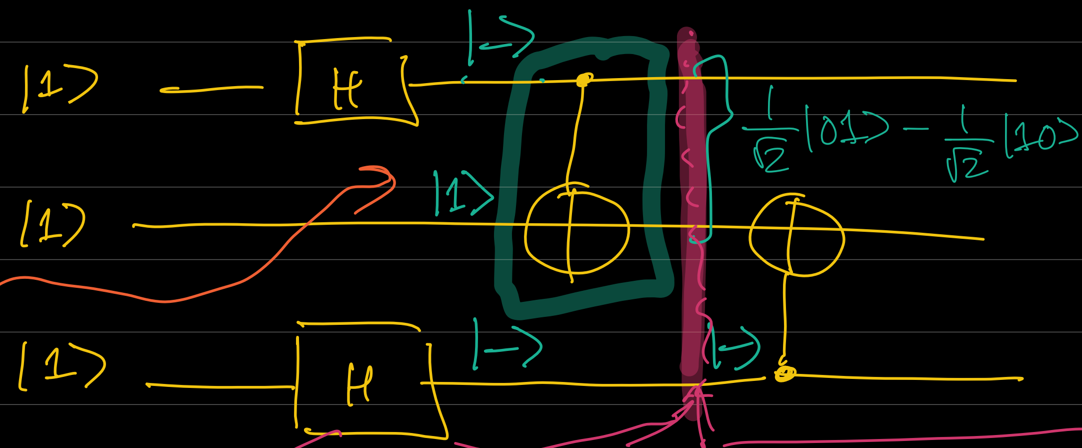
→ We'll start at 11:35 am

→ HW #2 is out, due Sun Oct. 23rd @ 11:59pm



$$\frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$$

$$\frac{1}{2}|010\rangle - \frac{1}{2}|100\rangle - \frac{1}{2}|011\rangle + \frac{1}{2}|101\rangle$$

$$|-\rangle \otimes |1\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \otimes |1\rangle$$

$$= \frac{1}{\sqrt{2}}|0\rangle \otimes |1\rangle - \frac{1}{\sqrt{2}}|1\rangle \otimes |1\rangle$$

$$= \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|11\rangle$$

$$CNOT(|-\rangle \otimes |1\rangle) = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$$

$$\left(\frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle\right) \otimes |-\rangle$$

$$= \left(\frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$= \frac{1}{2}|010\rangle - \frac{1}{2}\overbrace{|100\rangle}^{|10\rangle\otimes|0\rangle} - \frac{1}{2}|011\rangle + \frac{1}{2}|101\rangle$$

$$H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \qquad |0\rangle \longmapsto |+\rangle \qquad |1\rangle \longmapsto |-\rangle$$

CNOT:

$|00\rangle \to |00\rangle$

$|01\rangle \to |01\rangle$

$|10\rangle \to |11\rangle$

$|11\rangle \to |10\rangle$

$$(I_2 \otimes A)(|0\rangle \otimes |01\rangle) = |0\rangle \otimes A|01\rangle$$

$$\underbrace{I \otimes A}_{8\times 8} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes A$$

$|000\rangle = |0\rangle \otimes |00\rangle \to |000\rangle$

$|001\rangle = |0\rangle \otimes |01\rangle \to |001\rangle$

$|010\rangle \to |011\rangle$

$|100\rangle \quad \vdots \quad \vdots$

$|011\rangle$

$|101\rangle \to |101\rangle$

$|110\rangle = |1\rangle \otimes |10\rangle \to |111\rangle$

$|111\rangle = |1\rangle \otimes |11\rangle \to |110\rangle$

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$(A \otimes B)_{(i,j),(k,l)} = A_{ij} B_{kl}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes CNOT = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right.$$

$$|000\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$|001\rangle = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|0\rangle \otimes |0\rangle =$$

$$u \in \mathbb{C}^m, \quad v \in \mathbb{C}^n$$

$$u \otimes v \in \mathbb{C}^{mn}$$

$$\boxed{(u \otimes v)_{ij} = u_i v_j}$$

$$(\lambda_1 \vec{u}_1 + \lambda_2 \vec{u}_2) \otimes \vec{v} = \lambda_1 (\vec{u}_1 \otimes \vec{v}) + \lambda_2 (\vec{u}_2 \otimes \vec{v})$$

