Muscles

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Motor units

Muscle fiber: specialized multi-nucleated cell, 50-100 μm diameter and 2-6 cm length

1 motoneuron → 100-1000 muscle fibers

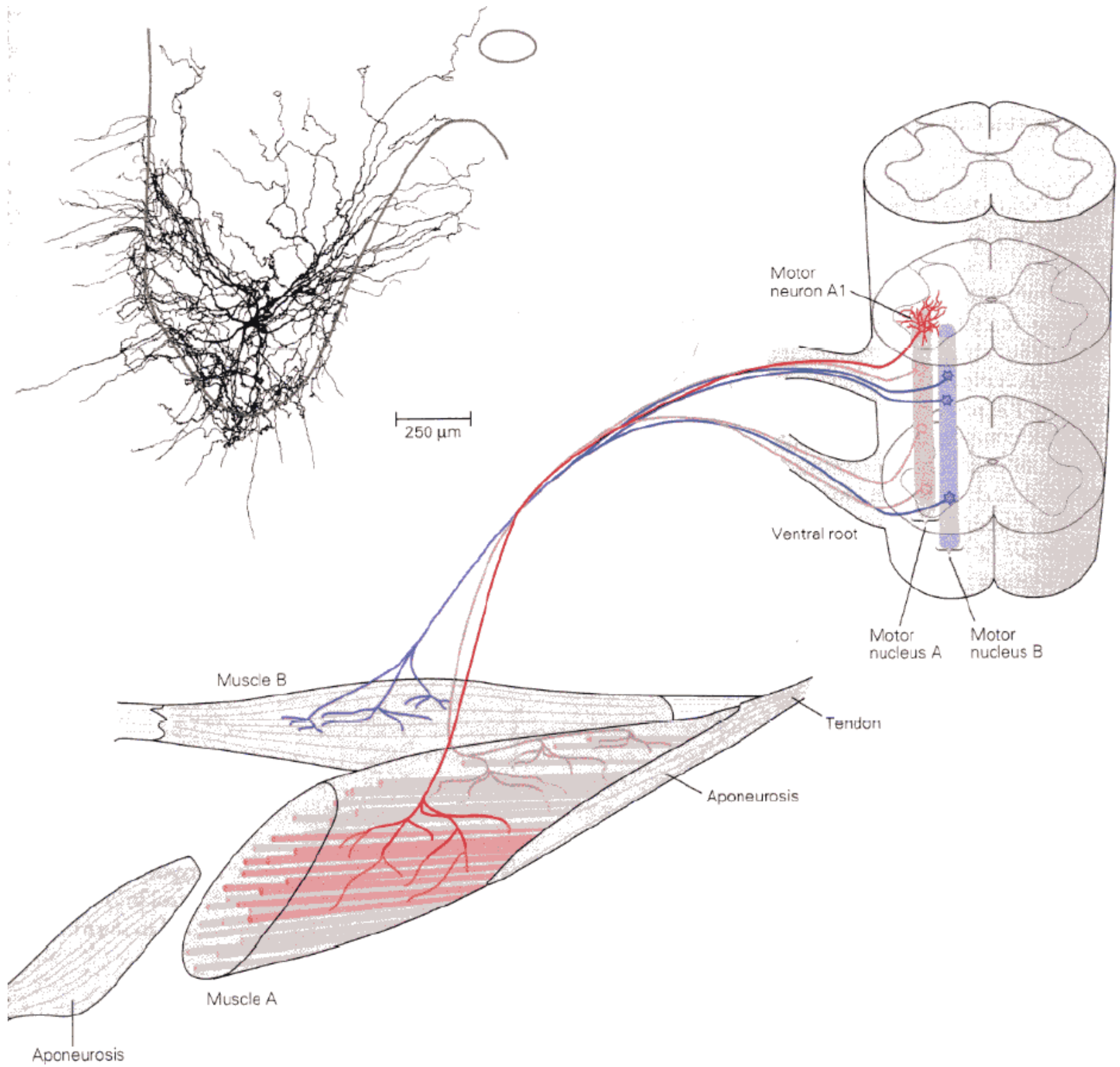
1 muscle fiber ← 1 motoneuron

Number of motor units per muscle varies greatly

Motoneuronal pool: thin vertical column in the ventral horn of the spinal cord

Motor axons exit the spinal cord through ventral roots;
sensory axons enter the spinal cord through dorsal roots

ACh synapse, called "endplate";
every action potential causes large but slowly propagating muscle spike (easily recorded: EMG)



