging to “sequentialize” back into CFG form)

Example



An example with a loop



Operators as nodes

Before: nodes in CFG were simple assignments

- could have operations on r.h.s.
- used variable names to refer to other values

Alternative: treat the operators themselves as the nodes

- refer directly other other nodes for their operands

Node ::= *Constant*	// 0 operands
| *Var*	// 0 operands
| &*Var*	// 0 operands
| *Unop*	// 1 operand
| *Binop*	// 2 operands
| `*` (ptr deref)	// 1 operand
| `.` (field deref)	// 1 operand
| `[]` (array deref)	// 2 operands
| ϕ	// *n* operands
| *Fn*`()`	// *n* operands
| *Var*`:=` (var assn)	// 1 operand
| `*:=` (ptr assn)	// 2 operands

Flow of data captured directly in operand dataflow edges

Also have control flow edges sequencing these nodes

- or some more refined control dependence edges

Example

```
p := &r;
x := *p;
a := x * y;
w := x;
x := a + a;
v := y * w;
a := v * 2;
```


An improvement

Bypass variable stores and loads

- i.e., build def/use chains

Treat variable names as (temporary) labels on nodes

- a variable reference implemented by an edge from the node with that label
- a variable assignment shifts the label

The nodes themselves become
the subscripted variables of SSA form

Each computation has its own name (i.e., itself)

Another improvement

"Value numbering":
merge all nodes that compute the same result

- same operator
- pure operator
- same data operands (recursively)
- same control dependence conditions

Implements (local) CSE

Can do this bottom-up as nodes are initially constructed

- "hash cons'ing"

In face of possibly cyclic data dependence edges, an optimistic algorithm can get better results [Alpern *et al.* 88]

Would like to support algebraic identities, too, e.g.

- commutative operators
- `x+x` = `x*2`
- associativity, distributivity

Another example

```
y := p + q;
if m > 1 then
  a := y * x;
  b := a;
else
  b := x - 2;
  a := b;
endif
if m < 1 then
  d := y * x;
else
  d := x - 2;
endif
w := a / r;
u := b / r;
t := d / r;
if m > 1 then
  c := y * x;
else
  c := x - 2;
endif
z := c / r;
```


The example, in SSA form

$$
\begin{array}{l}
\texttt{y := p + q;} \\
\texttt{if m > 1 then} \\
\quad a_1 \texttt{ := y * x; } b_1 \texttt{ := } a_1\texttt{;} \\
\texttt{else} \\
\quad b_2 \texttt{ := x - 2; } a_2 \texttt{ := } b_2\texttt{;} \\
a_3 \texttt{ := } \phi(a_1,a_2)\texttt{;} \\
b_3 \texttt{ := } \phi(b_1,b_2)\texttt{;} \\
\texttt{if m < 1 then} \\
\quad d_1 \texttt{ := y * x;} \\
\texttt{else} \\
\quad d_2 \texttt{ := x - 2;} \\
d_3 \texttt{ := } \phi(d_1,d_2)\texttt{;} \\
\texttt{w := } a_3 \texttt{ / r;} \\
\texttt{u := } b_3 \texttt{ / r;} \\
\texttt{t := } d_3 \texttt{ / r;} \\
\texttt{if m > 1 then} \\
\quad c_1 \texttt{ := y * x;} \\
\texttt{else} \\
\quad c_3 \texttt{ := x - 2;} \\
c_3 \texttt{ := } \phi(c_1,c_2)\texttt{;} \\
\texttt{z := } c_3 \texttt{ / r;}
\end{array}
$$

An improvement

ϕ-functions are treated poorly

- impure, since don't know when they're the same
  - even if they have the same operands
    and are in the same control equivalent region!

Fix: give them an additional input: the selector value
(now called select nodes, sometimes written as γ)

- e.g., a boolean, for a 2-input ϕ
- e.g., an integer for an *n*-input ϕ

ϕ-functions now are pure functions!

An approximation, due to Click:
use merge node in CFG as proxy for selector input

− fewer equivalences
+ easier to translate back into CFG form

Value dependence graphs

[Weise, Crew, Ernst, & Steensgaard, POPL 94]

Idea: represent all dependences,
including control dependences, as data dependences

+ simple, direct dataflow-based representation
  of all "interesting" relationships
  - analyses become easier to describe & reason about
− harder to sequentialize into CFG

Control dependences as data dependences:

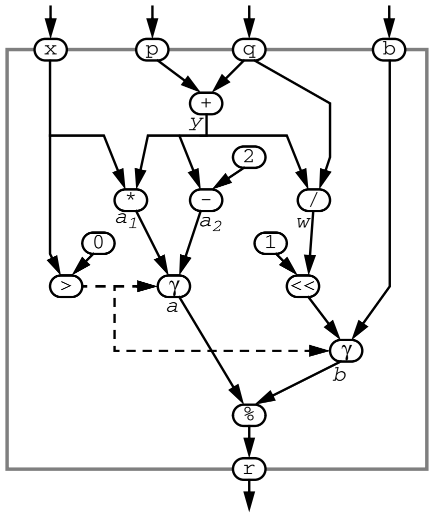
- control dependence on order of side-effects
  ⇒ data dependence on reading & writing to global Store
  - optimizations to break up accesses to single Store into separate independent chunks
    (e.g. a single variable, a single data structure)
- control dependence on outcome of branch
  ⇒ a select node, taking test, then, and else inputs
  ⇒ demand-driven evaluation model

Loops implemented as tail-recursive calls to local procedures

Apply CSE, folding, etc. as nodes are built/updated

VDG for example, after store splitting

```
y := p + q;
if x > 0 then a := x * y else a := y - 2;
w := y / q;
if x > 0 then b := 1 << w;
r := a % b;
```



Sequentialization

How to generate code for a soup of operator nodes?

- need to *sequentialize* back into regular CFG

Find an ordering that respects dependences (data and control)
Hard with arbitrary graph

- can get cycles with full PDG, VDG transforms
- may need to duplicate code to get a legal schedule

Click's representation: keeps original CFG around as a guide

– limits transformations/optimizations possible
+ turns sequentialization problem into simpler placement problem

Placement

Goal: assign each operation to
the least-frequently-executed basic block
that respects its data dependences

- ϕ-nodes tied to their original basic block

Hoist operations out of loops where possible
Push operations into conditionals where possible

Example

```
i := 0;
while ... do
  x := i * b;
  if ... then
    w := c * c;
    y := x + w;
  else
    y := 9;
  end
  print(y);
  i := i + 1;
end
```

Example, in SSA form

i_1 := 0;
while ... do
  i_3 := ϕ(i_1, i_2);
  x := i_3 * b;
  if ... then
    w := c * c;
    y_1 := x + w;
  else
    y_2 := 9;
  end
  y_3 := ϕ(y_1, y_2);
  print(y_3);
  i_2 := i_3 + 1;
end