$$\vec{u} \otimes (\lambda_1 \vec{v}_1 + \lambda_2 \vec{v}_2) = \lambda_1 (\vec{u} \otimes \vec{v}_1) + \lambda_2 (\vec{u} \otimes \vec{v}_2)$$

$$A \in \mathbb{C}^{m \times m}, \quad B \in \mathbb{C}^{n \times n}$$

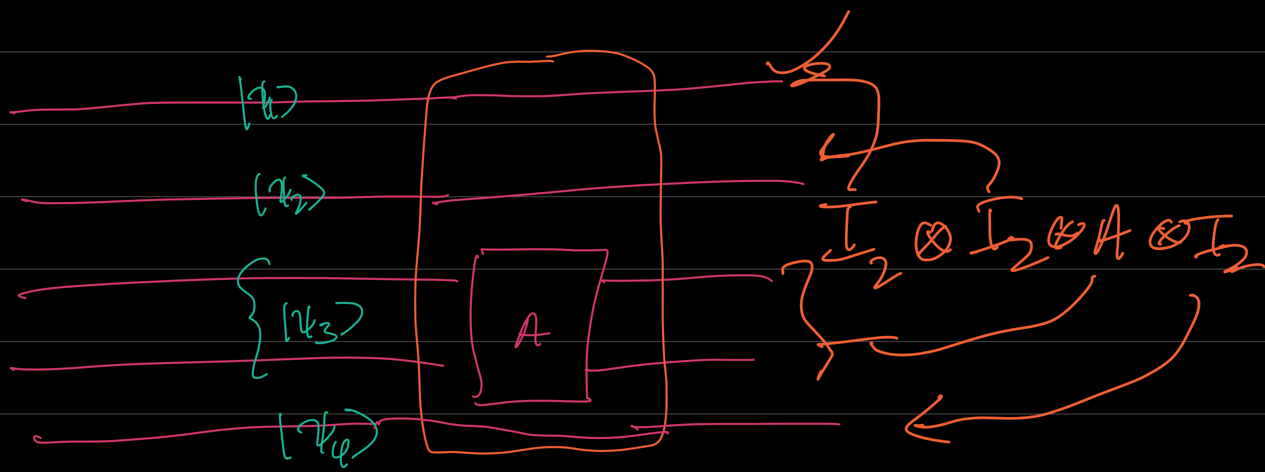
$$\rightarrow (A \otimes B)(\underline{u \otimes v}) = Au \otimes Bv$$

$\mathbb{C}^{mn \times mn}$ $\quad u_1, \ldots, u_m$ basis for $\mathbb{C}^m$

$\in \mathbb{C}$

$v_1, \dots, v_n$ basis for $\mathbb{C}^n$

$\left[\{ u_i \otimes v_j \} \right.$ basis for $\mathbb{C}^{mn}$

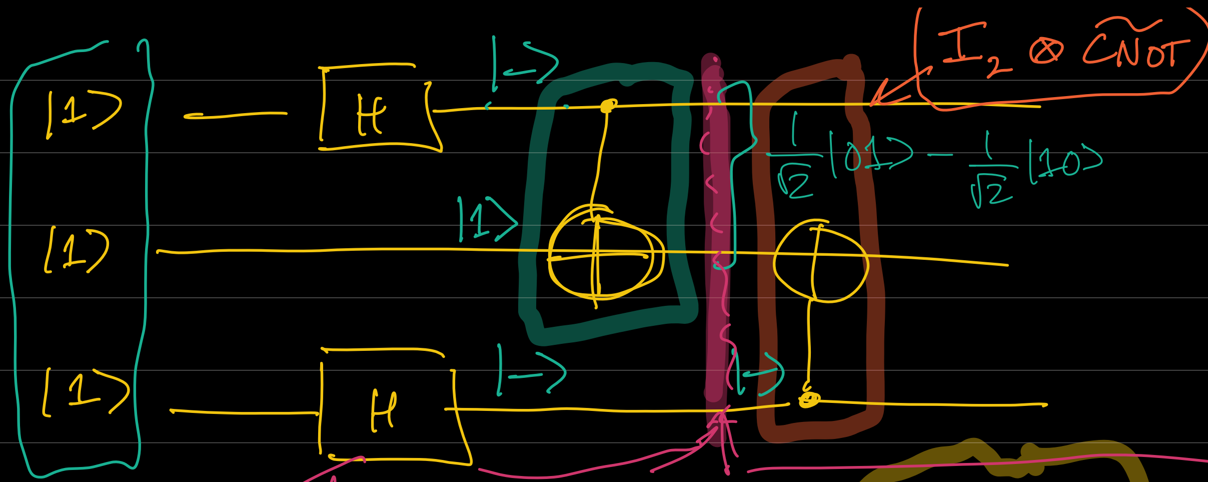


$$\boxed{\underbrace{I_4}_{4\times 4} \otimes \underbrace{A}_{4\times 4}} \quad 16 \times 16$$