Muscle fibers

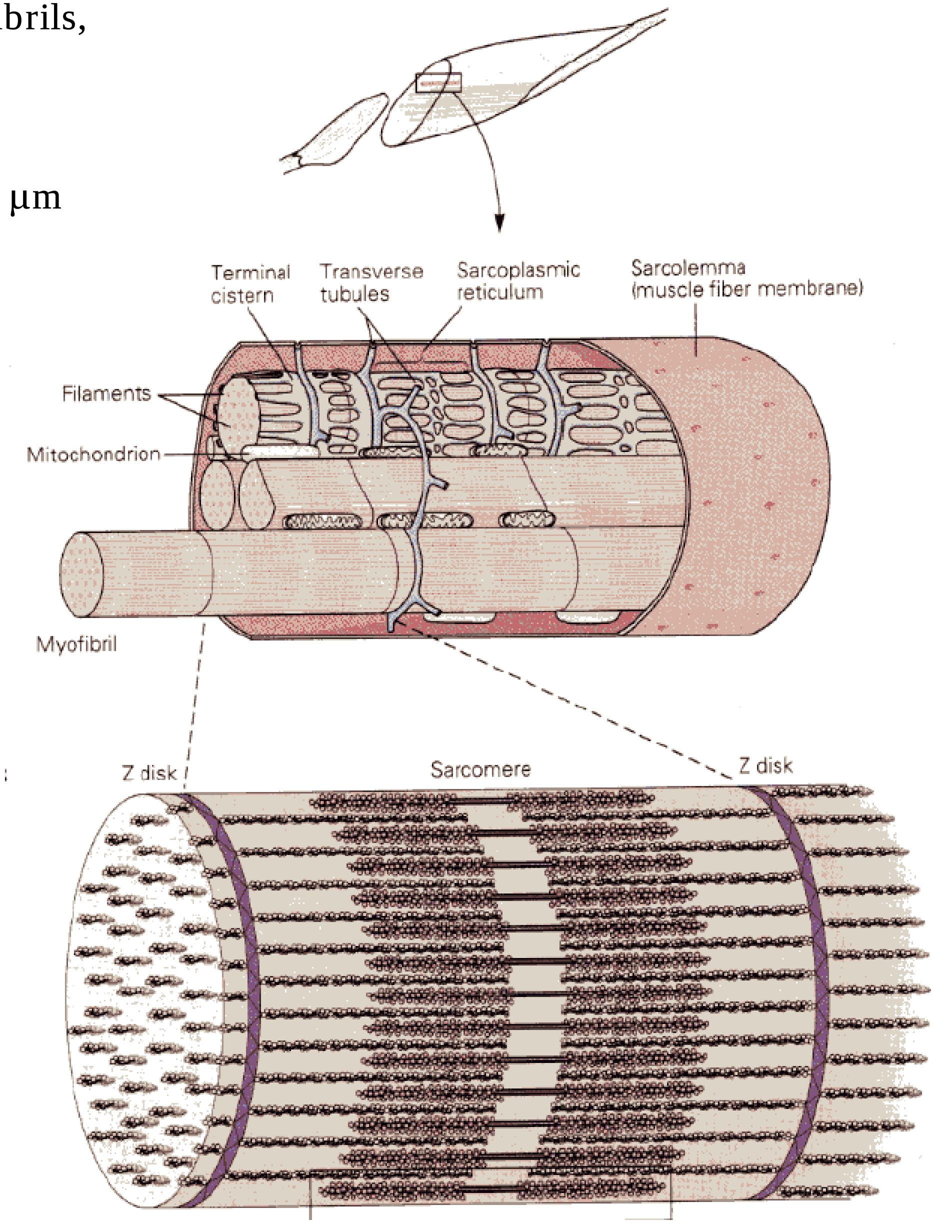
Muscle fiber contains parallel myofibrils,
CA-delivery mechanisms,
and stabilizing connective tissues

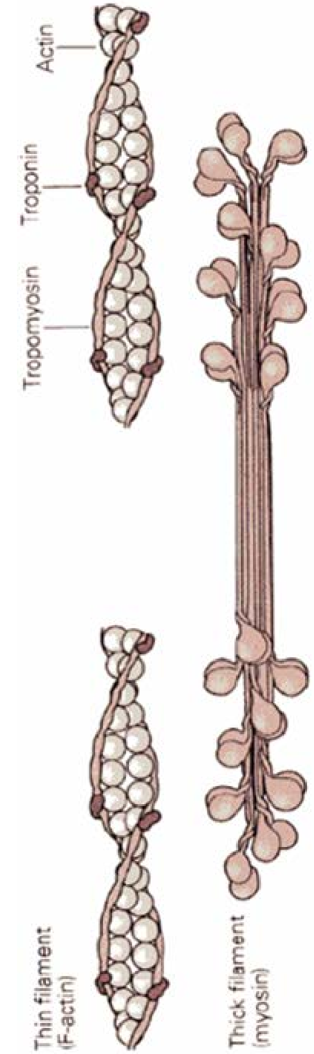
Miofibrils contain a series of 1.5-3.5 μm sacromeres, separated by Z-dsisk

Sacromeres have thin filaments on each end, and thick filaments in the middle

Thin filament: actin, troponin, tropomyosin

Thick filament: myosin





Force generation mechanism

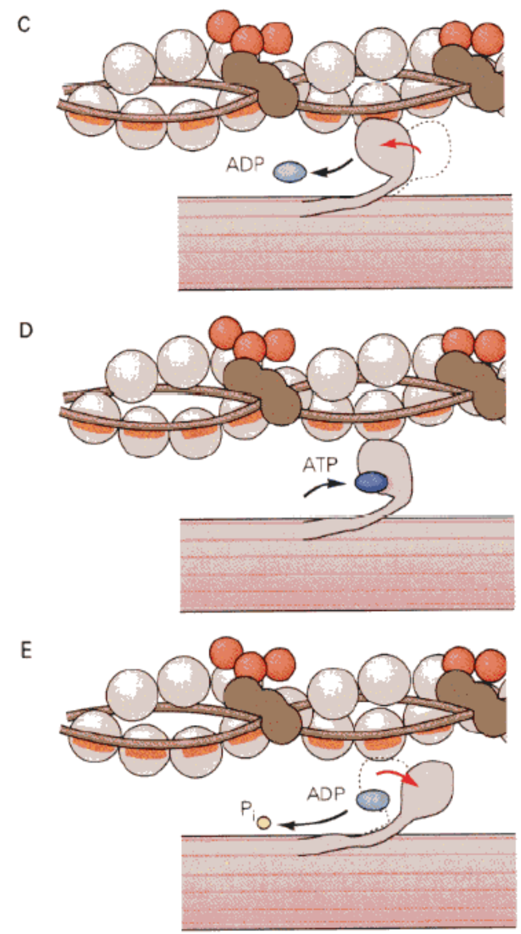
A: Myosin heads loaded with ADP attached; Actin binding sites blocked by troponin-tropomyosin

