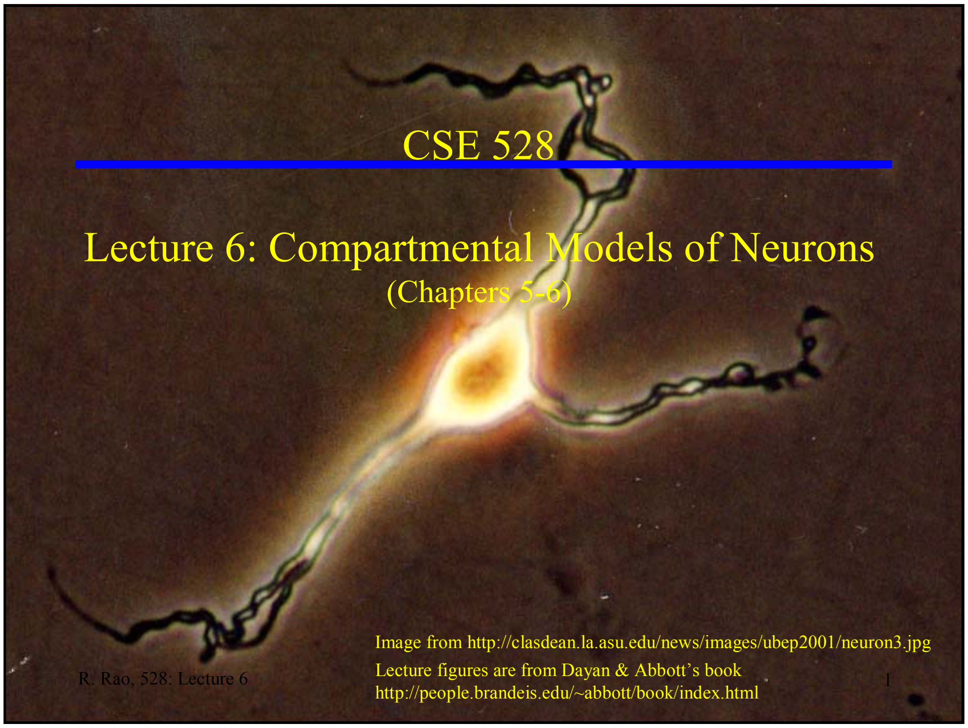


CSE 528

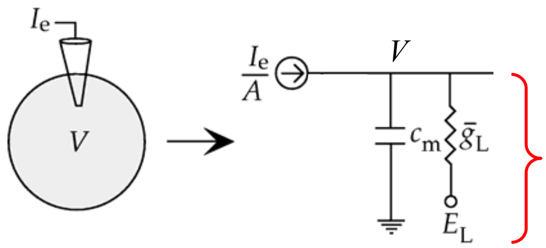
Lecture 6: Compartmental Models of Neurons
(Chapters 5-6)

Image from http://clasdean.la.asu.edu/news/images/ubep2001/neuron3.jpg
Lecture figures are from Dayan & Abbott's book
http://people.brandeis.edu/~abbott/book/index.html

Flashback (What did we do last time?)

- Passive "RC" membrane model
- Integrate-and-Fire Model
- Modeling Synaptic Inputs

The Integrate-and-Fire Model



Models a passive leaky membrane

$$\tau_m \frac{dV}{dt} = -(V - E_L) + I_e R_m$$

If V > $V_{threshold}$ → Spike

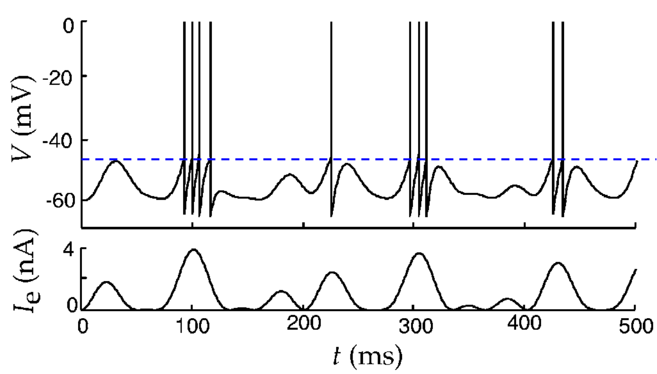
Then reset: V = V_{reset}

$E_L \approx -70$ mV (resting potential)

$V_{threshold} \approx -50$ mV

$V_{reset} \approx E_L$

The Integrate-and-Fire Model in Action

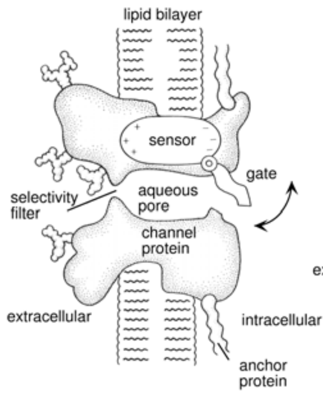


Output of the neuron

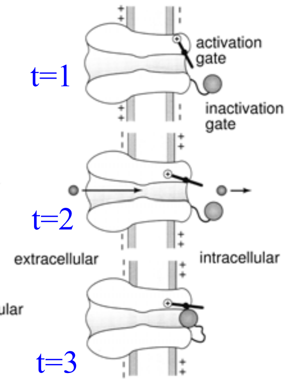
$V_{threshold}$

Input current

Wait! Aren't Neural Membranes Active?



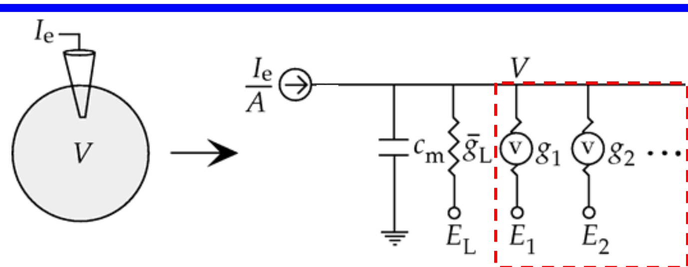
"Persistent conductance"



"Transient conductance"

Channels in membranes open and close based on local membrane potential value ➔ called voltage-dependent channels (= conductances)

Modeling Active Membranes



$$\tau_m \frac{dV}{dt} = -(V - E_L) - r_m g_1 (V - E_1) \ldots + I_e R_m$$

$$g_1 = g_{1,\max} P_1$$

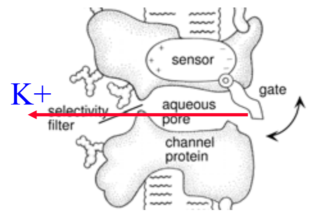
P_1 ← Fraction of type 1 channels open (probability of an open channel)

$g_{1,\max}$ ← Maximum possible conductance

Example 1: "Delayed-rectifier" K+ Channel

$$g_K = g_{K,\max} P_K$$

$$P_K = n^4$$ ← 4 independent "subunits" need to open



$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

$$\text{closed} \xrightarrow{\alpha_n(V)} \text{open}$$

$$\text{open} \xrightarrow{\beta_n(V)} \text{closed}$$

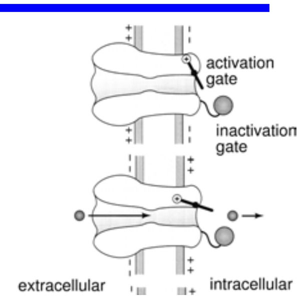
- $\alpha_n(V)$: Opening rate
- $(1-n)$: Fraction of channels closed
- $\beta_n(V)$: Closing rate
- n: Fraction of channels open

Example 2: Transient Na+ Channel

$$g_{Na} = g_{Na,\max} P_{Na}$$

$$P_{Na} = m^3 h$$

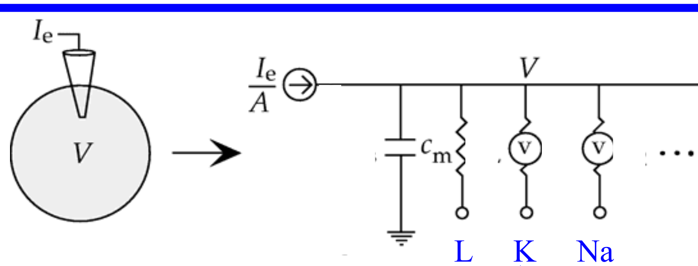
- m^3: Activation
- h: Inactivation



with equations for $\frac{dm}{dt}$ and $\frac{dh}{dt}$

(see text for details)