$$I_4 = I_2 \otimes I_2$$

$$(I_4 \otimes A)(\underbrace{|\psi_1\rangle}_{\mathbb{C}^2} \otimes \underbrace{|\psi_2\rangle}_{\mathbb{C}^2} \otimes \underbrace{|\psi_3\rangle}_{\mathbb{C}^4})$$

$$= (|\psi_1\rangle \otimes |\psi_2\rangle \otimes A|\psi_3\rangle)$$



$$\frac{1}{2}|010\rangle - \frac{1}{2}|100\rangle - \frac{1}{2}|011\rangle + \frac{1}{2}|101\rangle$$

$$\frac{1}{2}|010\rangle - \frac{1}{2}|100\rangle - \frac{1}{2}|001\rangle + \frac{1}{2}|111\rangle$$

$$(I_2 \otimes \widetilde{CNOT})|011\rangle = (I_2 \otimes \widetilde{CNOT})(|0\rangle \otimes |11\rangle)$$

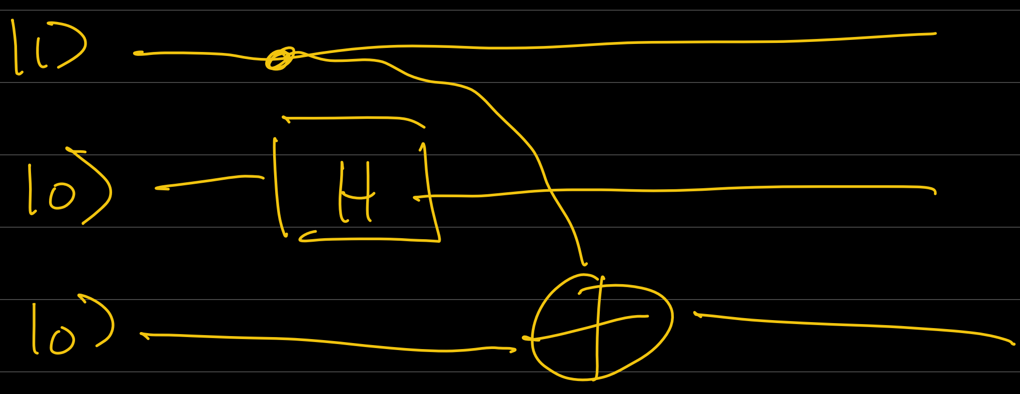
$$= I_2|0\rangle \otimes \widetilde{CNOT}|11\rangle$$

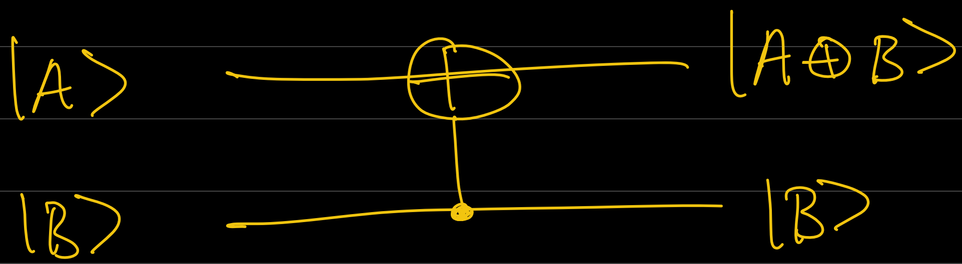
$$= |0\rangle \otimes |01\rangle$$

$$= |001\rangle$$

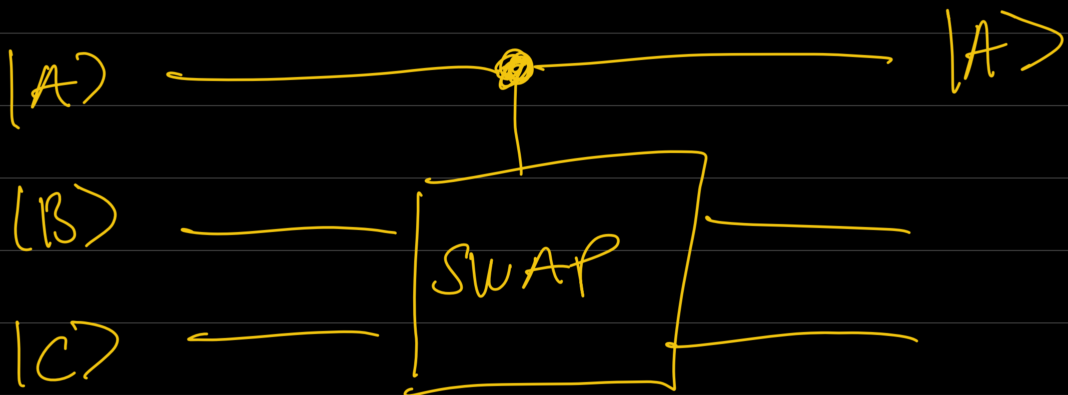
$$|010\rangle = |0\rangle \otimes \boxed{|1\rangle \otimes |0\rangle}$$

$$= |0\rangle \otimes \boxed{|10\rangle}$$





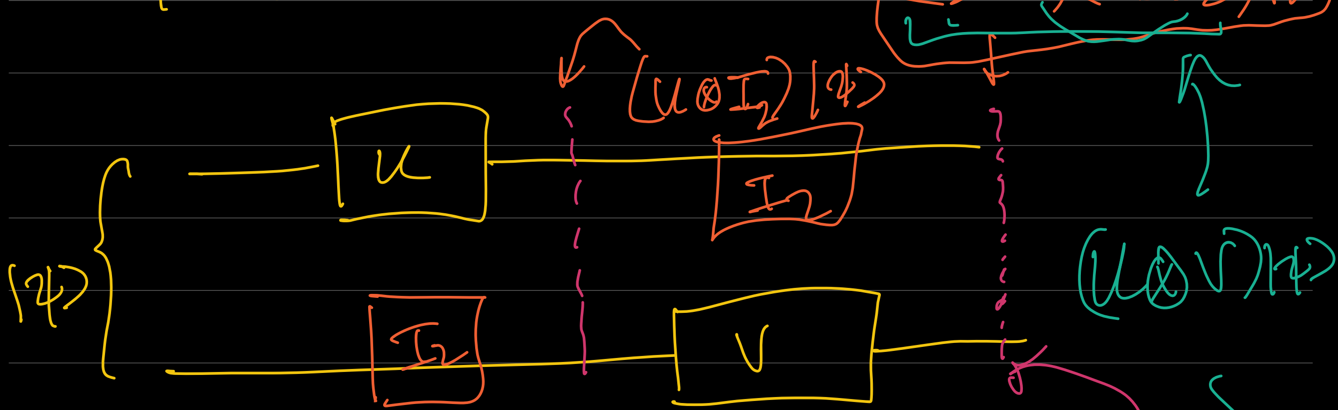
CSWAP



$|101\rangle \longrightarrow |110\rangle$

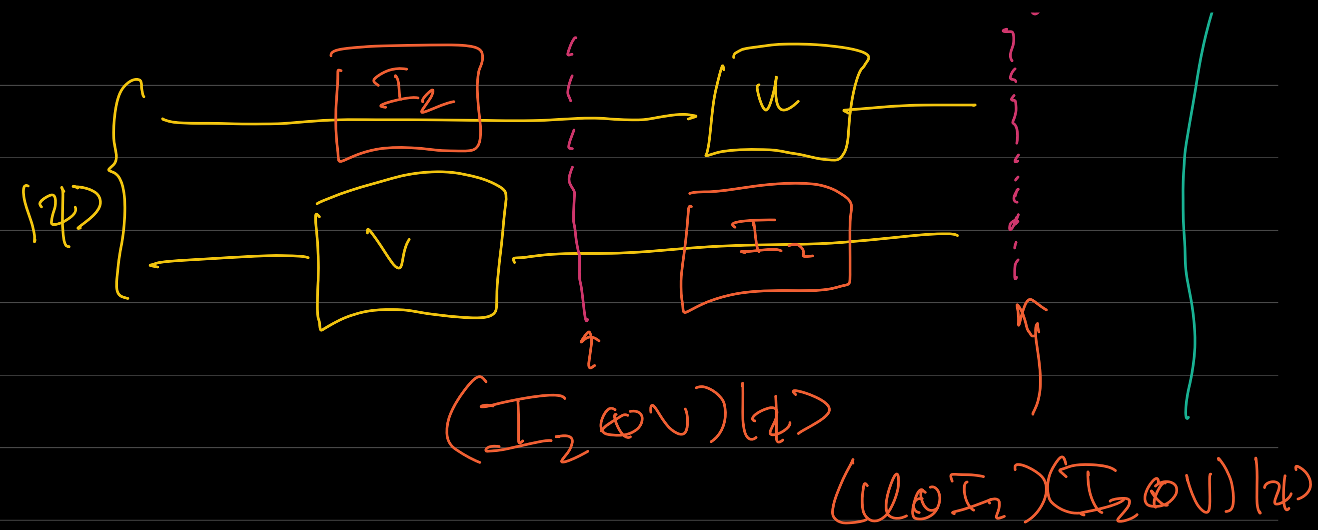
$|001\rangle \longrightarrow |001\rangle$

$(I_2 \otimes V)(U \otimes I_2)|\psi\rangle$

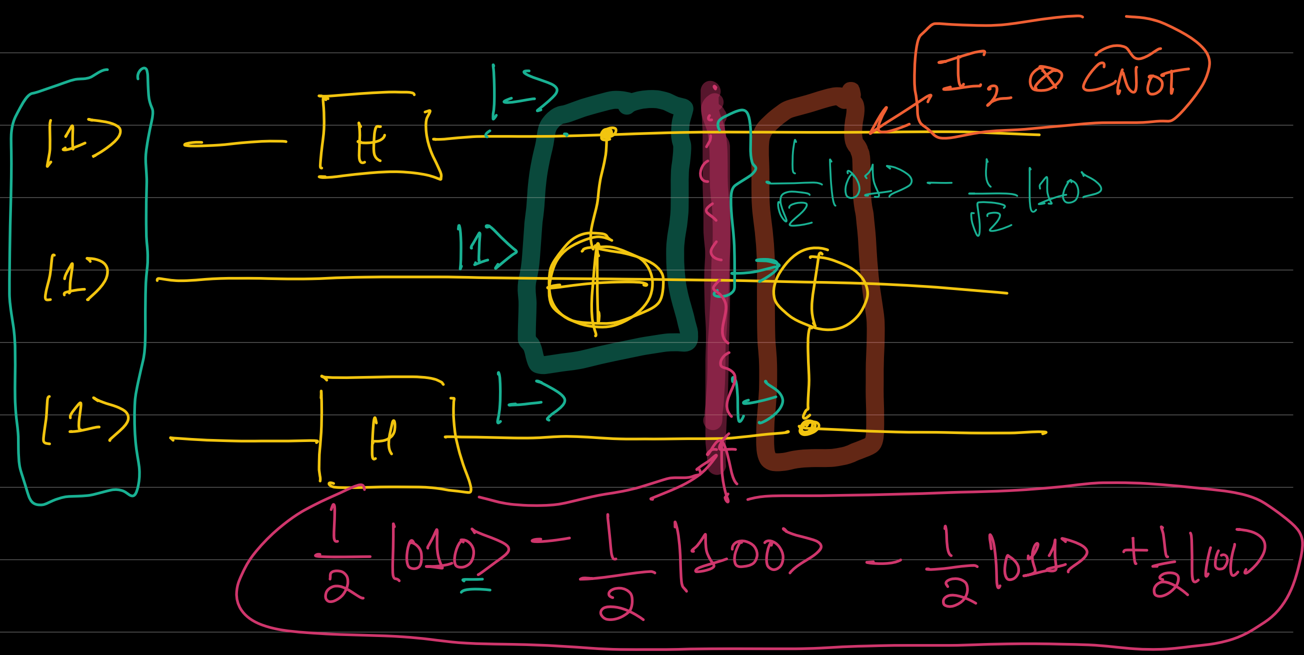


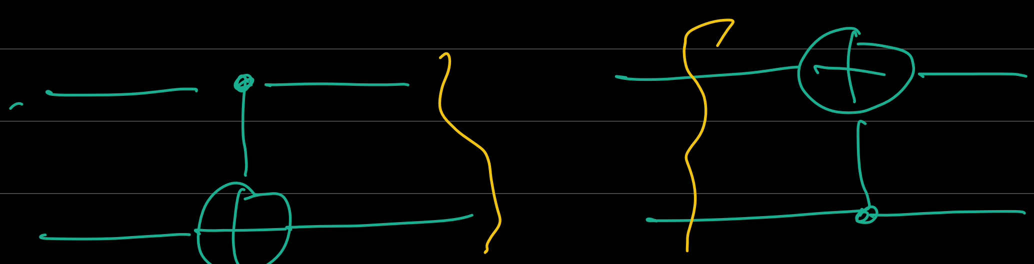
$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

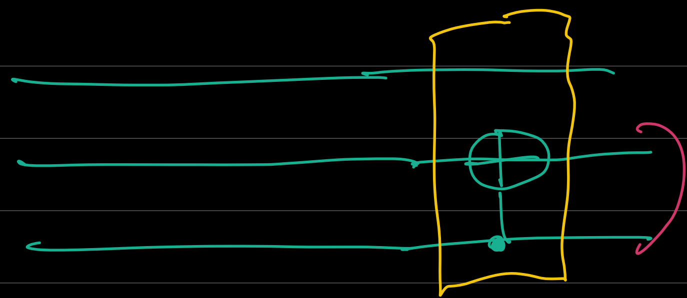
$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$



$$(A \otimes B)(\tilde{A} \otimes \tilde{B}) = (A\tilde{A} \otimes B\tilde{B})$$







$|0\rangle \otimes |00\rangle \rightarrow |0\rangle \otimes |00\rangle$

$|0\rangle \otimes |01\rangle \rightarrow |0\rangle \otimes |11\rangle$

$|0\rangle \otimes |10\rangle \rightarrow |0\rangle \otimes |10\rangle$

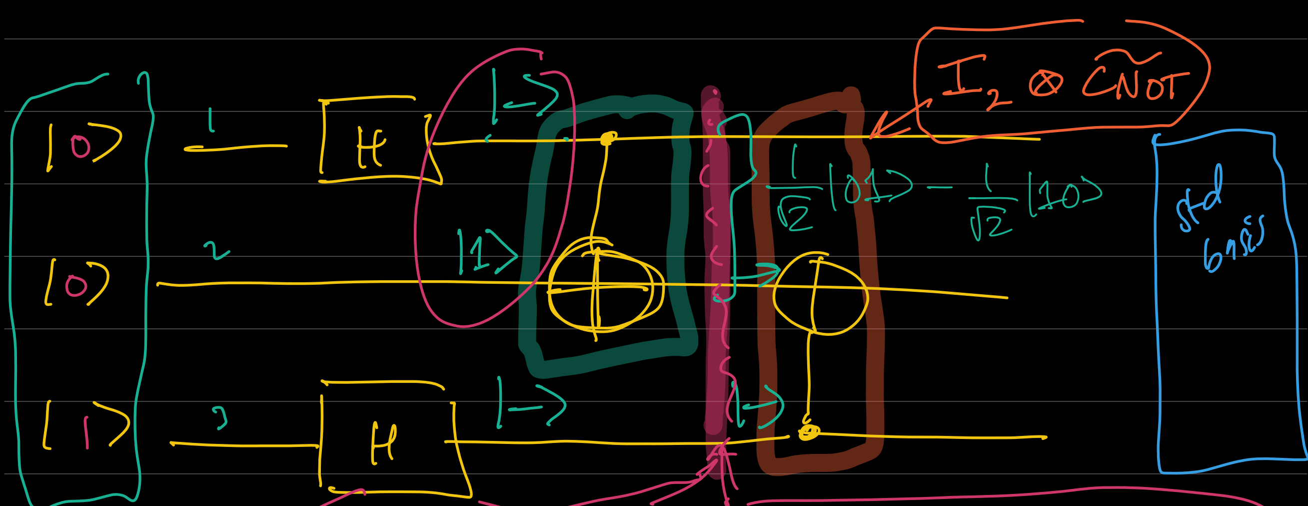
$|0\rangle \otimes |11\rangle \rightarrow |0\rangle \otimes |01\rangle$

$|1\rangle \otimes |00\rangle \rightarrow |1\rangle \otimes |00\rangle$

$|1\rangle \otimes |01\rangle \rightarrow |1\rangle \otimes |11\rangle$

$|1\rangle \otimes |10\rangle \rightarrow |1\rangle \otimes |10\rangle$

$|1\rangle \otimes |11\rangle \rightarrow |1\rangle \otimes |01\rangle$



$|\overset{1}{0}\overset{2}{0}\overset{3}{1}\rangle$ $\left(\frac{1}{2}|010\rangle - \frac{1}{2}|100\rangle - \frac{1}{2}|011\rangle + \frac{1}{2}|101\rangle\right)$

$$|\psi_1\rangle \otimes |\psi_2\rangle \neq |\psi_2\rangle \otimes |\psi_1\rangle$$

$$\alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \dots + \alpha_{111}|111\rangle$$

$$\mathbb{P}[\text{measure "}|001\rangle\text{"}] = |\alpha_{001}|^2$$

$$|-\rangle \otimes |1\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \otimes |1\rangle$$

$$= \frac{1}{\sqrt{2}}|0\rangle \otimes |1\rangle - \frac{1}{\sqrt{2}}|1\rangle \otimes |1\rangle$$

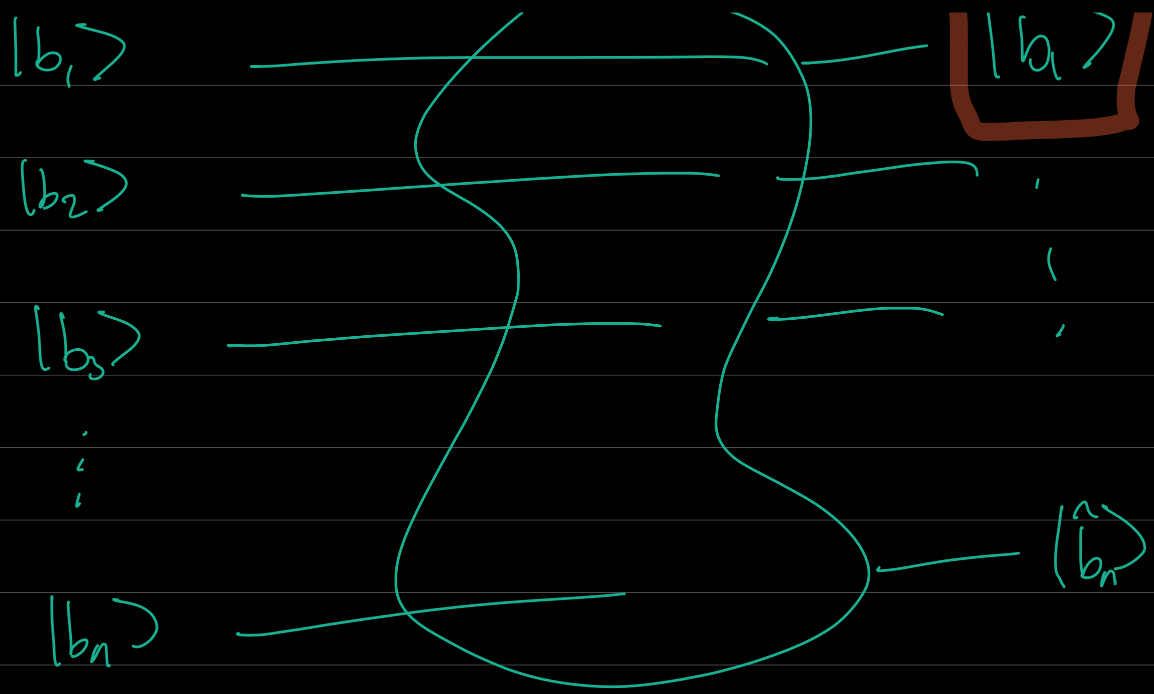
$$= \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|11\rangle$$

not equal

$$|1\rangle \otimes |-\rangle = |1\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}|10\rangle - \frac{1}{\sqrt{2}}|11\rangle$$

$$|01\rangle \neq |10\rangle$$



$|b_1\rangle$
$|b_2\rangle$
$|b_3\rangle$
$\vdots$
$|b_n\rangle$
$|b_1\rangle$
$\vdots$
$|b_n\rangle$