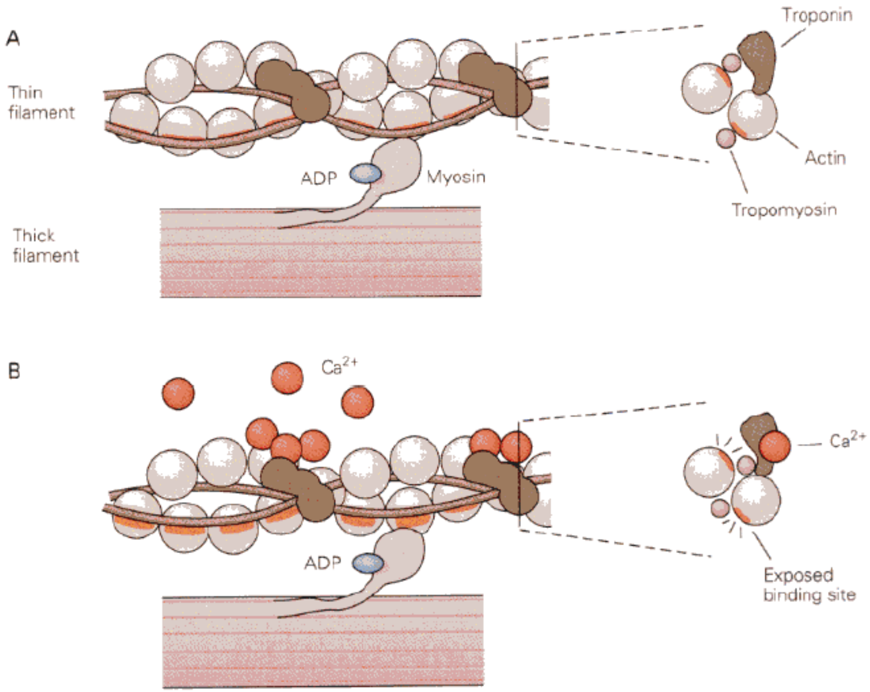
B: CA flows in, attaches to tropomyosin, and exposes binding sites

C: Cross-bridges form, and the mechanical energy stored in the myosin head produces shortening force; sliding motion of 0.06 μm

D: ATP binds to myosin and detaches cross-bridge

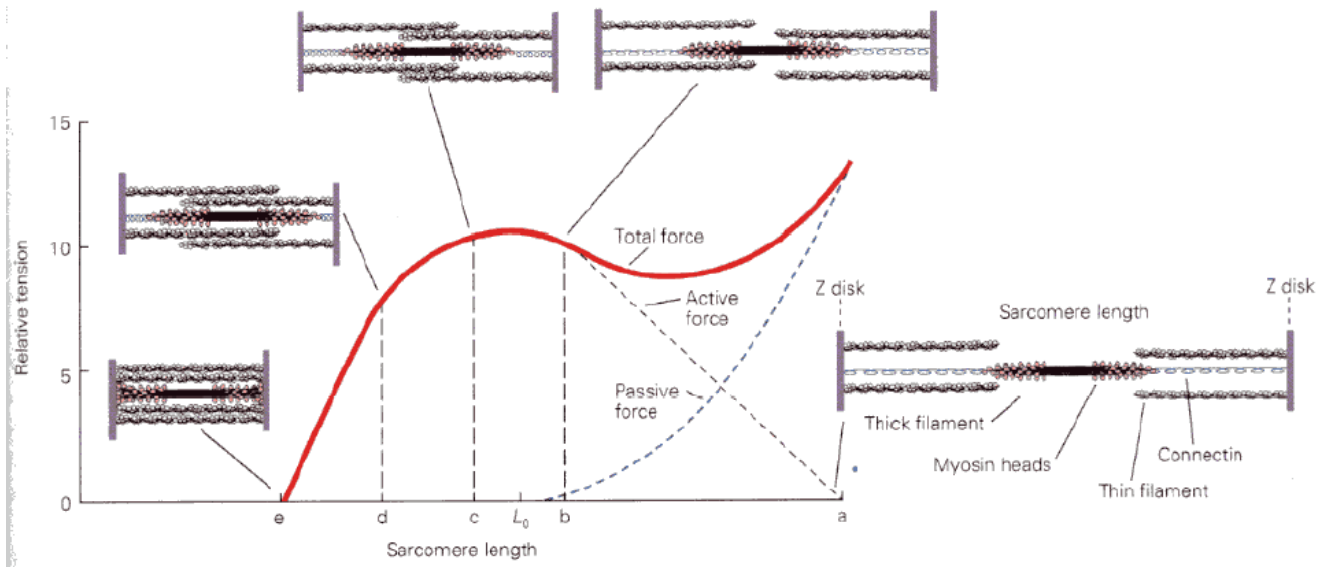
E: Converting ATP to ADP releases energy, which re-loads myosin head



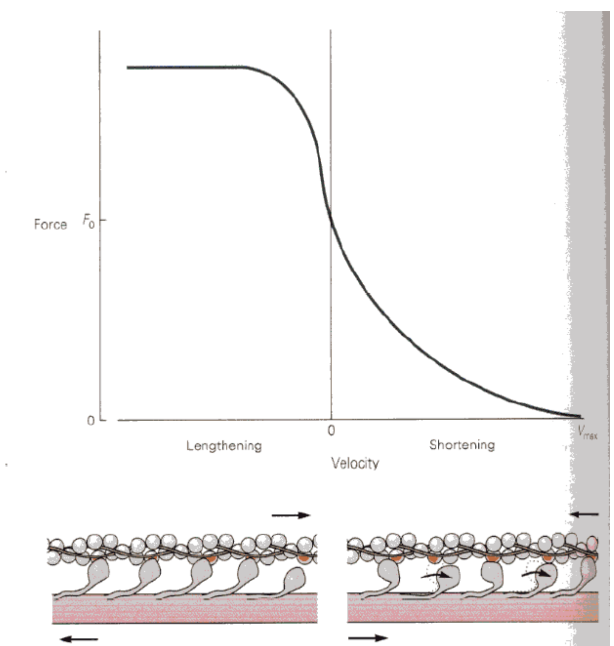


Muscle force depends on length and velocity

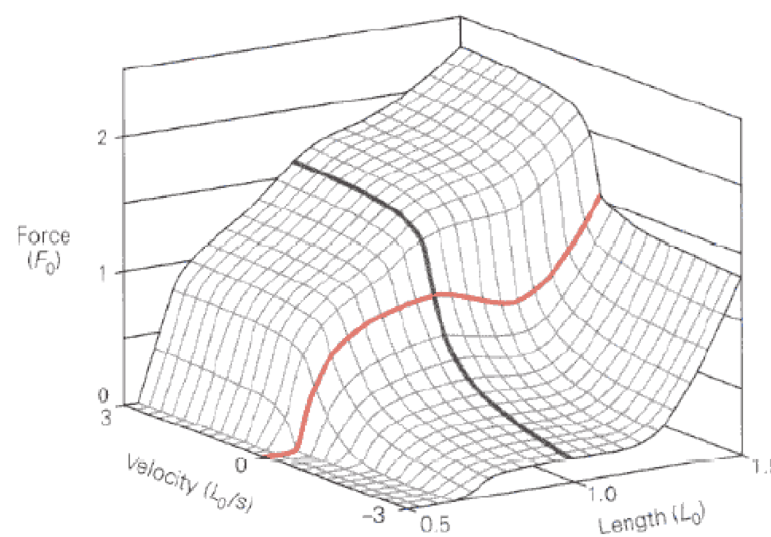
Length dependence:



Velocity dependence:

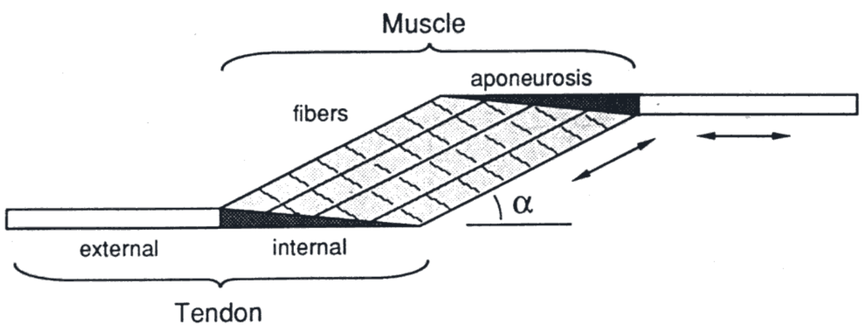


Force-length-velocity surface:

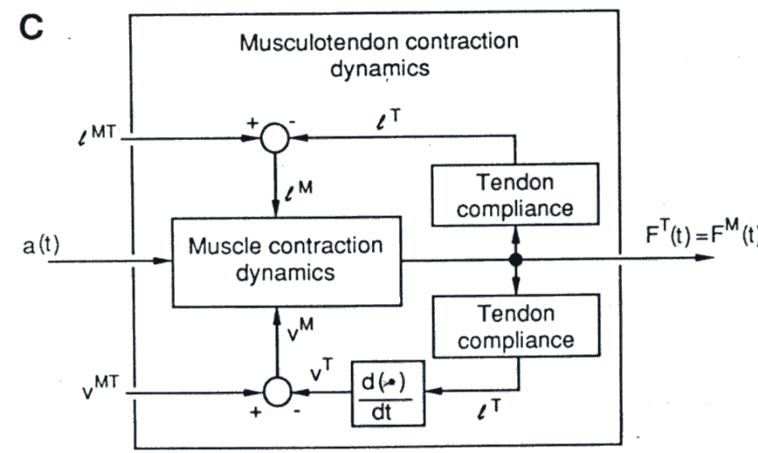


The muscle-tendon actuator

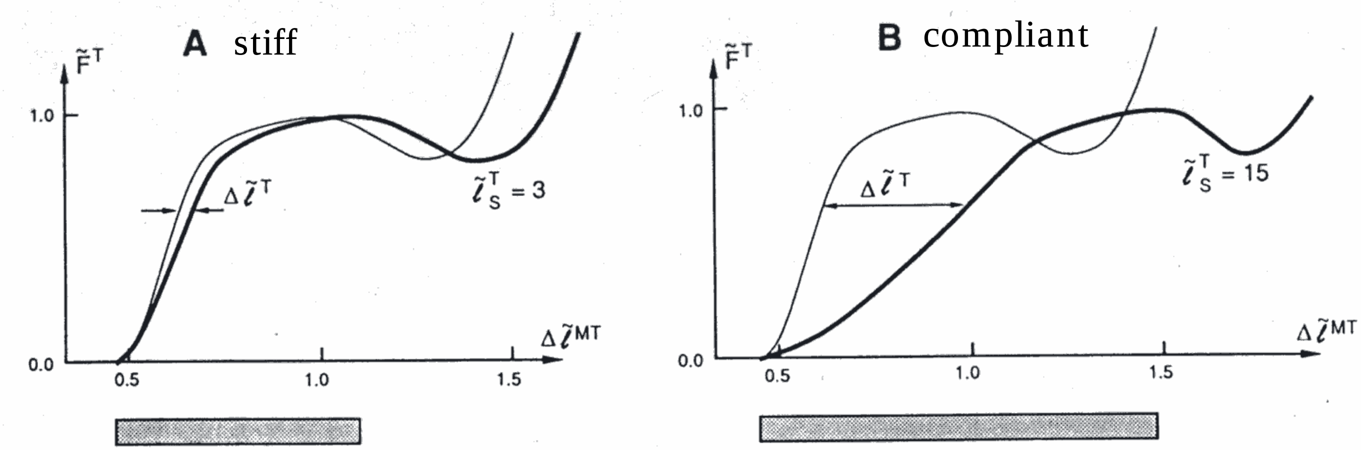
Muscle and tendon work as a unit



Interactions



Tendon compliance "stretches" muscle length-tension curve



Dynamics of force generation

CA release and re-uptake are different mechanisms

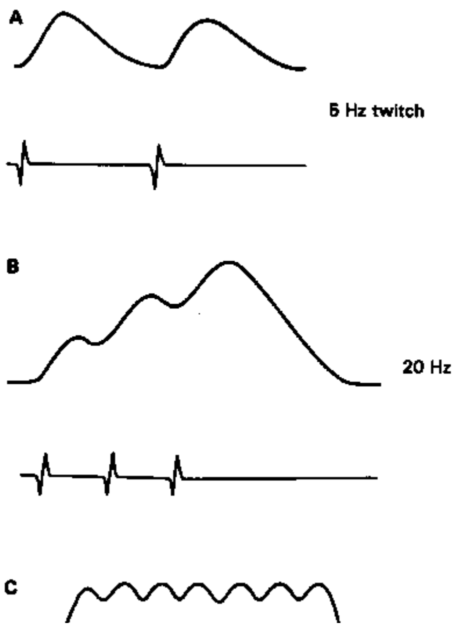
1st-order nonlinear filter model:

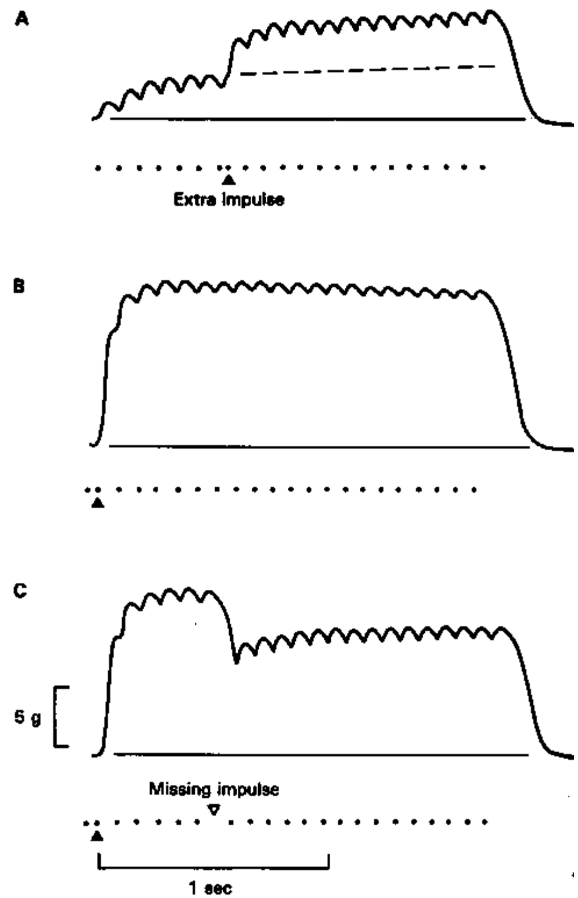
$$\dot{a}(t) = -a(t)/\tau_{deact} + \big(u(t) - a(t)\big)a(t)/\tau_{act}$$

$\tau_{act} \approx 20ms$

$\tau_{deact} \approx 60ms$

Temporal summation



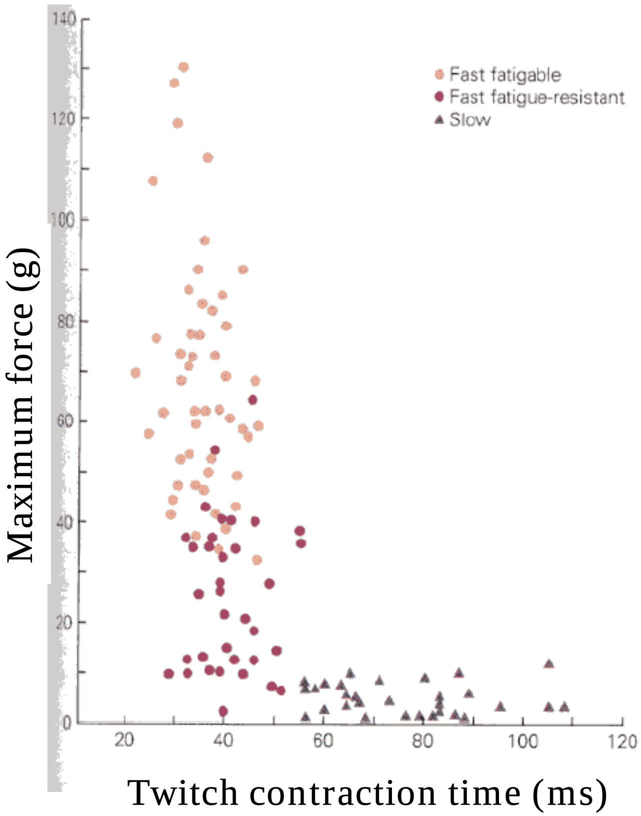


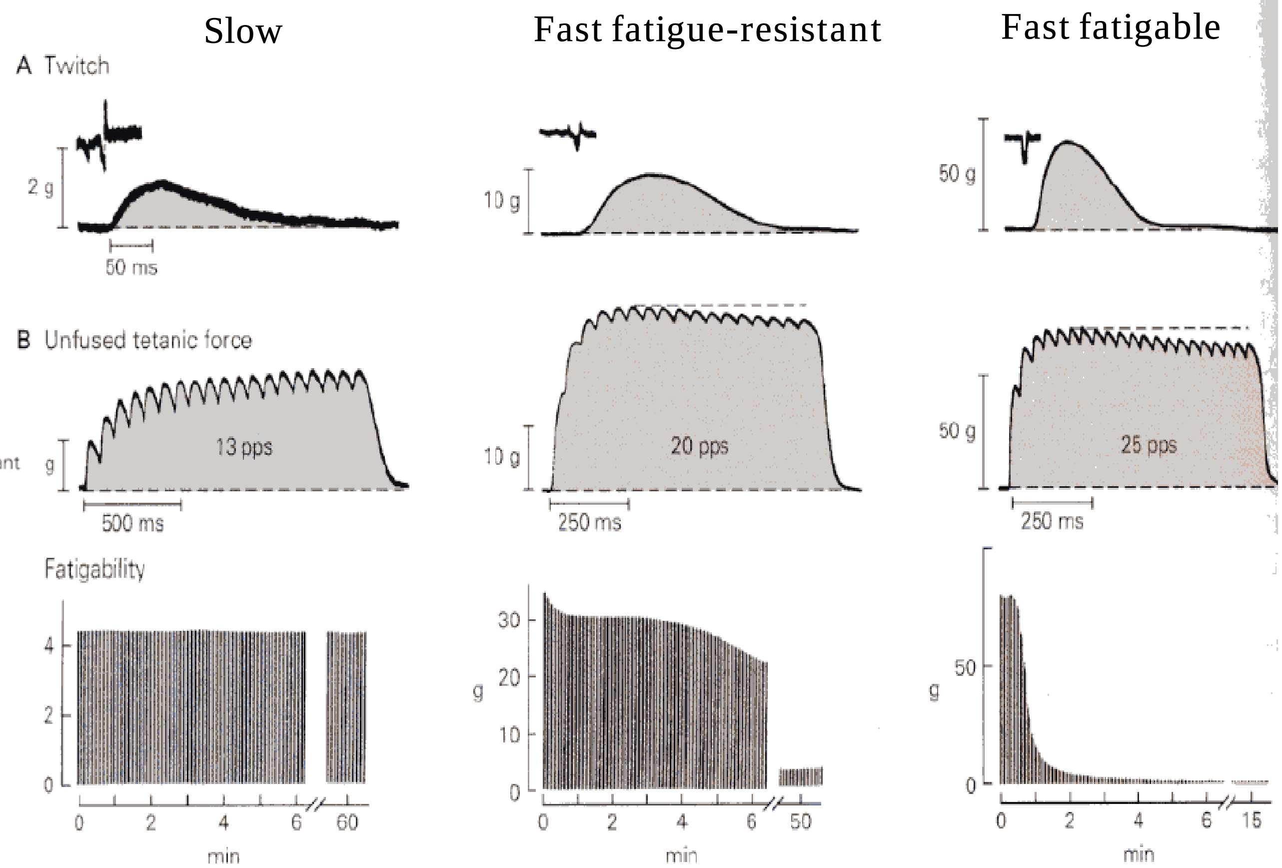
Sensitivity to spike pattern:

motorneurons: cv ~ 0.1, no baseline

other neurons: cv ~ 1, some baseline

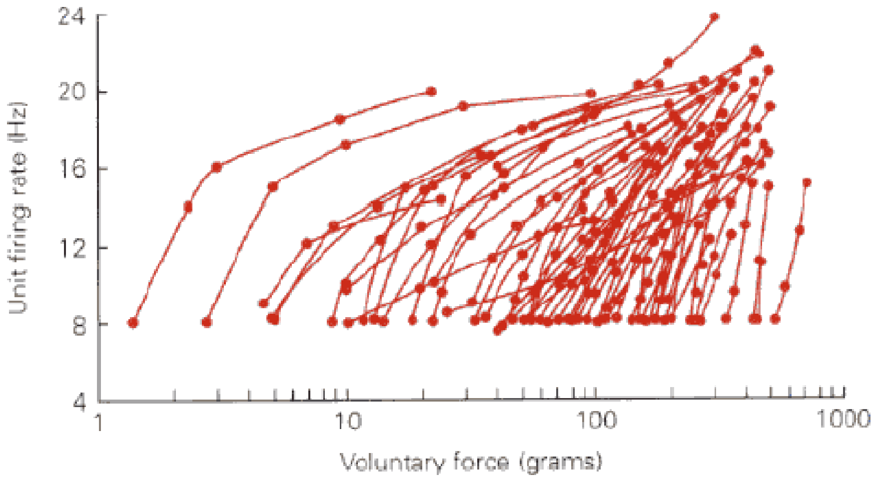
Types of motor units



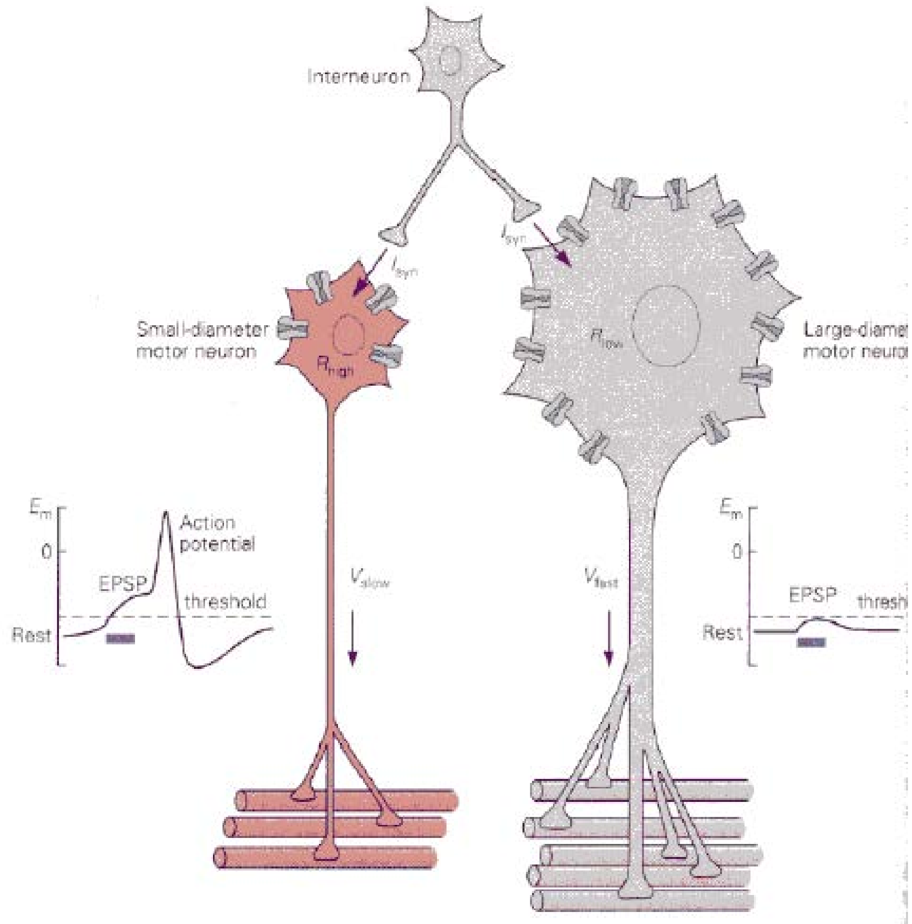


Recruitment order

Slow units are recruited first, minimizing fatigue and allowing better control of small forces



Simple mechanism for recruitment ordering



Muscle models

Complete, but hard to simulate:

Sliding filament hypothesis (Huxley) specifies how the distribution of cross-bridges evolves as a function of length, velocity, and neural input. This leads to a PDE. Keeping track of that distribution explicitly is not computationally feasible.

The right mix of detail and speed:

Distribution moment model (Zahalak) approximates the cross-bridge distribution with a Gaussian, and derives ODEs for the mean and variance of the Gaussian. Fast and surprisingly accurate, but people rarely use it (the math is unusual for this community).

