Intelligent Tutoring Systems
A Logical Perspective

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Microsoft Research

Background: Program Synthesis

- Application domain
  - Software Developers
  - **End-Users**
  - **Education**
- Specification of intent
  - Logic
  - **Examples**
  - **Natural Language**
- Search techniques
  - SAT/SMT solvers (Formal Methods)
  - A*-style goal-directed search (AI)
  - Version space algebras (Machine Learning)

WAMBSE 2012: "Synthesis from Examples", Gulwani.

Value Proposition

- Interactive Feedback
  - Answer-scripts graded immediately.
  - Students provided feedback during solution generation
- Exploration
  - Interactive Visualization Tools
  - Simulation Tools
- Social
  - Students working on same problem at same time can come together.
  - Learning experience for past students can be used to benefit the learning experience for future students.

Intelligent Tutoring Systems

- Various Aspects
  - Solution Generation
  - Grading/Feedback
  - Problem Generation
  - Content Entry

- Various Domains
  - Geometry, Algebra, Physics, Chemistry, Logic
    - Partnership with UW and IITK for deployment in schools
  - Programming, Automata, Excel, SQL
    - Partnership with MIT, UPenn, UIUC for EdX/internal deployment
  - Language Learning
    - Interest from Office Education Team

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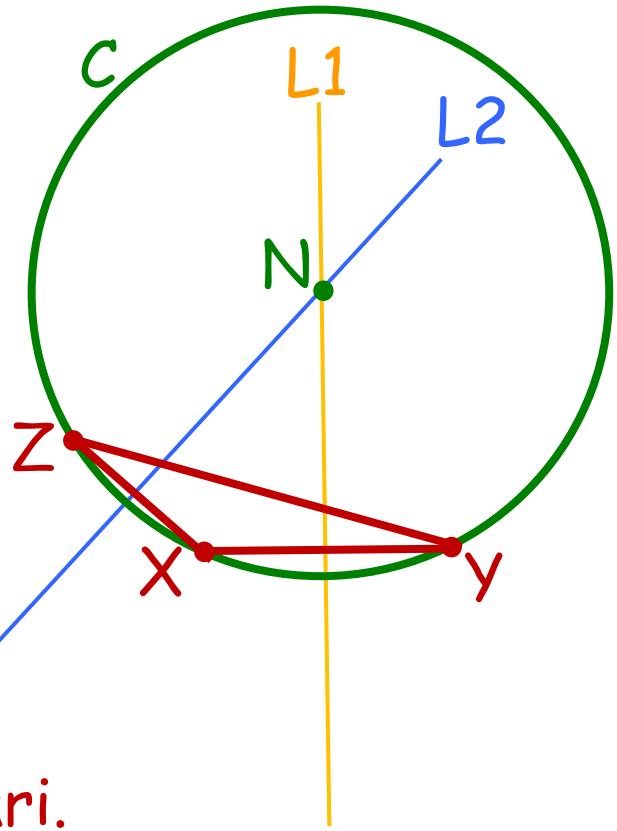
Ruler/Compass based Geometry Constructions

Given a triangle XYZ,
construct circle C such that
C passes through X, Y, and Z.



PLDI 2011: Gulwani, Korthikanti, Tiwari.

Formal specification of the problem

Given a triangle XYZ, construct circle C such that C passes through X, Y, and Z.

Precondition:

$Slope(X,Y) \neq Slope(X,Z) \ \wedge Slope(X,Y) \neq Slope(Z,X)$

Postcondition:

$LiesOn(X,C) \wedge LiesOn(Y,C) \wedge LiesOn(Z,C)$

Where $LiesOn(X,C) \equiv$

$Distance(X,Center(C)) = Radius(C)$

Solution is a program!

Geometry Program: A straight-line composition of geometry methods.

Geometry Types: Point, Line, Circle

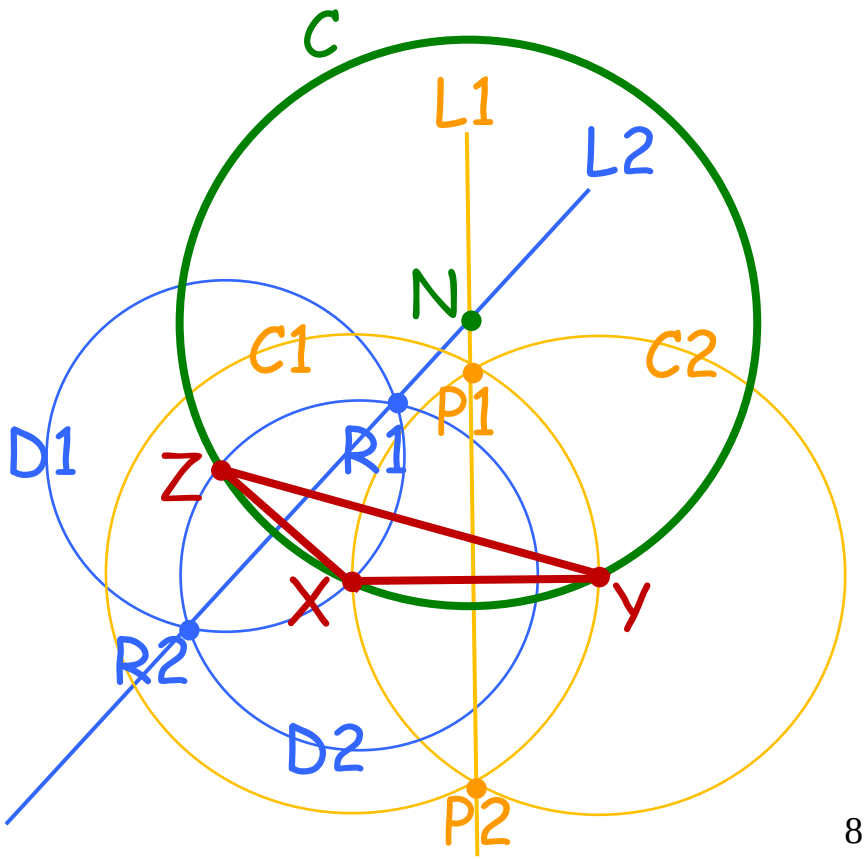
Geometry Methods:

- Ruler(Point, Point) -> Line
- Compass(Point, Point) -> Circle
- Intersect(Circle, Circle) -> Pair of Points
- Intersect(Line, Circle) -> Pair of Points
- Intersect(Line, Line) -> Point

Example of Geometry Program

Given a triangle XYZ, construct circle C such that C passes through X, Y, and Z.

1. C1 = Compass(X,Y);
2. C2 = Compass(Y,X);
3. <P1,P2> = Intersect(C1,C2);
4. L1 = Ruler(P1,P2);
5. D1 = Compass(Z,X);
6. D2 = Compass(X,Z);
7. <R1,R2> = Intersect(D1,D2);
8. L2 = Ruler(R1,R2);
9. N = Intersect(L1,L2);
10. C = Compass(N,X);



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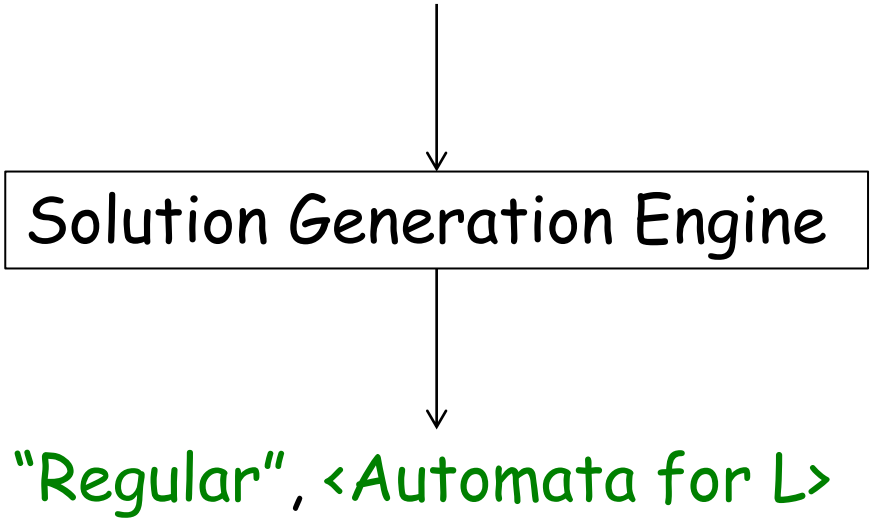
Classic Problem in Automata Theory Course

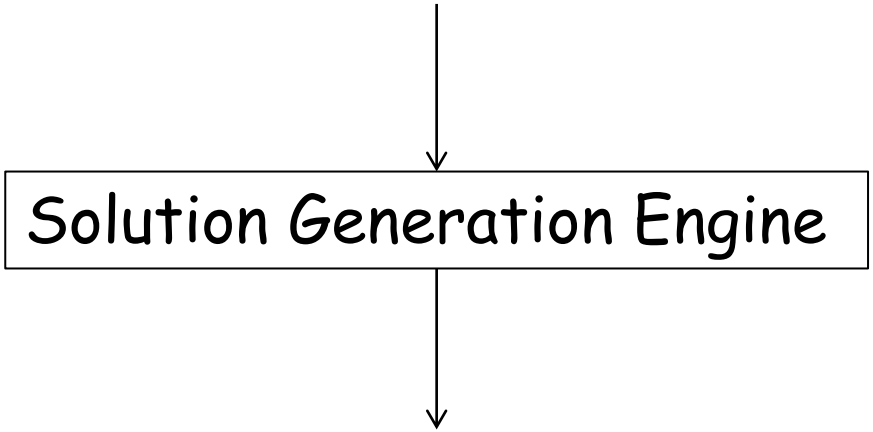
Let L be the language containing all strings over {a,b} that have the same number of occurrences of "ab" as occurrences of "ba". Construct an automata that accepts L, or prove that L is non-regular.



Joint work with Cerny, Henzinger, Zufferey, Radhakrishna

Classic Problem in Automata Theory Course

Let L be the language containing all strings over {a,b} that have the same number of occurrences of "a" as occurrences of "b". Construct an automata that accepts L, or prove that L is non-regular.



"Non-regular", <Proof of non-regularity>

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Joint work with Jain, Singh, Sharang, Sharma, Karkare, Roy

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Solution Generation by Example

Division method

Example 2 : Find the LCM of 144, 96, 160

Step 1 : Write the numbers as shown. Divide all numbers by a prime number which divides at least two of the numbers.

Step 2 : Write the quotient in each case below the number. If a number cannot be divided exactly, write the number as it is in the next row.

Step 3 : Keep dividing by prime numbers until the last row has coprime numbers with no common factors.

2	144, 96, 160
2	72, 48, 80
2	36, 24, 40
2	18, 12, 20
2	9, 6, 10
3	9, 3, 5
	3, 1, 5

Step 4 : Multiply all divisors and all numbers left in the last row. The product gives the LCM of the given numbers.

LCM = 2 × 2 × 2 × 2 × 2 × 3 × 3 × 5 = 1440.

Exercise 3.4

A. Find the LCM of the following sets of numbers.

1) 108, 144	2) 72, 90	3) 39,195
4) 98, 147	5) 105, 70	6) 85, 51
7) 76, 57	8) 252, 144	9) 256, 64
10) 75,90,125	11) 93, 62, 120	12) 75, 90
13) 48, 60, 84	14) 65, 115, 130	15) 39, 52, 65
16) 255, 340, 765, 425		

Joint work with Erik Andersen and Zoran Popovic

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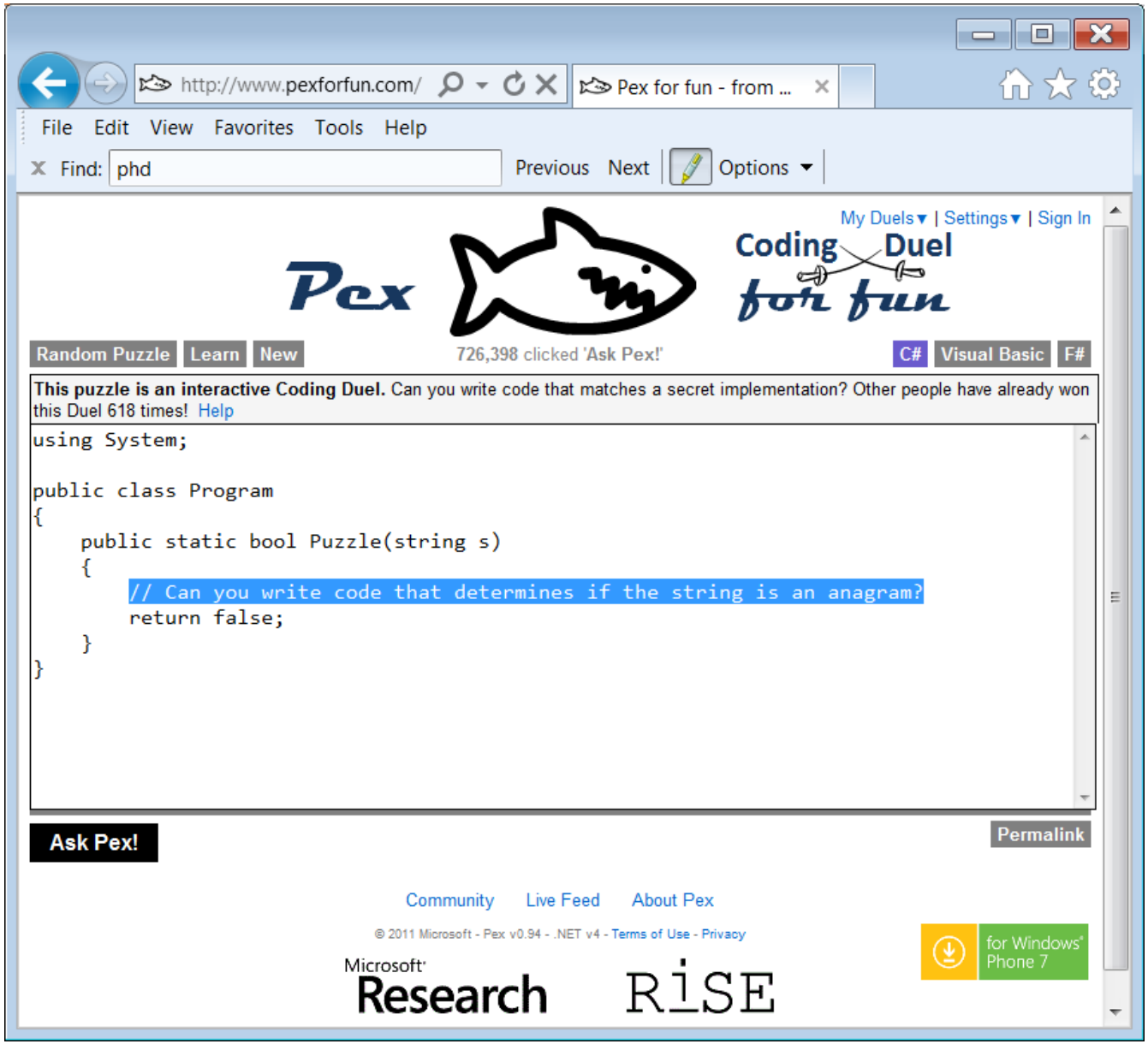
Background: PexForFun



Buggy Program for Array Reverse

6:28::50 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i < a.Length; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

Buggy Program for Array Reverse

6:32::01 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length-1; i < a.Length-1; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

Buggy Program for Array Reverse

6:32::32 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length-1; i < a.Length-1; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

No change! Sign of Frustation?

Buggy Program for Array Reverse

6:33::19 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i <= a.Length; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

Buggy Program for Array Reverse

6:33::55 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i < a.Length; i--)
        {       Console.Writeline(i);
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

Same as initial attempt except Console.Writeline!

Buggy Program for Array Reverse

6:34::06 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i < a.Length; i--)
        {       Console.Writeline(i);
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

No change! Sign of Frustation?

Buggy Program for Array Reverse

6:34::56 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i <= a.Length; i--)
        {       Console.Writeline(i);
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

The student has tried this before!

Buggy Program for Array Reverse

6:36::24 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i < a.Length; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

Same as initial attempt!

Buggy Program for Array Reverse

6:37::39 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length-1; i < a.Length-1; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

The student has tried this before!

Buggy Program for Array Reverse

6:38::11 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i > 0; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

Almost correct! (a[i-1] instead of a[i] in loop body)

Buggy Program for Array Reverse

6:38::44 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i >= 0; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

Student going in wrong direction!

Buggy Program for Array Reverse

6:39::33 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i < a.Length; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

Back to bigger error!

Buggy Program for Array Reverse

6:39::45 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i < a.Length; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

No change! Frustation!

Buggy Program for Array Reverse

6:40::27 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i < a.Length; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

No change! More Frustation!!

Buggy Program for Array Reverse

6:40::57 AM

```
using System;
public class Program {
public static int[] Puzzle(int[] a) {
        int[] b = new int[a.Length];
        int count = 0;
        for(int i=a.Length; i < a.Length; i--)
        {
                b[count] = a[i];
                count++;
        }
        return b;
} }
```

No change! Too Frustated now!!! Gives up.

Proposal: Semantic Grading

Provides additional value over counterexample feedback.

- More friendly feedback.
  - Helpful for students who give up after several tries (with only counterexample feedback).
- Grading
  - Counterexample feedback does not distinguish between a slightly incorrect solution and one that is very far off from being correct.

Example Errors

Array Index: v[a] -> v[{a+1, a-1}]

Increment: v++ -> { ++v, v--, --v }

Conditional: a op b -> a ops b

where ops = { <, >, <=, >=, ==, != }

Initialization: v=n -> v={n+1, n-1, 0}

Return Value: return v -> return ?v

Joint work with Rishabh Singh and Armando Solar-Lezama

Automated Grading (Array Reverse)

```
using System;
public class Program {
  public static int[] Puzzle(int[] a) {

    int [] b= new int[a.Length];
    for (int i = 0; i < a.Length; i++)
      {
      b[a.Length-i]=a[i-1];
    }
    return b;

  }
}
```

i = 1

i <= a.Length

```
using System;
public class Program {
  public static int[] Puzzle(int[] a) {
    int front, back, temp;
    front = 0;
    back = a.Length-1;
    temp = a[back];
    while (front > back)
    { a[back] = a[front];
      a[front] = temp;
      ++back;
      ++front;
      temp = a[back];
    }
    return a;
  }
}
```

front <= back

--back

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Trigonometry Problem

Example Problem: $(\sec x + \cos x)(\sec x - \cos x) = \tan^2 x + \sin^2 x$

Query: $(T_1(x) \pm T_2(x))(T_3(x) \pm T_4(x)) = T_5^2(x) \pm T_6^2(x)$

$T_1 \neq T_5$

New problems generated:

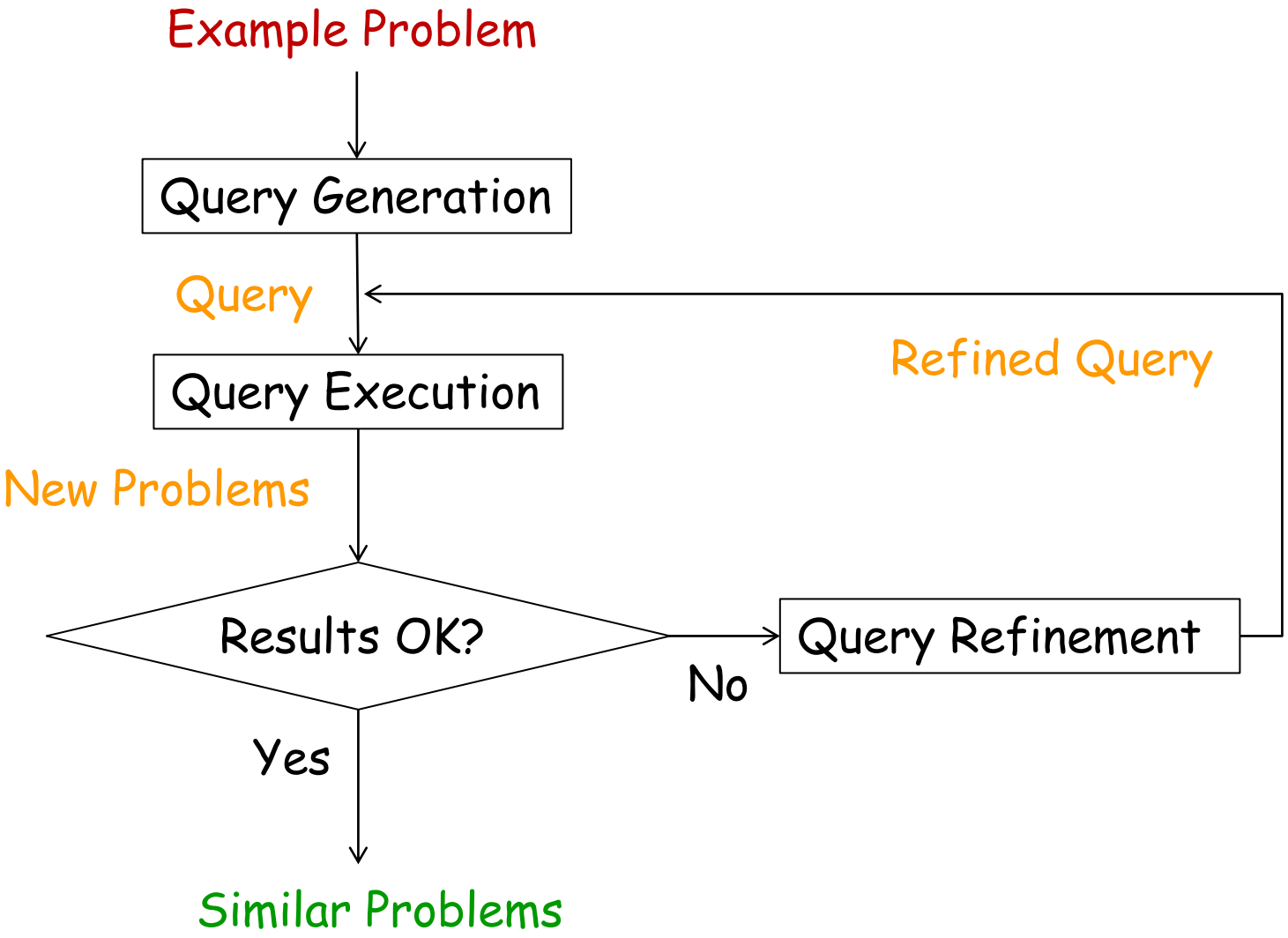
$(\csc x + \cos x)(\csc x - \cos x) = \cot^2 x + \sin^2 x$

$(\csc x - \sin x)(\csc x + \sin x) = \cot^2 x + \cos^2 x$

$(\sec x + \sin x)(\sec x - \sin x) = \tan^2 x + \cos^2 x$

:

Algebra Problem Generation



Limits/Series Problem

Example Problem: $\lim_{n\to\infty} \sum_{i=0}^{n} \frac{2i^2+i+1}{5^i} = \frac{5}{2}$

Query: $\lim_{n\to\infty} \sum_{i=0}^{n} \frac{C_0 i^2 + C_1 i + C_2}{{C_3}^i} = \frac{C_4}{C_5}$

$$C_0 \neq 0 \ \wedge \ \gcd(C_0, C_1, C_2) = \gcd(C_4, C_5) = 1$$

New problems generated:

$$\lim_{n\to\infty} \sum_{i=0}^{n} \frac{3i^2+2i+1}{7^i} = \frac{7}{3}$$

$$\lim_{n\to\infty} \sum_{i=0}^{n} \frac{3i^2+3i+1}{4^i} = 4$$

$$\lim_{n\to\infty} \sum_{i=0}^{n} \frac{i^2}{3^i} = \frac{3}{2}$$

$$\lim_{n\to\infty} \sum_{i=0}^{n} \frac{5i^2+3i+3}{6^i} = 6$$

Integration Problem

Example Problem: $\int (\csc x)\,(\csc x - \cot x)\,dx = \csc x - \cot x$

Query: $\int T_0(x)(T_1(x) \pm T_2(x))dx = T_4(x) \pm T_5(x)$

$T_1 \neq T_2 \;\wedge\; T_4 \neq T_5$

New problems generated:

$\int (\tan x)\,(\cos x + \sec x)\,dx = \sec x - \cos x$

$\int (\sec x)\,(\tan x + \sec x)\,dx = \sec x + \cot x$

$\int (\cot x)\,(\sin x + \csc x)\,dx = \sin x - \csc x$

Determinant Problem

Ex. Problem $\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (y+z)^2 & xy \\ yz & xy & (z+x)^2 \end{vmatrix} = 2xyz\,(x+y+z)^3$

Query $\begin{vmatrix} F_0(x,y,z) & F_1(x,y,z) & F_2(x,y,z) \\ F_3(x,y,z) & F_4(x,y,z) & F_5(x,y,z) \\ F_6(x,y,z) & F_7(x,y,z) & F_8(x,y,z) \end{vmatrix} = C_{10}F_9(x,y,z)$

$F_i := F_j[x \to y; y \to z; z \to x]$ $where\ (i,j) \in \{(4,0),(8,4),(5,1),\dots\}$

New problems generated:

$\begin{vmatrix} y^2 & x^2 & (y+x)^2 \\ (z+y)^2 & z^2 & y^2 \\ z^2 & (x+z)^2 & x^2 \end{vmatrix} = 2(xy+yz+zx)^3$

$\begin{vmatrix} yz+y^2 & xy & xy \\ yz & zx+z^2 & yz \\ zx & zx & xy+x^2 \end{vmatrix} = 4x^2y^2z^2$

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Sentence Completion Problems

1. The principal characterized his pupils as _____ because they were pampered and spoiled by their indulgent parents.

2. The commentator characterized the electorate as _____ because it was unpredictable and given to constantly shifting moods.

(a) cosseted
(b) disingenuous
(c) corrosive
(d) laconic
(e) mercurial

Joint work with Geoffrey Zweig and Vikram Rao Sudarshan

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Mathematical (Syntactic) Intellisense

$$\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$$

$$\begin{pmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{pmatrix}$$

$$\begin{pmatrix} A_1 \sin^3 \alpha & B_1 \sin^3 \beta & C_1 \sin^3 \gamma \\ A_2 \sin \alpha & B_2 \sin \beta & C_2 \sin \gamma \end{pmatrix}$$

Joint work with Alex Polozov and Sriram Rajamani

Mathematical (Semantic) Intellisense

Prove $(\csc x - \sin x)(\sec x - \cos x)(\tan x + \cot x) = 1$

$$\text{L.H.S.} = \left(\frac{1}{\sin x} - \sin x\right)\left(\frac{1}{\cos x} - \cos x\right)\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \left(\frac{1 - \sin^2 x}{\sin x}\right)\left(\frac{1 - \cos^2 x}{\cos x}\right)\left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)$$

$$= \left(\frac{\cos^2 x}{\sin x}\right)\left(\frac{\sin^2 x}{\cos x}\right)\left(\frac{1}{\cos x \sin x}\right)$$

$$= 1$$

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CHI 2012: Cheema, Gulwani, LaViola

Conclusion

- Technical Perspective
  - Aspects: Solution+Problem Generation, Grading, Content Entry
  - Domains: Math/Science, Programming, Language Learning

- Value Proposition:
  - Short term: Interactive Feedback, Exploration, Social
  - Long Term: Ultra-intelligent computer, Model of human mind, Inter-stellar travel ☺

- Crowdsourcing Opportunities
  - Solution Generation: Natural Language to Logic translation
  - Problem Generation: Rate difficulty and ambiguity level
  - Automated Grading: Provide natural explanations for errors