The (Nobel-Worthy) Hodgkin-Huxley Model

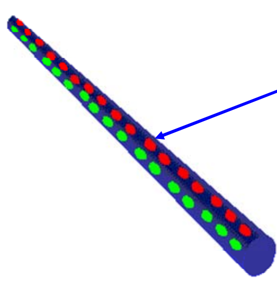


$$c_m \frac{dV}{dt} = -i_m + \frac{I_e}{A}$$

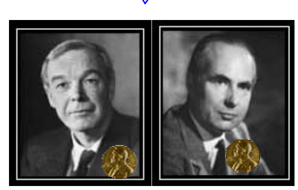
$$i_m = g_{L,\max}(V - E_L) + g_{K,\max} n^4 (V - E_K) + g_{Na,\max} m^3 h (V - E_{Na})$$

E_L = -54 mV, E_K = -77 mV, E_{Na} = +50 mV

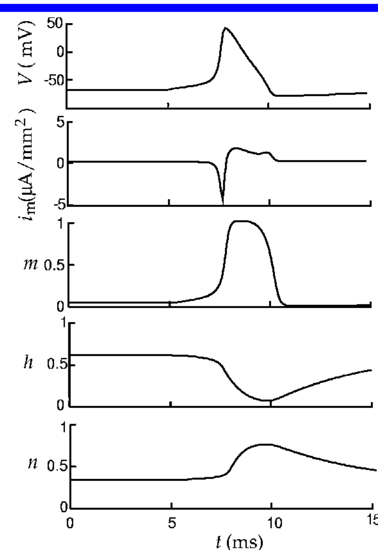
Hodgkin-Huxley in Action



Our equation describes what happens at a fixed point on an axon



Hodgkin-Huxley Dissected



Action Potential (spike)

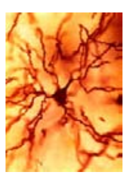
Membrane current

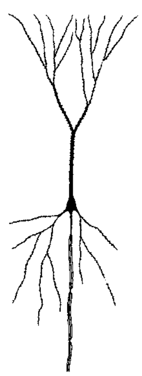
Na+ activation (m)

Na+ inactivation (h)

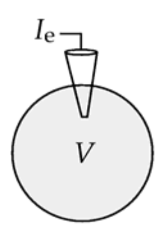
K+ activation (n)

That's fine but what about a neuron's structure?



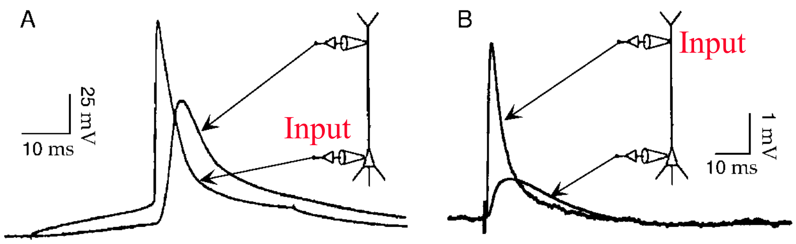


Real Neurons



Our Model

Voltage Attenuation in Dendrites of Real Neurons

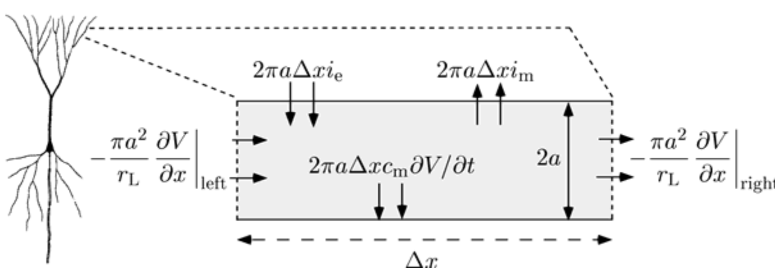


Inject current at the cell body and record effect in a dendrite

Inject current in a dendrite and record effect at the cell body

Voltage decays with distance in passive membranes (How?)

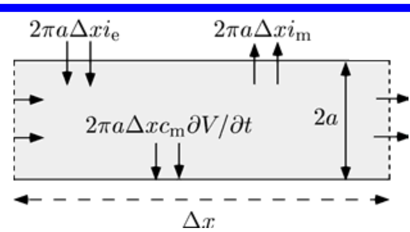
Idea: Model Dendrites as Cables (Cylinders)



Voltage V is a function of *both x and t*

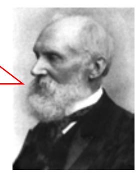
Problem: Derive an equation for $V(x,t)$

The Cable Equation



$$c_m \frac{\partial V}{\partial t} = \frac{1}{2ar_L}\frac{\partial}{\partial x}\left(a^2 \frac{\partial V}{\partial x}\right) - i_m + i_e$$

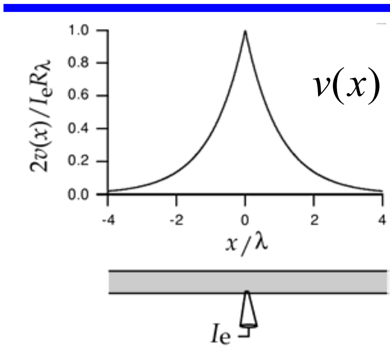
New twist to our old equation



Invented theory for undersea telephone cables

Lord Kelvin (1824 - 1907)

Example: Voltage Decay over Space



$$v(x) \propto e^{\left(-\frac{|x|}{\lambda}\right)}$$

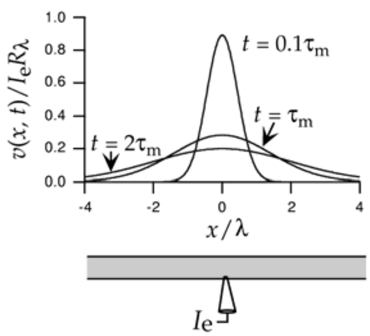
Potential decays exponentially from x = 0

Infinite Cable, Constant current at x = 0

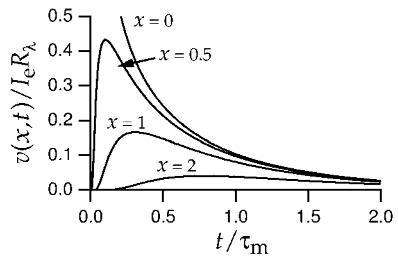
$$\lambda = \sqrt{\frac{r_m a}{2r_L}} = \text{electrotonic length constant of the cable}$$

(analogous to time constant of a membrane)

Example: Voltage Decay over Space and Time

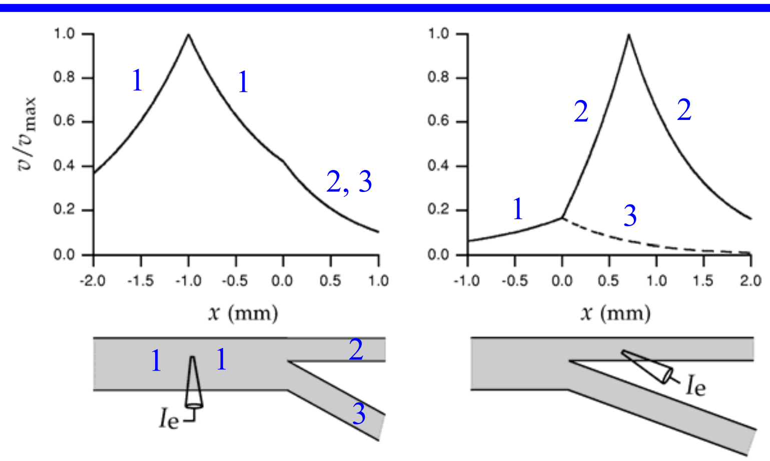


Infinite Cable,
Current pulse at t = 0, x = 0



Potential peaks later (and at lower values) for points further away from input

Example: Voltage in Branching Dendrites



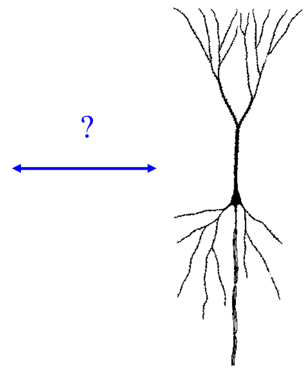
3 semi-infinite cables,
Constant current injection

But What about Modeling an Entire Neuron?

Cable Equation

$$c_m \frac{\partial V}{\partial t} = \frac{1}{2ar_L} \frac{\partial}{\partial x}\left(a^2 \frac{\partial V}{\partial x}\right) - i_m + i_e$$

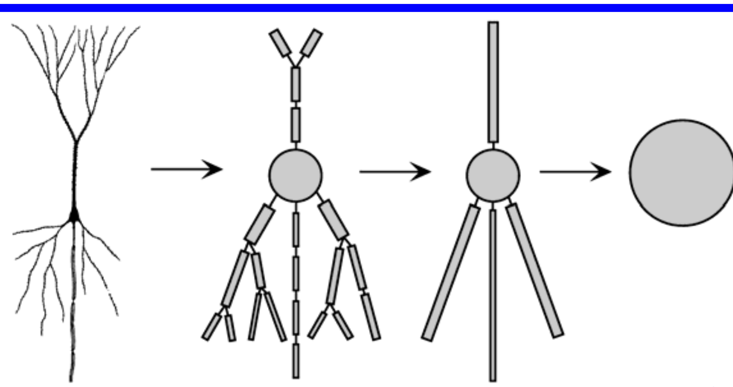
?



Quickly becomes intractable to solve analytically for realistic neurons

Solution: "Divide-and-Conquer" Strategy

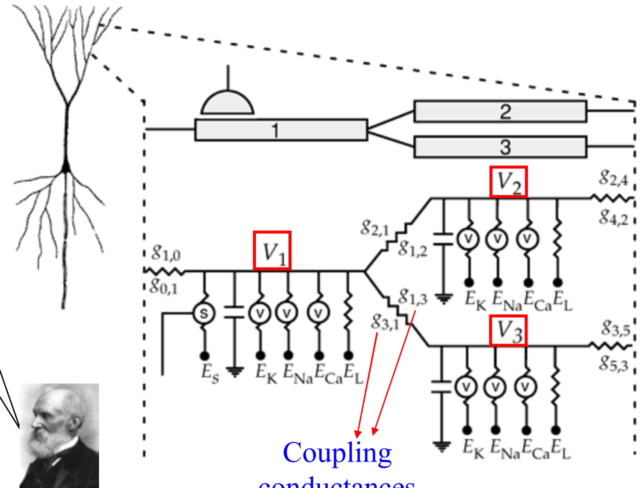
Enter: Compartmental Models



Decreasing number of "compartments"

Each compartment = one dV/dt equation (usually no dependence on x)

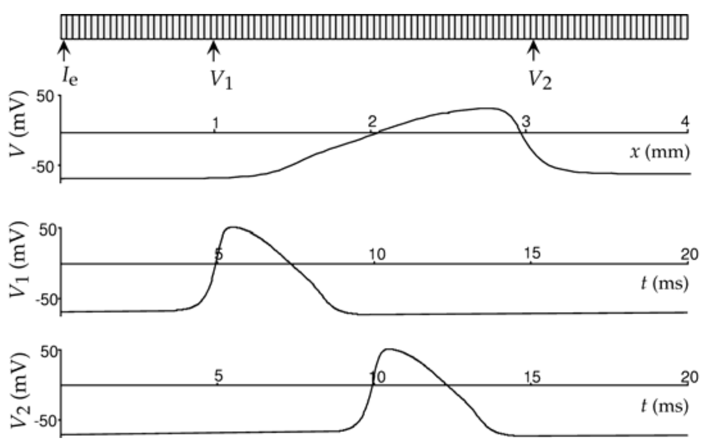
The Gory Details…



Where's that desktop supercomputer they've been promising?

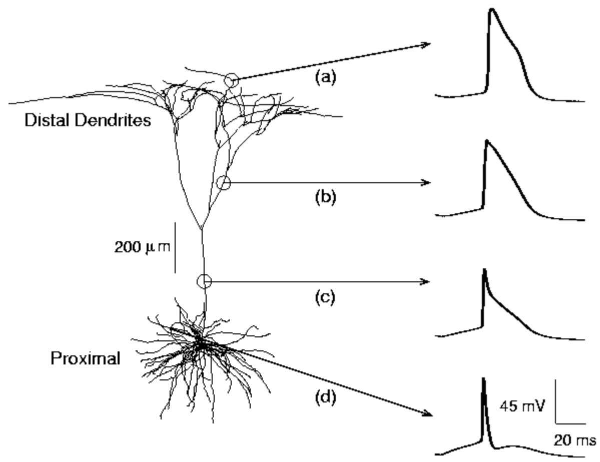
Coupling conductances

Example 1: Spike Propagation along an Axon



(at t = 9.75 ms)

Example 2: Backpropagation of Spikes



Backpropagation of a spike that was initiated in the cell body of a multi-compartmental model neuron

Next Class: Population Coding and Decoding

Things to do:

- Read Chapter 6
- Do Homework #2 on class website