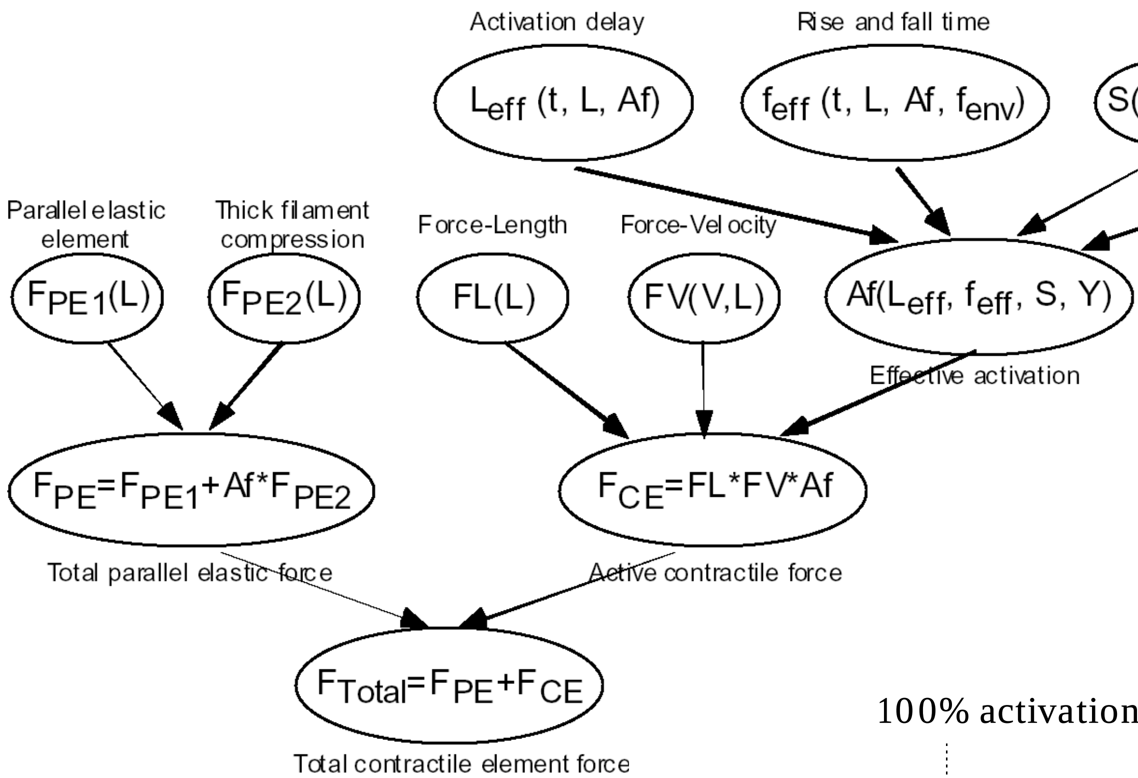
Curve-fitting:

For most application a phenomenological (Hill-type) model is sufficient.
Many such models exist, usually in the form:

t – tension
a – activation
u – neural input / EMG

$$t(a,q,\dot{q}) = a\left(f_L(q) + f_V(q,\dot{q})\right) + f_P(q)$$
$$\dot{a} = g(a,u)$$

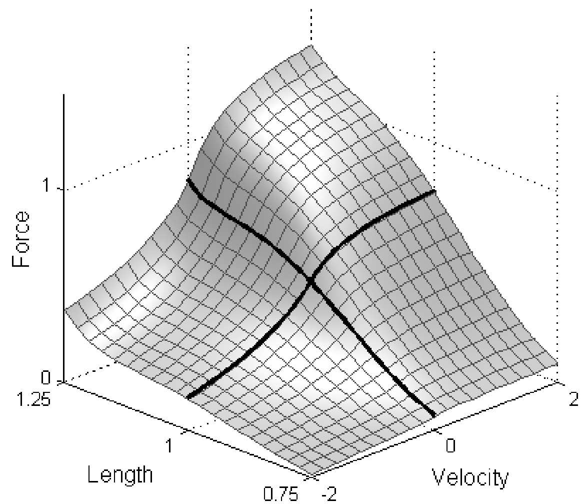
Virtual muscle: A state-of-the-art Hill-type model



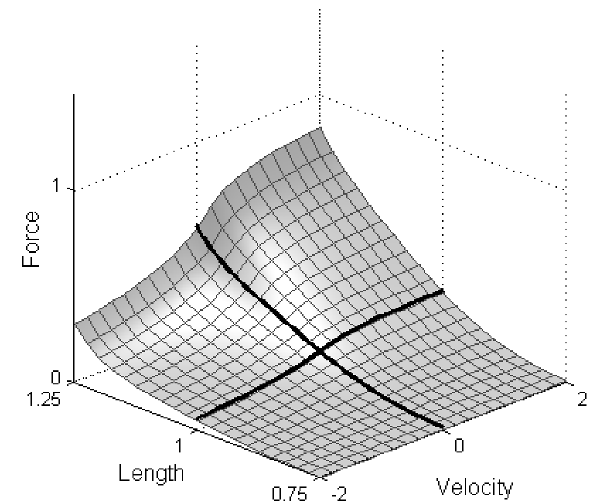
MATLAB Simulink implementation available on Jerry Loeb's website:

http://mddf.usc.edu/software.html

100% activation



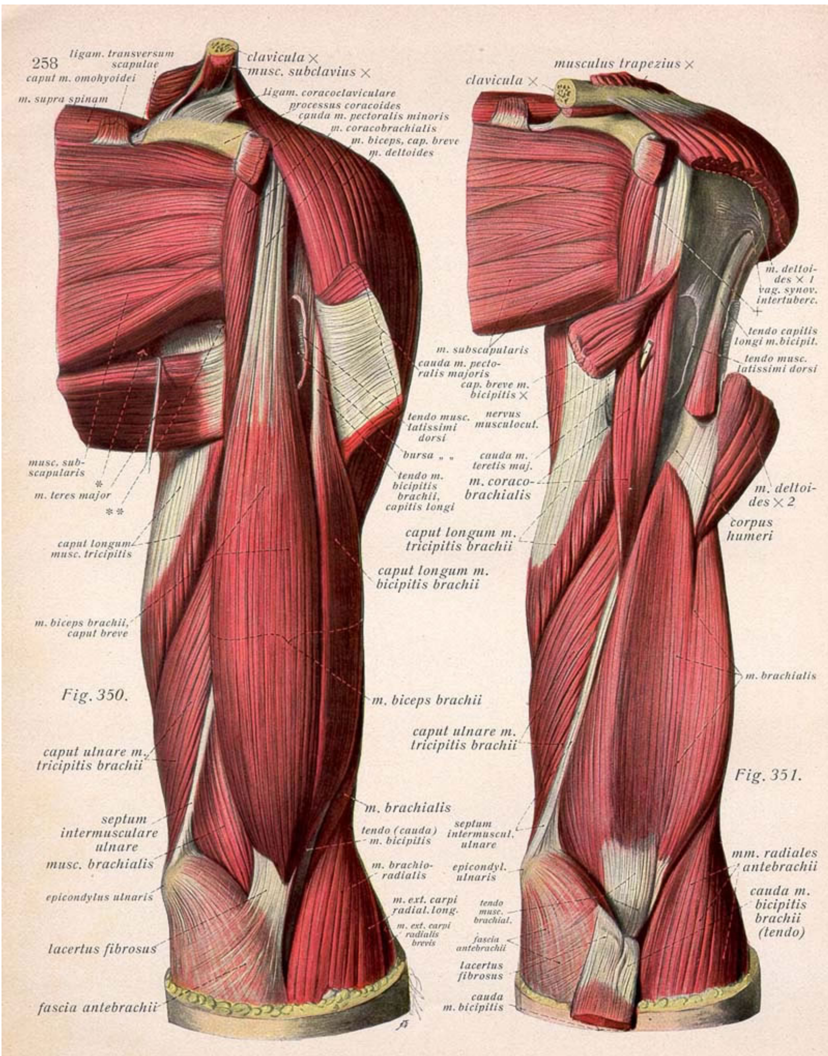
50% activation

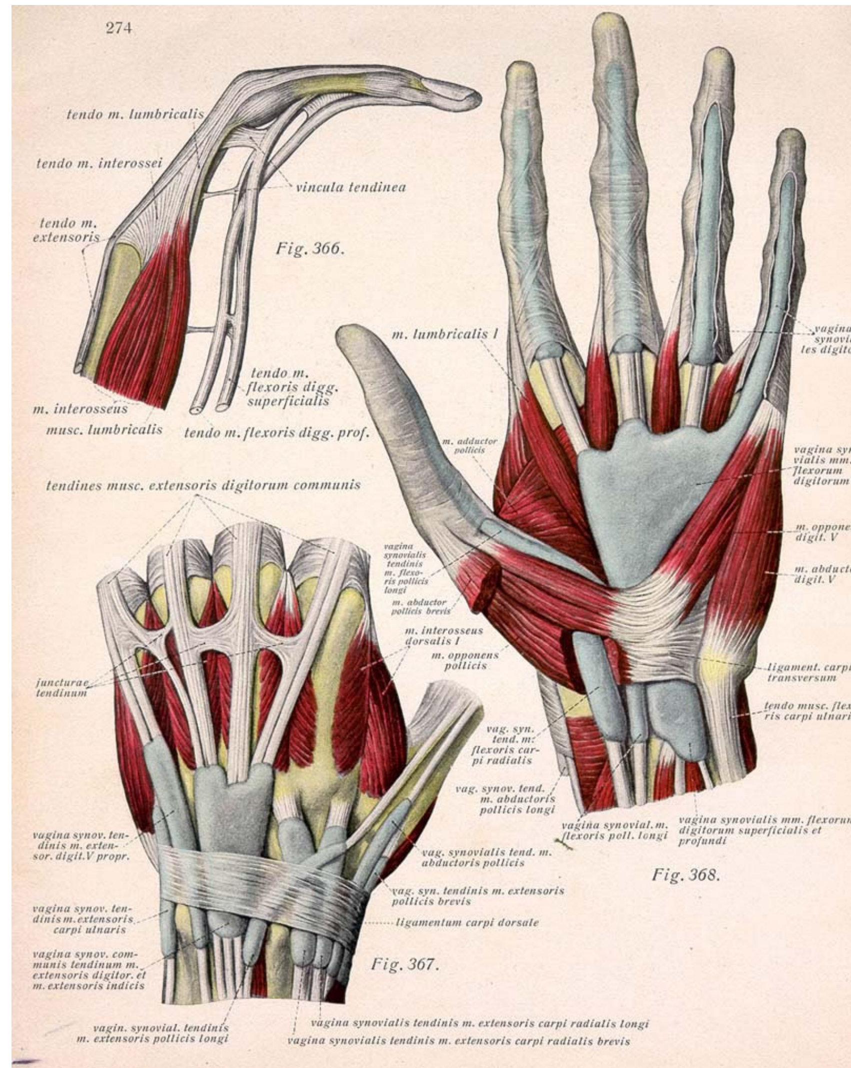


Muscle geometry

As the skeleton moves the muscle moment arms change, which changes the mapping from tension to joint torque.

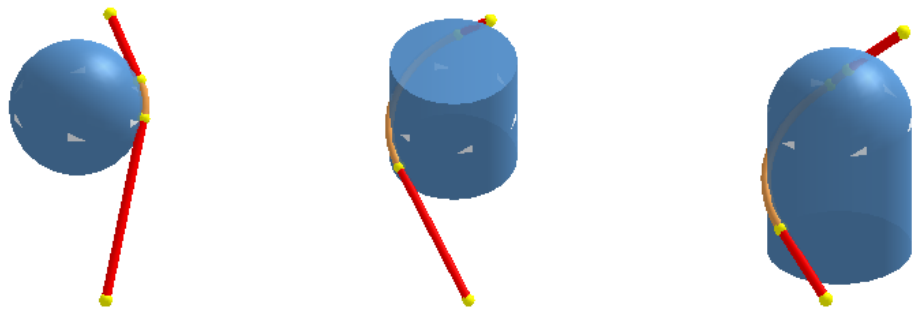
$$\text{moment arm}(q) = \frac{\partial\ \text{length}(q)}{\partial q}$$





Geometric modeling using virtual obstacles

Obstacle set:



A complete arm model: